OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4768:** Statistics 3

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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OCR Publications
PO Box 5050
Annesley
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NG15 0DL

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Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure Statistics strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		A paired sample is used in this context in order to eliminate any effects due to the surfaces used.	E1 **[1]**	Must refer to (differences between) surfaces.
1	(ii)		A *t* test might be used since … … the sample is small and … the population variance is not known (it must be estimated from the data). Must assume: Normality of population … … of differences.	 E1 E1 B1 B1 **[4]**	 Allow use of "σ", otherwise insist on "population". Allow "underlying" or "distribution" to imply "population".
1	(iii)		H_0: $\mu_D = 0$ H_1: $\mu_D > 0$ Where μ_D is the (population) mean reduction/difference in drying time. MUST be PAIRED COMPARISON *t* test. Differences (reductions) (before – after) are: 0.7 0.7 0.2 –0.3 0.8 –0.1 0.3 –0.1 0.1 0.5 $\bar{x} = 0.28$ $s_{n-1} = 0.3852(84)$ $(s_{n-1}^2 = 0.1484(44))$ Test statistic is $\frac{0.28 - 0}{\frac{0.3853}{\sqrt{10}}}$ = 2.298. Refer to t_9. Single-tailed 5% point is 1.833. Significant. Seems mean drying time has fallen.	B1 B1 B1 M1 A1 M1 A1 A1 A1 **[9]**	Both. Accept alternatives e.g. $\mu_D < 0$ for H_1, or $\mu_B - \mu_A$ etc provided adequately defined. Hypotheses in words only must include "population". Do NOT allow "$\bar{X} = ...$" or similar. unless $\bar{X}$ is clearly and explicitly stated to be a population mean. For adequate verbal definition. Allow absence of "population" if correct notation μ is used. Allow "after – before" if consistent with alternatives above. Do not allow $s_n = 0.3655$ ($s_n^2 = 0.1336$) Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 0 + (c's 1.833) × $\frac{0.3853}{\sqrt{10}}$ (= 0.2233) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's 1.833) × $\frac{0.3853}{\sqrt{10}}$ (= 0.0566) for comparison with 0.) c.a.o. but ft from here in any case if wrong. Require 3/4 sf; condone up to 6. Use of $0 - \bar{x}$ scores M1A0, but ft. No ft from here if wrong. $P(t > 2.298) = 0.02357$. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. "Non-assertive" conclusion in context to include "on average" oe.

Question			Answer	Marks	Guidance
1	(iv)		CI is given by $0.28 \pm$ 2.262 $\times \frac{0.3853}{\sqrt{10}}$ $= 0.28 \pm 0.2756 = (0.0044, 0.5556)$	M1 B1 M1 A1 **[4]**	Allow c's $\bar{x}$. Allow c's s_{n-1}. c.a.o. Must be expressed as an interval. Require 3/4 dp; condone 5. If the final answer is centred on a negative sample mean then do not award the final A mark. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1 B0 M1 A0. Recovery to t_9 is OK.
2	(a)	(i)	For example, need to take a sample because the population might be too large for it to be sensible to take a complete census. Because the sampling process might be destructive.	E1 E1 **[2]**	 Reward 1 mark each for any two distinct, sensible points.
2	(a)	(ii)	For example Sample should be unbiased. Sample should be representative (of the population).	 E1 E1 **[2]**	 Reward 1 mark each for any two distinct, sensible points that the sample/data should be fit for purpose. Further examples include: data should not be distorted by the act of sampling; data should be relevant.
2	(a)	(iii)	A random sample … enables proper statistical inference to be undertaken …… because we know the probability basis on which it has been selected	E2 **[2]**	Award E2, 1, 0 depending on the quality of response.
2	(b)	(i)	A Wilcoxon signed rank test might be used when nothing is known about the distribution of the background population. Must assume symmetry (about the median).	E1 E1 **[2]**	 Do not allow "sample", or "data" unless it clearly refers to the population. Do not allow if "Normality" forms part of the assumption.

Question			Answer	Marks	Guidance
2	(b)	(ii)	H_0: $m = 28.7$ H_1: $m > 28.7$ where m is the population median	B1 B1	Both. Accept hypotheses in words. Adequate definition of m to include "population".
			Speeds / −28.7 / Rank of |diff| 32.0 / 3.3 / 8 29.1 / 0.4 / 3 26.1 / −2.6 / 6 35.2 / 6.5 / 12 34.4 / 5.7 / 11 28.6 / −0.1 / 1 32.3 / 3.6 / 9 28.5 / −0.2 / 2 27.0 / −1.7 / 5 33.3 / 4.6 / 10 28.2 / −0.5 / 4 31.9 / 3.2 / 7	M1 M1 A1	for subtracting 28.7. for ranks. ft if ranks wrong. If candidate has tied ranks then penalise A0 here but ft from here.
			$W_- = 1 + 2 + 4 + 5 + 6 = 18$ Refer to Wilcoxon single sample tables for $n = 12$. Lower 5% point is 17 (or upper is 61 if 60 used). Result is not significant. No evidence to suggest that the median speed has increased.	B1 M1 A1 A1 A1 **[10]**	($W_+ = 3 + 7 + 8 + 9 + 10 + 11 + 12 = 60$) No ft from here if wrong. ie a 1-tail test. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. "Non-assertive" conclusion in context to include "on average" oe.
3			$S \sim N(11.07,\ 2.36^2)$ $C \sim N(57.33,\ 8.76^2)$ $R \sim N(24.23,\ 3.75^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only.
	(i)		$P(10 < S < 13)$ $= P\left(\frac{10-11.07}{2.36} < Z < \frac{13-11.07}{2.36}\right)$ $= P(-0.4534 < Z < 0.8178)$ $= 0.7931 - (1 - 0.6748)$ $= 0.4679$	M1 A1 A1 **[3]**	For standardising. Award once, here or elsewhere. Cao Accept 0.468(0), 0.4681, 0.4682, but not 0.4683.

Question			Answer	Marks	Guidance
3	(ii)		Want P($R > S + 10$) i.e. P($R - S > 10$) $R - S \sim N(24.23 - 11.07 = 13.16,$ $3.75^2 + 2.36^2 = 19.6321)$ $P(\text{this} > 10) = P(Z > \frac{10 - 13.16}{\sqrt{19.6321}} = -0.7132)$ $= 0.7621$	M1 B1 B1 A1 **[4]**	Allow $S - R$ provided subsequent work is consistent. Mean. Variance. Accept sd = $\sqrt{19.6321} = 4.4308...$ cao
3	(iii)		Want P($S + R > \frac{2}{3}C$) i.e. P($S + R - \frac{2}{3}C > 0$) $S + R - \frac{2}{3}C \sim N(11.07 + 24.23 - \frac{2}{3} \times 57.33 = -2.92,$ $2.36^2 + 3.75^2 + (\frac{2}{3} \times 8.76)^2 = 53.7377)$ $P(\text{this} > 0) = P(Z > \frac{0 - (-2.92)}{\sqrt{53.7377}} = 0.3983)$ $= 1 - 0.6548 = 0.3452$	M1 B1 B1 A1 **[4]**	Allow $\frac{2}{3}L - (S + R)$ provided subsequent work is consistent. Mean Variance. Accept sd = $\sqrt{53.7377} = 7.3306...$ cao
3	(iv)		$\bar{x} = 98.484$, $s_{n-1} = 10.1594$ CI is given by $98.484 \pm$ $2.201 \times \frac{10.1594}{\sqrt{12}}$ $= 98.484 \pm 6.455 = (92.03, 104.94)$	B1 M1 B1 M1 A1 **[5]**	Do not allow $s_n = 9.7269$. ft c's $\bar{x} \pm$. From t_{11}. ft c's s_{n-1}. cao Must be expressed as an interval. Require 1 or 2 dp; condone 3dp.
3	(v)		Normality is unlikely to be reasonable – times could well be (positively) skewed. Independence is unlikely to be reasonable – e.g. a competitor who is fast in one stage may well be fast in all three.	E1 E1 **[2]**	Discussion required. Accept any reasonable point. Accept "reasonable" provided an adequate explanation is given. Discussion required. Accept any reasonable point. This is independence between stages for a particular competitor, not between competitors.

Question			Answer	Marks	Guidance
4	(i)		H_0: The model for the number of callouts fits the data H_1: The model for the number of callouts does not fit the data.	B1 B1	Do not allow "Data fit the model" o.e for either hypothesis.
			<table><tr><td>Obs'd frequency</td><td>145</td><td>79</td><td>22</td><td>6</td><td>3</td><td>0</td></tr><tr><td>Exp'd frequency</td><td>139.947</td><td>83.968</td><td>25.190</td><td>5.038</td><td>0.756</td><td>0.101</td></tr></table>		
			Merge last 3 cells. Obs 9 Exp 5.895	M1	
			X^2 = 0.1824 + 0.2939 + 0.4040 + 1.6355 = 2.515(8)	M1 A1	Calculation of X^2. Cao Require 3/4 sf; condone up to 6.
			Refer to χ^2_2.	M1	Allow correct df (= cells – 2) from wrongly grouped table and ft. Otherwise, no ft if wrong. P($X^2 > 2.5158$) = 0.2842.
			Upper 5% point is 5.991. Not significant. Suggests it is reasonable to suppose that the model fits the data.	A1 A1 A1 **[9]**	No ft from here if wrong. ft only c's test statistic. ft only c's test statistic. "Non-assertive" conclusion in words (+context). Do not allow "Data fit model" o.e.
4	(ii)		Mean = 5/3 $\therefore \lambda = 0.6$	B1 **[1]**	
4	(iii)		$F(t) = \int_0^t 0.6e^{-0.6x}dx$ $= \left[-e^{-0.6x}\right]_0^t$ $= \left(-e^{-0.6t} - (-e^0)\right) = 1 - e^{-0.6t}$	M1 A1 A1 **[3]**	Correct integral with limits (which may be implied subsequently). Allow use of "+ c" accompanied by a valid attempt to evaluate it. Correctly integrated. Limits used or c evaluated correctly. Accept unsimplified form. If final answer is given in terms of λ then allow max M1A1A0.
4	(iv)		$P(T > 1) = 1 - F(1)$ $= 1 - \left(1 - e^{-0.6}\right) = 0.5488$	M1 A1 **[2]**	ft c's F(t). cao Allow any exact form of the correct answer.
4	(v)		$F(m) = \frac{1}{2}$ $\therefore 1 - e^{-0.6m} = \frac{1}{2}$ $\therefore e^{-0.6m} = \frac{1}{2}$ $\therefore -0.6m = -\ln 2$ $\therefore m = \frac{\ln 2}{0.6}$ m = 1.155 (days)	M1 M1 A1 **[3]**	Use of definition of median. Allow use of c's F(t). Convincing attempt to rearrange to "m = …", to include use of logs. Cao obtained only from the correct F(t). Must be evaluated. Require 2 to 4 sf; condone 5.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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