OCR
RECOGNISING ACHIEVEMENT

Monday 25 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4777 Numerical Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 8 page Answer Booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** x_0, x_1, x_2 are three terms in a first order iteration converging to α. Given that the error in x_0 is ε and the error in x_1 is $k\varepsilon$ (where ε is small), what can you say about the error in x_2?

Show that α may be estimated as $x_0 - \dfrac{(\Delta x_0)^2}{\Delta^2 x_0}$. **[6]**

The equation $x = \cos(bx)$, where x is in radians and $0 \leqslant b \leqslant 3$, has a root α which depends upon b.

(ii) Use a spreadsheet to show that the iteration $x_{r+1} = \cos(bx_r)$, with $x_0 = 1$,

(A) converges slowly when $b = 1$,

(B) diverges when $b = 2$.

Show that the formula obtained in part **(i)** may be used to give more rapid convergence when $b = 1$. What does the use of this formula achieve when $b = 2$? **[10]**

(iii) Obtain a graph of α against b for $0 \leqslant b \leqslant 3$. Find, correct to 4 decimal places, the value of b for which α is closest to 0.5. [Hint: you may find it convenient to use starting values other than $x_0 = 1$ for some values of b.] **[8]**

2 The Gaussian 3-point integration formula has the form

$$\int_{-h}^{h} \mathrm{f}(x)\,\mathrm{d}x = a\mathrm{f}(-\alpha) + b\mathrm{f}(0) + a\mathrm{f}(\alpha).$$

(i) Obtain the three equations that determine a, b and α. Verify that these equations are satisfied by

$$\alpha = \sqrt{\tfrac{3}{5}}\,h, \qquad a = \tfrac{5}{9}h, \qquad b = \tfrac{8}{9}h.$$ **[8]**

(ii) Taking $h = \frac{\pi}{4}$ initially, use the Gaussian 3-point rule to estimate the value of

$$\int_0^{\frac{\pi}{2}} (\sin x + 2\cos x)^{\frac{1}{2}}\,\mathrm{d}x.$$

Repeat the process, halving h as necessary, in order to establish the value of the integral correct to 6 decimal places. **[12]**

(iii) Determine, correct to 3 decimal places, the value of k such that

$$\int_0^{\frac{\pi}{2}} (\sin x + 2\cos x)^k\,\mathrm{d}x = 2.$$ **[4]**

3 The second order differential equation

$$\frac{d^2y}{dx^2} - 2y\frac{dy}{dx} = x^2$$

with initial conditions $x = 1$, $y = 1$, $\frac{dy}{dx} = -1$, is to be solved using finite difference methods.

(i) Show that, in the usual notation,

$$y_{r+1}(1 - hy_r) = h^2 x_r^{\,2} + 2y_r - y_{r-1} - hy_r y_{r-1}$$

and

$$y_1 = 1 - h - \tfrac{1}{2}h^2. \qquad \textbf{[8]}$$

(ii) Obtain a solution from $x = 1$ to $x = 3$ with $h = 0.1$. Use your spreadsheet to produce a graph of this solution. **[8]**

(iii) Halving h as necessary, find the values of y at $x = 2$ and at $x = 3$, each correct to 3 significant figures.

Show that this method of solution is second order. **[8]**

4 **(i)** Describe the conditions for convergence of the Gauss-Jacobi and Gauss-Seidel methods for the solution of a system of linear equations. **[3]**

(ii) A system of linear equations is represented by the following augmented matrix.

$$\left(\begin{array}{cccc|c} k & 1 & 2 & 0 & 1 \\ 1 & 3 & 0 & 2 & 0 \\ 2 & 0 & 3 & 1 & 0 \\ 0 & 2 & 1 & 3 & 0 \end{array}\right)$$

Investigate the convergence of the Gauss-Jacobi method applied to this system of equations in the cases $k = 1$, $k = 3$, $k = 5$.

Relate your results to your answer to part **(i)**. **[12]**

(iii) Modify your routine from part **(ii)** to find the inverse of the coefficient matrix in the case $k = 5$. **[9]**