OCR
RECOGNISING ACHIEVEMENT

Monday 24 June 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 This question concerns curves with parametric equations

$$x = a\cos t + \cos at\,,\quad y = a\sin t - \sin at$$

where a is a positive integer and $0 \leqslant t < 2\pi$.

(i) Sketch the curves for the cases $a = 2, a = 3$ and $a = 4$ and describe two of the common features of these three curves. **[7]**

(ii) For the case $a = 2$, find the values of t for the points where the curve intersects the axes and hence find the coordinates of the points of intersection with the axes. **[5]**

(iii) For the case $a = 2$, confirm the feature of the curve at the point where $t = \frac{2\pi}{3}$ by investigating the gradient as $t \to \frac{2\pi}{3}$. **[5]**

(iv) Sketch the curve

$$x = k\cos^3 t,\quad y = k\sin^3 t$$

where k is positive and $0 \leqslant t < 2\pi$.

You are given that, for the case $a = 3$, the curve

$$x = a\cos t + \cos at,\quad y = a\sin t - \sin at$$

can be written in the form

$$x = k\cos^3 t,\quad y = k\sin^3 t$$

for a particular positive value of k and $0 \leqslant t < 2\pi$. Find this value of k and obtain a cartesian equation for the curve in this case. **[6]**

2 This question concerns the function $f(z) = \sin z$ for $z \in \mathbb{C}$, with derivative $f'(z) = \cos z$.

(i) Find, with real and imaginary parts given to 3 decimal places, the values of z_1, z_2 and z_3, where $z_1 = f(3 + 2i)$, $z_2 = f(3.1 + 2i)$ and $z_3 = f'(3 + 2i)$.

Plot these points on an Argand diagram.

Express $\dfrac{z_2 - z_1}{0.1}$ and z_3 in the form $re^{i\theta}$ and explain why they are approximately equal. **[8]**

(ii) Construct a spreadsheet to demonstrate that

$$\lim_{h \to 0}\left(\frac{f(z+h) - f(z)}{h}\right) = \cos z \text{ for } z = 3 + 2i \text{ and } h \in \mathbb{R}.$$

State which values of h you have used and the expression(s) you have evaluated. Quoting sufficient values from your spreadsheet, explain how the result is demonstrated.

Find, correct to 1 significant figure, the largest value of h, $h \in \mathbb{R}$, such that

$$\left|\frac{f((3 + 2i) + h) - f(3 + 2i)}{h} - \cos(3 + 2i)\right| < 0.01.$$

[6]

(iii) Find, correct to 1 significant figure, the largest value of h, $h \in \mathbb{R}$, such that

$$\left|\frac{f((3 + 2i) + hi) - f(3 + 2i)}{hi} - \cos(3 + 2i)\right| < 0.01.$$

[2]

(iv) Use your software to find the roots of the equation $\cos z = 0$, where $-2\pi < \mathrm{Re}(z) < 2\pi$. Plot these roots on an Argand diagram.

Use the real and imaginary parts of $\cos z$ to show algebraically that all the roots of the equation $\cos z = 0$ are real. **[8]**

3 This question concerns arithmetic modulo 17. The unknowns x and y are integers such that $0 \leqslant x < 17$, $0 \leqslant y < 17$.

(i) Create a program to find all the solutions, x and y, to the congruence

$$ax + by \equiv c \pmod{17}$$

where a, b and c are positive integers. You should write out your program in full.

Find the number of solutions to the congruence

$$x + 5y \equiv 13 \pmod{17}.$$

State the solution for which the sum $x + y$ is largest. **[8]**

(ii) Edit your program to find the solutions, x and y, to the simultaneous congruences

$$ax + by \equiv c \pmod{17}$$
$$dx + ey \equiv f \pmod{17}$$

where a, b, c, d, e and f are positive integers. Indicate clearly all the changes to your program.

Use the edited program to solve the simultaneous congruences

$$3x + 5y \equiv 7 \pmod{17}$$
$$2x + 7y \equiv 1 \pmod{17}$$

and state the solution.

Check the solution by calculating the values of $3x + 5y$ and $2x + 7y$. **[6]**

(iii) Explain how you would investigate the number of solutions, x and y, to the simultaneous congruences

$$kx + 5y \equiv 7 \pmod{17}$$
$$2x + 7y \equiv 1 \pmod{17}$$

for different integer values of k where $0 \leqslant k < 17$.

State the value of k for which the simultaneous congruences do not have a solution. Explain why the congruences do not have a solution for this value of k. **[6]**

(iv) Find the number of solutions, x and y, to the simultaneous congruences

$$7x + y \equiv 6 \pmod{17}$$
$$x + 5y \equiv 13 \pmod{17}.$$

Explain your result. **[5]**