OCR RECOGNISING ACHIEVEMENT

Friday 17 May 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

1 A particle is attached to a spring and suspended vertically from a point P which is made to oscillate vertically. The vertical displacement, x, of the particle from a fixed point at time t is modelled by the differential equation

$$2\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = \cos t.$$

(i) Find the general solution of the differential equation. **[8]**

Initially the displacement and velocity of the particle are both zero.

(ii) Find the particular solution and sketch its graph for large positive values of t. **[6]**

(iii) Find approximate values of the displacement and velocity at $t = 10\pi$. **[3]**

The point P stops oscillating at $t = 10\pi$ and the subsequent motion of the particle is modelled by

$$2\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = 0.$$

(iv) Determine the type of damping present. **[2]**

(v) Using the values obtained in part **(iii)**, find the particular solution for this motion. **[5]**

2 **In this question take $g = 10$.**

A rocket of mass 500 kg is launched from rest from the sea bed at a depth of 124 m. It travels vertically upwards. After t s it has risen x m and its velocity is $v\,\mathrm{m\,s^{-1}}$.

In a simple model, for all stages of its motion, the mass of the rocket is constant and the only forces acting on it are its weight, a driving force of 10 000 N and a resistance force.

When in the sea, the magnitude of the resistance force is modelled by kv N, where k is a constant.

(i) Write down and solve a differential equation to show that $v = \frac{5000}{k}\left(1 - e^{-\frac{kt}{500}}\right)$. **[8]**

(ii) Find x in terms of t and k. **[3]**

The time for the rocket to reach the surface of the sea is 5 s.

(iii) Verify that $k \approx 2.5$ is consistent with this information and hence estimate the speed of the rocket when it reaches the surface. **[3]**

After the rocket reaches the surface it travels vertically upwards through the air and the magnitude of the resistance force is now modelled by $0.4v^2$ N.

(iv) Show that $v\frac{dv}{dx} = 10 - 0.0008v^2$. **[2]**

(v) Solve this differential equation to find the particular solution for v in terms of x. Sketch a graph of this solution, showing the asymptote. **[8]**

3 **(a)** The differential equation $\frac{dy}{dx} + 2y = \sin 2x$ is to be solved.

(i) Find the complementary function and a particular integral. Hence write down the general solution. **[7]**

(ii) Find the particular solution subject to the condition $y = 2$ when $x = 0$. Sketch the solution curve for $x \geqslant 0$. **[4]**

(b) The differential equation $\frac{dy}{dx} + 2y = e^{-x}$ is to be solved.

(i) Use the integrating factor method to find the general solution for y in terms of x. **[5]**

(ii) Find the particular solution subject to the condition $y = 2$ when $x = 0$. **[2]**

(c) The differential equation $\frac{dy}{dx} + 2y = \tan x$ is to be solved subject to the condition $y = 2$ when $x = 0$.

Use an integrating factor and the approximation $\int_0^1 e^{2x} \tan x \, dx \approx 2.71862$ to calculate an approximate value of y when $x = 1$. **[6]**

4 The simultaneous differential equations

$$\frac{dx}{dt} = x - 2y - z$$
$$\frac{dy}{dt} = x + 3y + z$$
$$\frac{dz}{dt} = -z$$

are to be solved. When $t = 0$, $x = 1$, $y = 0$ and $z = 2$.

(i) Use the third equation to find the particular solution for z in terms of t. **[2]**

(ii) Using part **(i)** eliminate y and z to obtain a second order differential equation for x. Hence find the general solution for x in terms of t. **[12]**

(iii) Find the corresponding general solution for y. **[3]**

(iv) Find the particular solutions for x and y. **[4]**

(v) Show that $x = y$ when $3 \sin t = e^{-3t}$. Deduce that $x = y$ occurs infinitely often. **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.