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OCR
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GCSE (9–1)
Transition Guide

MATHEMATICS

J560
For first teaching in 2015

KS3–KS4 focus
Theme: Pythagoras

Version 2

www.ocr.org.uk/maths

GCSE (9–1) MATHEMATICS

Key Stage 3 to 4 Transition Guides focus on how a particular topic is covered at the different key stages and provide information on:

- Differences in the demand and approach at the different levels;
- Useful ways to think about the content at Key Stage 3 which will help prepare students for progression to Key Stage 4;
- Common student misconceptions in this topic.

Transition guides also contain links to a range of teaching activities that can be used to deliver the content at Key Stages 3 and 4 and are designed to be of use to teachers of both key stages. Central to the transition guide is a Checkpoint task which is specifically designed to help teachers determine whether students have developed deep conceptual understanding of the topic at Key Stage 3 and assess their 'readiness for progression' to Key Stage 4 content on this topic. This checkpoint task can be used as a summative assessment at the end of Key Stage 3 teaching of the topic or by Key Stage 4 teachers to establish their students' conceptual starting point.

Key Stage 3 to 4 Transition Guides are written by experts with experience of teaching at both key stages.

Key Stage 3 Content

Key Stage 3 National Curriculum Content*

- Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' theorem, and use known results to obtain simple proofs
- Use Pythagoras' theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles

This document refers to the following elements of these statements:

- Apply triangle congruence, similarity and properties of quadrilaterals to derive Pythagoras' theorem, and use known results to obtain simple proofs
- Use Pythagoras' theorem to solve problems involving right-angled triangles

* https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/239058/SECONDARY_national_curriculum_-_Mathematics.pdf

Key Stage 4 Content

GCSE (9–1) content Ref.	Subject content	Initial learning for this qualification will enable learners to…	Foundation tier learners should also be able to…	Higher tier learners should additionally be able to…	DfE Ref.
10.05	**Triangle mensuration**				
10.05a	Pythagoras' theorem		Know, derive and apply Pythagoras' theorem $a^2 + b^2 = c^2$ to find lengths in right-angled triangles in 2D figures.	Apply Pythagoras' theorem in more complex figures, including 3D figures.	G6, G20

Comment

Difference between the level of demand and KS3 and KS4

The essential difference in approach between Key Stages 3 and 4 is a move from concrete application in given triangles towards a more abstract application in general right-angled triangles; full achievement at Key Stage 3 involves Pythagoras' theorem being investigated and understood numerically and proved by demonstration; the Key Stage 4 content requires students to be guided towards a wider application with more difficult numbers and contexts, and an appropriate formal understanding of the theorem. There is potentially a substantial leap in demand for weaker students who may at Key Stage 3 have only worked with straight-forward problems that involve finding the hypotenuse and may not have used the theorem to find a short side.

Pre-required knowledge before starting KS3/Delivering the KS3 content (with a focus on setting things up well conceptually)

Before starting this topic at Key Stage 4 students should have already worked on right-angled triangles in terms of finding area, measuring angles, or producing scale drawings. A conceptual grounding for introducing Pythagoras' theorem could include reminding students about rounding and approximation, the terms used to describe triangles and the use of the right-angle symbol in diagrams. Work may also need to be done on squaring and finding roots, leading to solving simple quadratic equations. [Lipowsky *et al*., 2009]

Leading up to Key Stage 4 weaker students might be expected to understand that Pythagoras' theorem describes the relationship between the areas of the squares on the sides of a right-angled triangle, and this can be reinforced by a practical demonstration.

The concept can be reinforced by drawing and measuring some examples, looking at the relationship between the squares of the sides and conjecturing that it might apply in general. More highly attaining students may well have already become very familiar with Pythagoras' theorem in purely numeric applications, including substituting into and rearranging the formula to find the length of a short side.

Delivering the KS4 content

Through Key Stage 4 your students will need to solidify their understanding of Pythagoras' theorem as a rule that can be applied to the lengths of particular triangles and begin to understand that it is a general property of all right-angled triangles which can be applied in algebraic and abstract geometric contexts. This understanding can be best developed through classroom discussion which includes asking students to describe their reasoning and avoiding specifying the procedures to be followed. Some students will need considerable reinforcement of Key Stage 3 material in order to have a solid foundation of concrete knowledge on which to build subsequent abstraction. [Liposky *et al*., 2009] [Presmeg, 2006]

As students progress, they should be able to recall the formula and begin to engage with it, moving away from a "square and add" rule to substitution and rearrangement, learning to set out their working clearly. They will start to apply the theorem to triangles in real-world and geometric contexts, including those where the lengths are given as fractions or decimals, and where suitable approximations have to be made to irrational values. The effects of approximation can be considered in terms of error intervals and compound error. They will also be expected to have a conceptual understanding of the theorem and to have seen and understood a simple proof.

There is potential here to add in some context and real world applications, though questions about diagonals of shapes may be more purposeful than inauthentic contexts which may not have a clear purpose. However, linking the concepts to real world examples will help to avoid the misconception about the algebraic use of the square root that leads students to move from $a^2 = b^2 + c^2$ to $a = \sqrt{b^2} + \sqrt{c^2}$ to $a = b + c$. Visualisation of real objects provides simple counterexamples to this statement that lead to better understanding of the algebraic process. [Bagni, G., 2000]

More highly achieving students will be expected to be completely fluent with the formula, including applying it in cases where lengths are given as algebraic expressions or numbers in surd form. They will know and understand at least one simple proof of the theorem, be able to critique a given proof and know that the converse can be used to demonstrate that a given triangle is right-angled. They will begin to apply the theorem in a variety of geometric contexts, including in multi-step and/or three dimensional contexts such as the length of the diagonal of a cuboid and the slant height of a cone.

Comment

References

Bagni, G. (2000). "Simple" rules and general rules in some High School students' mistakes, Journal für Mathematik Didaktik, 21 (2), 124-138.

Lipowsky, F., Rakoczy, K., Pauli, C., Drollinger-Vetter, B., Klieme, E., and Reusser, K. (2009). Quality of geometry instruction and its short-term impact on students' understanding of the Pythagorean Theorem, Learning and Instruction, Volume 19, Issue 6, December 2009, Pages 527-537.

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics: Emergence from psychology. In A.Gutierrez & P. Boero (Eds.), Handbook of Research on the Psychology of Mathematics Education (pp. 205–235). Dordrecht, NL: Sense Publishers.

Activities

An interactive introduction to Pythagoras' theorem, with plenty of examples of the different ways it can be used and an animation of a visual proof.

Resources: http://www.mathsisfun.com/pythagoras.html

Some examples of how Pythagoras' theorem can be used in everyday life.

Resources: http://www.brighthubeducation.com/homework-math-help/36639-applications-of-pythagoras-theorem-in-real-life/

Sections 1 and 2 provide some interactive and practical demonstrations suitable for KS3.

Resources: http://www.teachmathematics.net/activities/proof-pythagoras-theorem.htm

BBC Two Learning Zone presents the Key Stage 3 - Rugby Pythagoras Challenge.

Resources: http://sport.maths.org/content/3-2-1-go-rugby-pythagoras-challenge-ks3

Checkpoint Task

Redesigning the Square: The checkpoint task is set in the context of a company redesigning a tiled courtyard.

Task 1 refreshes some of the essential knowledge about right-angled triangles.

Task 2 explores the processes of squaring and square rooting in a concrete example.

Task 3 reviews some of the simple algebra that is needed for a full treatment of Pythagoras' theorem and a common algebraic/geometric proof. It also includes consideration of approximation when applying the theorem in context.

Task 4 uses Pythagoras' theorem to find missing lengths in a right-angled triangle. It also includes an algebraic treatment with which some students will be familiar.

Resources: Redesigning the Square worksheet.

Checkpoint Task:

KS3_KS4_transition guide - Pythagoras checkpoint task

Activities

Proving Pythagoras' theorem: section 3 follows the practical demonstration with an initial look at proof.

Resources: http://www.teachmathematics.net/page/15514/proof-pythagoras-theorem

Generating Triples: this investigation looks at right-angled triangles with integer length sides. Students have to apply their knowledge of Pythagoras' theorem and number sequences, then make some conjectures and prove them algebraically.

Resources: http://nrich.maths.org/7282

Pythagoras Proofs: this activity presents three sketch proofs of Pythagoras' theorem and asks students to make sense of the idea behind each proof, to decide which is the most convincing and to suggest ways of explaining the idea to others. Some solutions are given with comments and points for discussion.

Resources: http://nrich.maths.org/6553

An introduction to Pythagoras himself and the basics of his theorem.

Resources: http://nrich.maths.org/2721

An interesting physical demonstration using a water wheel.

Resources: http://www.youtube.com/watch?v=CAkMUdeB06o

Vi Hart's version of the origami proof of Pythagoras' theorem. This would make an excellent basis for a kinematic approach to understanding the concepts.

Resources: http://www.youtube.com/watch?v=z6lL83wl31E

Activities

Thinking Inside the Box: an investigation about fitting pencils into a box. It is a good introduction to applying Pythagoras' theorem in three dimensions and includes extension material which considers the pencils as hexagonal prisms.

Resources: http://www.ocr.org.uk/Images/150944-total-maths-issue-4-winter-2013.pdf

Wheel of Theodorus: the wheel provides a natural context for Pythagoras' theorem and exploring irrational numbers. This link gives a good introduction and provides an art project that might be suitable across the ability range.

Resources:
http://www.ldlewis.com/Teaching-Mathematics-with-Art/documents/MTMS2007-04-442a.pdf

Pythagorean Triples: some further articles on Pythagorean triples that might be good reading and suggest some extension tasks.

Resources: http://nrich.maths.org/1332/1332 and http://nrich.maths.org/1309

Resources, Links and Support

Total Maths – Our termly update Total Maths is designed to keep you up-to-date with Mathematics here at OCR, as well as to share information, news and resources from the Maths community that you might find useful: http://www.ocr.org.uk/qualifications/by-subject/mathematics/total-maths/

Find resources and qualification information through our Maths page: www.ocr.org.uk/maths

Contact the team: maths@ocr.org.uk

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