RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755:** Further Concepts for Advanced Mathematics

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. **Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

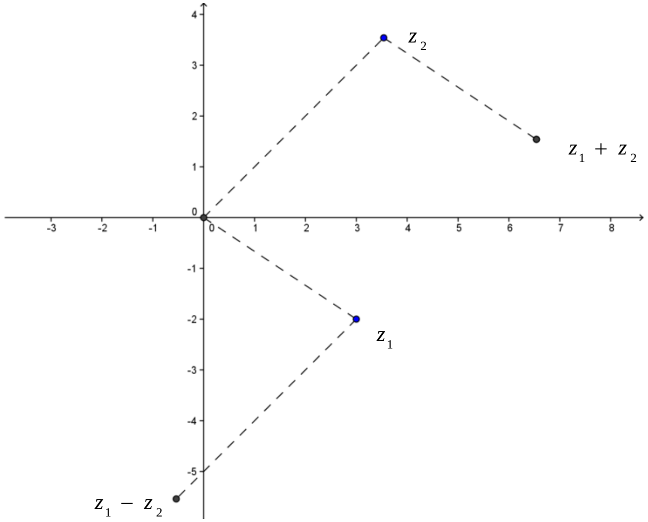
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

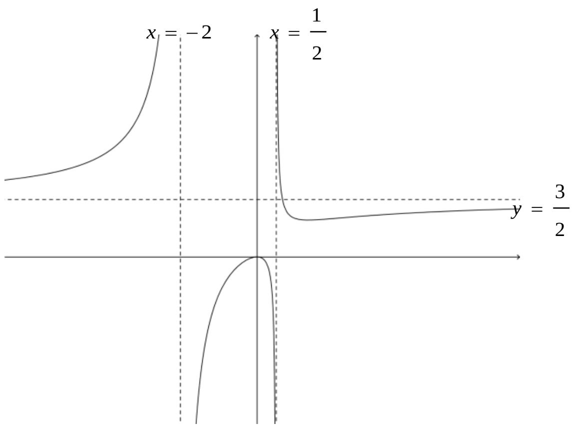
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

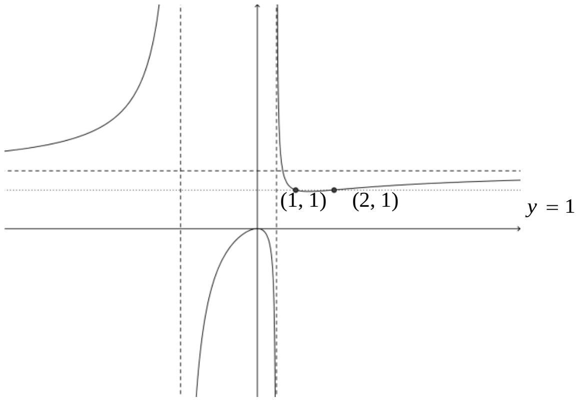
Question			Answer	Marks	Guidance	
1			$2x\left(x^2-5\right) \equiv \left(x-2\right)\left(Ax^2+Bx+C\right)+D$	M1	Evidence of comparing coefficients, or multiplying out the RHS, or substituting. May be implied by A = 2 or D = - 4	
			Comparing coefficients of x^3, $A=2$	B1		
			Comparing coefficients of x^2, $B-2A=0 \Rightarrow B=4$	B1		
			Comparing coefficients of x, $C-2B=-10 \Rightarrow C=-2$	B1		
			Comparing constants, $D-2C=0 \Rightarrow D=-4$	B1	Unidentified, max 4 marks.	
				[5]		
2			$z=\frac{3}{2}$ is a root $\Rightarrow$ $(2z-3)$ is a factor.	M1	Use of factor theorem, accept $2z+3, z\pm\frac{3}{2}$	
			$\Rightarrow (2z-3)\left(z^2+bz+c\right)=\left(2z^3+9z^2+2z-30\right)$	M1	Attempt to factorise cubic to linear x quadratic	
			Other roots when $z^2+6z+10=0$	M1	Compare coefficients to find quadratic (or other valid complete method leading to a quadratic)	
				A1	Correct quadratic	
			$z=\frac{-6\pm\sqrt{36-40}}{2}$	M1	Use of quadratic formula (or other valid method) in their quadratic	
			$=-3+\text{j}$ or $-3-\text{j}$	A1	oe for both complex roots FT their 3-term quadratic provided roots are complex.	
			OR $\frac{3}{2}+\beta+\gamma=-\frac{9}{2}, \frac{3}{2}\beta\gamma=15$, **or** $\frac{3}{2}\beta+\beta\gamma+\frac{3}{2}\gamma=1$	M1	Two root relations (may use α)	
			$\beta+\gamma=-6, \beta\gamma=10$	M1	leading to sum and product of unknown roots	
			$z^2+6z+10=0$,	M1	and quadratic equation	
				A1	which is correct	
			$z=\frac{-6\pm\sqrt{36-40}}{2}$	M1	Use of quadratic formula (or other valid method) in their quadratic	
			$=-3+\text{j}$ or $-3-\text{j}$	A1	oe For both complex roots FT their 3-term quadratic provided roots are complex.	
			or roots must be complex, so $a\pm bj, 2a=-6, 9+b^2=10$	M1	SCM0B1 if conjugates not justified	
			$z=-3+j, z=-3-j$	A1		
				[6]		

Question			Answer	Marks	Guidance	
3	**(i)**		$-2-4p=0$	M1	Any valid row x column leading to *p*	
			$\Rightarrow p=-\frac{1}{2}$	B1		
				[2]		
3	**(ii)**		$\begin{pmatrix}x\\y\\z\end{pmatrix}=\mathbf{N}^{-1}\begin{pmatrix}-39\\5\\22\end{pmatrix}$	M1	Attempt to use $\mathbf{N}^{-1}$	Correct solution by means of simultaneous equations can earn full marks.
			$=\begin{pmatrix}1&0&2\\2&1&3\\\frac{-7}{2}&\frac{-1}{2}&-6\end{pmatrix}\begin{pmatrix}-39\\5\\22\end{pmatrix}$	M1	Attempt to multiply matrices (implied by 3x1 result)	M1 elimination of one unknown, M1 solution for one unknown
			$=\begin{pmatrix}5\\-7\\2\end{pmatrix}$	A1	One element correct	A1 one correct, A1 all correct
				A1	All 3 correct. FT their *p*	
				[4]		
4	**(i)**		$z_2=5\left(\cos\frac{\pi}{4}+\mathrm{j}\sin\frac{\pi}{4}\right)$	M1	May be implied	
			$=\frac{5\sqrt{2}}{2}+\frac{5\sqrt{2}}{2}\mathrm{j}$	A1	oe (exact numerical form)	
				[2]		

Question			Answer	Marks	Guidance	
4	**(ii)**		$z_1 + z_2 = 3 + \frac{5\sqrt{2}}{2} + \left(-2 + \frac{5\sqrt{2}}{2}\right)\mathrm{j} = 6.54 + 1.54\,\mathrm{j}$ $z_1 - z_2 = 3 - \frac{5\sqrt{2}}{2} + \left(-2 - \frac{5\sqrt{2}}{2}\right)\mathrm{j} = -0.54 - 5.54\,\mathrm{j}$	M1	Attempt to add and subtract z_1 and their z_2 - may be implied by Argand diagram	
				B3	For points cao, -1 each error – dotted lines not needed.	
				[4]		
5			$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{1}{4}\sum_{r=1}^{n}\left[\frac{1}{4r-3} - \frac{1}{4r+1}\right]$	M1	For splitting summation into two. Allow missing 1/4	
			$= \frac{1}{4}\left[\left(\frac{1}{1} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{9}\right) + \ldots + \left(\frac{1}{4n-3} - \frac{1}{4n+1}\right)\right]$	M1	Write out terms (at least first and last terms in full)	
				A1	Allow missing 1/4	
			$= \frac{1}{4}\left[1 - \frac{1}{4n+1}\right]$	M1	Cancelling inner terms; SC insufficient working shown above,M1M0M1A1 (allow missing 1/4)	
				A1	Inclusion of 1/4 justified	
			$= \frac{1}{4}\left[\frac{4n+1-1}{4n+1}\right] = \frac{n}{4n+1}$	A1	Honestly obtained (AG)	
				[6]		

Question			Answer	Marks	Guidance	
6			$w=\frac{x}{3}+1\Rightarrow 3(w-1)=x$	M1		
			$x^3-5x^2+3x-6=0$			
			$\Rightarrow (3(w-1))^3-5(3(w-1))^2+3(3(w-1))-6=0$	M1 A1	Substituting Correct	
			$\Rightarrow 27(w^3-3w^2+3w-1)-45(w^2-2w+1)+9w-15=0$			
			$\Rightarrow 27w^3-126w^2+180w-87=0$	A3	FT $x=3w+3, 3w\pm 1$, -1 each error	
			$\Rightarrow 9w^3-42w^2+60w-29=0$	A1	cao	
			OR In original equation $\sum\alpha=5, \sum\alpha\beta=3, \alpha\beta\gamma=6$	M1A1	all correct for A1	
			New roots $\mathrm{A}, \mathrm{B}, \Gamma$			
			$\sum \mathrm{A}=\frac{\sum\alpha}{3}+3,\ \sum \mathrm{AB}=\frac{\sum\alpha\beta}{9}+\frac{2}{3}\sum\alpha+3$			
			$\mathrm{AB}\Gamma=\frac{\alpha\beta\gamma}{27}+\frac{\sum\alpha\beta}{9}+\frac{\sum\alpha}{3}+1$	M1 A3	At least two relations attempted Correct -1 each error FT their 5,3,6	
			Fully correct equation	A1 **[7]**	Cao, accept rational coefficients here	

Question			Answer	Marks	Guidance	
7	(i)		Vertical asymptotes at $x=-2$ and $x=\frac{1}{2}$ occur when	M1	Some evidence of valid reasoning – may be implied	
			$(bx-1)(x+a)=0$			
			$\Rightarrow a=2$ and $b=2$	A1 A1		
			Horizontal asymptote at $y=\frac{3}{2}$ so when x gets very large,	A1		
			$\frac{cx^2}{(2x-1)(x+2)}\to\frac{3}{2}\Rightarrow c=3$	**[4]**		
7	(ii)		Valid reasoning seen	M1	Some evidence of method needed e.g. substitute in 'large' values with result	
			Large positive x, $y\to\frac{3}{2}$ from below			
			Large negative x, $y\to\frac{3}{2}$ from above	A1	Both approaches correct (correct b,c)	
				B1 B1	LH branch correct RH branch correct Each one carefully drawn.	
				[4]		

Question			Answer	Marks	Guidance	
7	(iii)		$\frac{3x^2}{(2x-1)(x+2)} = 1 \Rightarrow 3x^2 = (2x-1)(x+2)$	M1	Or other valid method, to values of x (allow valid solution of inequality)	
			$\Rightarrow 0 = (x-2)(x-1)$		Explicit values of x	
			$\Rightarrow x = 1 \text{ or } x = 2$	A1	 	
			From the graph $\frac{3x}{(2x-1)(x+2)} < 1$			
			for $-2 < x < \frac{1}{2}$ or $1 < x < 2$	B1 B1	FT their x=1,2 provided >1/2.	
				[4]		

Question			Answer	Marks	Guidance	
8	(i)		$\sum_{r=1}^{n}[r(r-1)-1]=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r-n$	M1	Split into separate sums	
			$=\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)-n$	M1	Use of at least one standard result (ignore 3rd term)	
				A1	Correct	
			$=\frac{1}{6}n[(n+1)(2n+1)-3(n+1)-6]$	M1	Attempt to factorise. If more than two errors, M0	
			$=\frac{1}{6}n[2n^2-8]$			
			$=\frac{1}{3}n[n^2-4]$	A1	Correct with factor $\frac{1}{3}n$ oe	
			$=\frac{1}{3}n(n+2)(n-2)$		Answer given	
				[5]		
8	(ii)		When $n=1$,			
			$\sum_{r=1}^{n}[r(r-1)-1]=(1\times 0)-1=-1$			
			and $\frac{1}{3}n(n+2)(n-2)=\frac{1}{3}\times 1\times 3\times -1=-1$			
			So true for $n=1$	B1		
			Assume true for $n=k$	E1	Or "if true for n=k, then…"	
			$\sum_{r=1}^{k}[r(r-1)-1]=\frac{1}{3}k(k+2)(k-2)$			
			$\Rightarrow\sum_{r=1}^{k+1}[r(r-1)-1]=\frac{1}{3}k(k+2)(k-2)+(k+1)k-1$	M1*	Add (k + 1)th term to both sides	
			$=\frac{1}{3}k^3+k^2-\frac{4}{3}k+k-1$			
			$=\frac{1}{3}(k^3+3k^2-k-3)$			

Question			Answer	Marks	Guidance	
			$= \frac{1}{3}(k+1)(k^2+2k-3)$	M1dep*	Attempt to factorise a cubic with 4 terms	
			$= \frac{1}{3}(k+1)(k+3)(k-1)$	A1		
			$= \frac{1}{3}(k+1)((k+1)+2)((k+1)-2)$		Or $= \frac{1}{3}n(n+2)(n-2)$ where $n = k+1$; or target seen	
			But this is the given result with $n = k + 1$ replacing $n = k$. Therefore if the result is true for $n = k$, it is also true for $n = k+1$.	E1	Depends on A1 and first E1	
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1 **[7]**	Depends on B1 and second E1	
9	**(i)**		**Q** represents a rotation 90 degrees clockwise about the origin	B1 B1 **[2]**	Angle, direction and centre	
9	**(ii)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ $\text{P} = (-2, 2)$	M1 A1 **[2]**	Allow both marks for P(-2, 2) www	
9	**(iii)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix}$ l is the line $y = -x$	M1 A1 **[2]**	Or use of a minimum of two points Allow both marks for $y = -x$ www	
9	**(iv)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$ n is the line $y = 6$	M1 B1 **[2]**	Use of a general point or two different points leading to $\begin{pmatrix} -6 \\ 6 \end{pmatrix}$ y=6; if seen alone M1B1	

Question			Answer	Marks	Guidance	
9	**(v)**		$\det \mathbf{M} = 0 \Rightarrow \mathbf{M}$ is singular (or 'no inverse'). The transformation is many to one.	B1 E1 **[2]**	www Accept area collapses to 0, or other equivalent statements	
9	**(vi)**		$\mathbf{R} = \mathbf{QM} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix}$ q is the line $y = x$	M1 A1 **[2]**	Attempt to multiply in correct order Or argue by rotation of the line $y = -x$ $y = x$ SC B1 following M0	

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