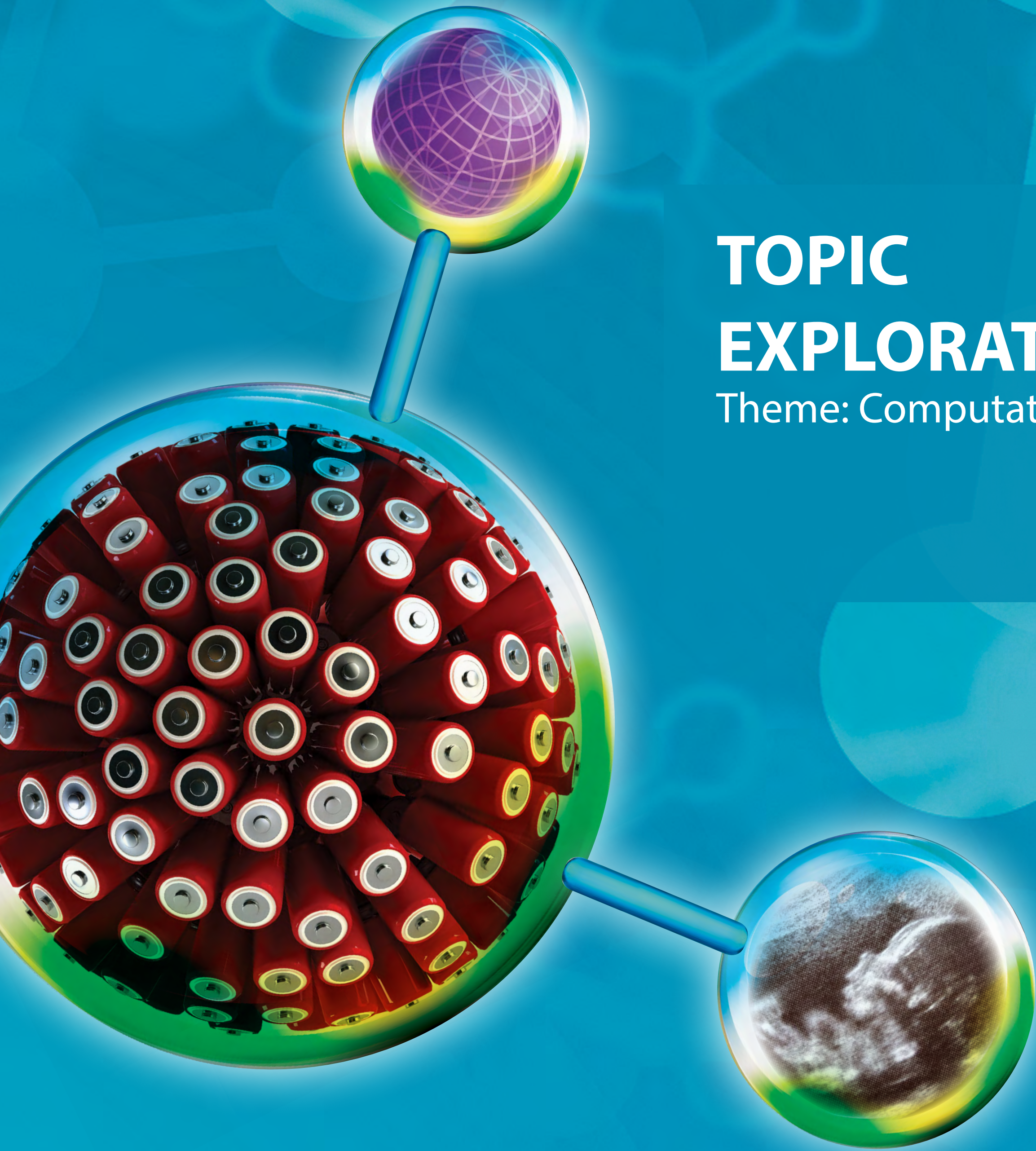
TOPIC EXPLORATION PACK

Theme: Computation



A LEVEL PHYSICS A AND B

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Oxford Cambridge and RSA

Contents

This Topic Exploration Pack should accompany the OCR resource ‘Arithmetical and Numerical Computation’ learner activities, which you can download from the OCR website.

This activity offers an opportunity for maths skills development.

Introduction

KS4 Prior Learning

- Calculations in Standard form
- Split a quantity into a given ratio
- Simplify a ratio to the form 1:n, where n is an integer
- Find a percentage of an amount
- Find percentage increases/decreases
- Round a number to a given number of significant figures.

KS5 Knowledge

- Apply the previous knowledge in unfamiliar settings.

Delivery

This is a large topic and there are a number of common misconceptions that you will have to address in order for the students to succeed. This topic can be broadly split up into two sections:

- Fractions
- Ratios and Percentages
- Decimal Calculations.

Fractions, Ratios and Percentages

Students will have done lots of work on these areas in the past and it is important that if you intend to spend some time working on these areas the resources and activities need to be fresh so to avoid student fatigue. Fractions have perhaps the worst reputation in KS4 Maths and need to be dealt with in a particular way so as not to 'turn-off' the students. In Physics, students will mostly be dealing with fractions when rearranging the formulae. A common example is the formula for resistances of circuits in parallel:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

A common mistake to rearrange for R_{total} say is to just 'flip' the fractions over:

$$R_{total} = R_1 + R_2$$

which is clearly incorrect.

A strategy to avoid this to encourage students to add the two fractions up immediately to form a single fraction and then the ‘flipping’ is valid. The ‘cross-multiplier’ approach is the most useful for a ‘memory’ method:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

The denominator is the product of the 2 individual denominators and then the numerator is found by ‘cross’ multiplying the numerators and denominators and then adding them. See **Activity 1** for a simple activity to help students practise this.

Another key area of difficulty is percentage increases and decreases. This is often because students are ‘over-taught’ the method on how to find percentage increases and decreases, actually the calculations are relatively easy and shouldn’t provide them with too much fear. The key is to understand that the multiplier 1 represents a change of 0%. A multiplier of 1.43 therefore represents an increase of 43% whilst a multiplier of 0.83 represents a decrease of 17% (note that it is the difference between the multiplier and 1 which is the change – it isn’t a percentage decrease of 83%). Amounts and percentages can then be found using a very simple formulae:

$$original\ amount\ \times multiplier = new\ amount$$

This formula can be stated in a formula triangle and then applied to situations where the percentage change is required. For example, if initially the mass of a radioactive isotope is 5.6 grams and after a week it is 4.7 grams, to work out the percentage decrease we have:

$$5.6 \times multiplier = 4.7$$

$$multiplier = \frac{4.7}{5.6} = 0.84$$

This represents a percentage decrease of (1 - 0.84) = 16% decrease. Also say we wanted to know the population of rabbits which were initially 82 and experienced an increase of 16% then:

$$82 \times multiplier = new\ population$$

$$82 \times 1.16 = New = 95.12$$

See **Activity 2** for a lesson example on percentage changes.

Standard form and decimals

The issues students have with dealing with standard form are largely due to the issues students have with dealing with indices. A key misconception is for students to 'over-use' the addition rule for the multiplication of indices. Another key mistake is for the students not to leave the first number in the form $a \times 10^b, 0 < a < 10, b$ is a +/- integer. For example they may write:

$$(2 \times 10^4) \times (8 \times 10^3) = 16 \times 10^7$$

Here the student has correctly dealt with the indices and has got the correct numerical answer but the number isn't in Standard form. The 16 is incorrect and has to be 'scaled' down by dividing by 10. However if we divide by 10 we have to multiply the 10^7 by 10 to keep the numerical value the same. Hence:

$$16 \times 10^7 = 1.6 \times 10^8$$

Students may also mistakenly multiply the indices which is only valid in an example like this:

$$(2 \times 10^{-3})^2$$

Students will make a number of errors here. Common mistakes are to forget squaring the 2, ignoring the negative power and adding the powers instead, incorrect responses could include:

$$2 \times 10^6$$
$$2 \times 10^{-1}$$

Whilst the correct response should be:

$$4 \times 10^{-6}$$

See **Activity 3** for an example of a classroom activity to help develop these skills.

Activity 1 – Adding Fractions

Resources: Activity Sheet 1 (2 copies per page)

Instructions: Hand out a copy of Activity Sheet 1 to the students. Ask them to rearrange the formula to find the letter in red. Encourage them to show every stage of their workings out.

Pedagogy: This is a very simple activity aimed at students to practise manipulating algebraic fractions in formulae. You may wish to use the example above as a teacher-led example before letting the students embark on this activity. This could be used as a starter activity or as a homework activity to help prepare them for the actual formula used in the context of AS Physics.

Timing: Depending on the proficiency of the students this should take about 10-15 minutes to complete. Early finishers can make their own questions up to test their peers.

Activity 2 – Percentage Changes

Resources: Activity Sheet 2

Instructions: Go over an example of how to calculate percentage increases/decreases on the board. Use mini-whiteboards to see if students have understood by going over a couple of other examples. When students are comfortable with the calculations hand out Activity Sheet 2 and explain the task. Students have to use percentage increases to find out the final investment plan. For example if we invest £1 and get 25% interest once every ¼ of a year we will have the final amount:

$$1 \times 1.25 \times 1.25 \times 1.25 \times 1.25 = 1 \times 1.25^4 = 2.44$$

Encourage students to develop their own strategies for working out the final amounts. You may wish to let them know of the indices shortcut or you may wish for them to discover this by themselves. At the end of the investigation students should realise the final amount appears to converge to approximately 2.7. In fact if we were to take an infinitesimally small percentage every infinitesimal time period over a year the final amount would converge on approximately 2.7182818 which is actually exponential e. This is a bonus of this activity as it gives a context as to why exponential e is used in formulae to describe growth and decay etc. The formal mathematical statement is:

$$\mathrm{e} = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

You could ask the very strong students to research exponential e and find a mathematical proof for this statement. This is probably only for the very able and the ones who are doing Further Mathematics at the same time. It could be left as a research task however.

Pedagogy: This is a very simple activity aimed at students to practise finding percentage increases effectively and quickly. It also introduces the concept of exponential e which is useful for the A2 section of the course.

Timing: Once students realise the trick with the calculations this activity doesn't take long. It should be used as a starter activity lasting 5-10 minutes to a lesson where percentage increases and decreases have to be found.

Extension: Tell the students to increase £1 by 10% and then to decrease the new amount by 10%. The answer doesn't go back to 1. Ask them to try it with different percentages, i.e. 15% or 20%. Can they explain what is happening mathematically?

Solution:

Let x be the percentage amount to increase and then decrease. If the initial amount is £1 then after the increase the new amount is:

$$1 \times \left(1 + \frac{x}{100}\right)$$

After the decrease it is:

$$1 \times \left(1 + \frac{x}{100}\right) \times \left(1 - \frac{x}{100}\right)$$

This can be simplified to:

$$1 - \frac{x^2}{10000}$$

And this explains the results the students would have got. So for 8% they would have got

$$1 - \frac{64}{10000} = 0.936$$

Activity 3 – Calculations

Resources: Activity Sheet 3

Instructions: Go over a couple of examples of some calculations involving standard form. You could use the examples:

- $(3.4 \times 10^7) \times (4.3 \times 10^{-5}) = 14.62 \times 10^2 = 1.462 \times 10^3$
- $\dfrac{3.42 \times 10^8}{1.21 \times 10^3} = 2.83 \times 10^5$
- $(8.2 \times 10^{-4})^3 = 551.368 \times 10^{-12} = 5.51368 \times 10^{-10}$

Give students Activity Sheet 3 and ask them to complete the task. Although this is A2 Physics material, the main objective is to get students practising using standard form and the operations involved. Say to students they are only allowed to use a basic calculator to work out the decimal multiplications etc. for example 3.4 x 4.4. The other part of the calculations must be done by hand involving working out the correct power of 10. Give the students the formula for the gravitational acceleration and inform them that all of the calculations must involve quantities in SI units, ie metres, kg etc. Inform students that they have to go to 2 significant figures.

Pedagogy: Students can practise the standard form calculations whilst also practising using a common Physics formulae.

Timing: This could be used as a main task in a lesson to practise using Standard Form. Alternatively it could be used as a homework task or used in the lesson material for the A2 syllabus.

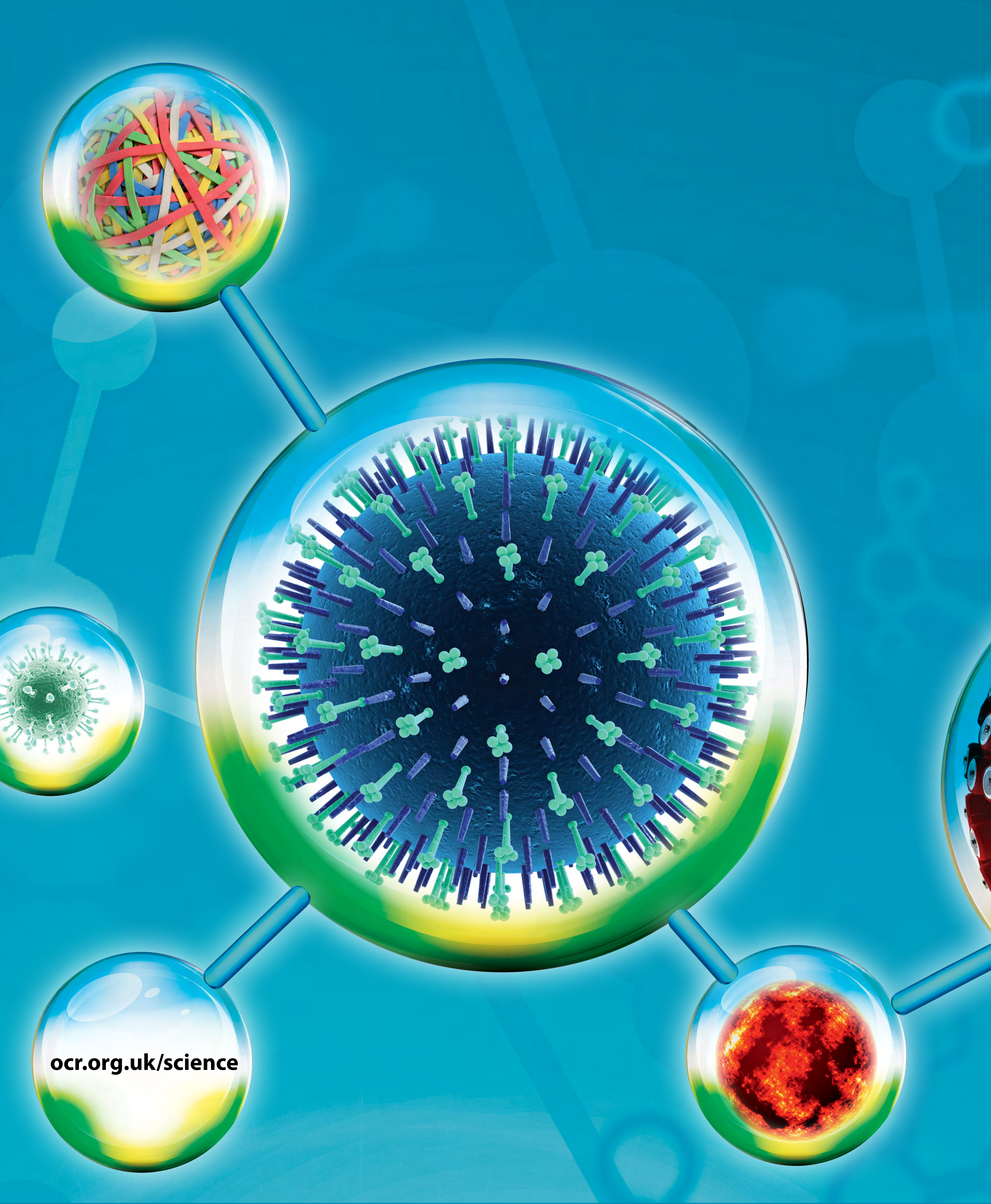
Extension: Ask students to calculate upper and lower bounds for their gravitational accelerations. For example $x = 3.2 \times 10^5$ to 2 significant figures has an upper bound of 3.25×10^5 and 3.15×10^5 as a lower bound. Similarly for $y = 4.2 \times 10^3$ with upper and lower bounds of 4.25×10^3 and 4.15×10^3. This means the upper bound of xy is given by $x_{upper} \times y_{upper}$. However the upper bound for $\frac{x}{y} = \frac{x_{upper}}{y_{lower}}$. Students could then calculate maximum possible percentage errors in their calculations using the formulae in the percentages section above.

Activity 4 – Rearranging equations

Resources: Activity Sheet 4, scientific calculator

Instructions: Go over a couple of examples of rearranging equations and substituting values involving standard form.

Pedagogy: Students are often unsure on how to use their calculators and may use the longer method of input with multiply, the number ten, y^x, then the power. On many calculators if this is used in continuing calculations then it will give an erroneous answer. These questions are intended to identify learners who are not skilled with standard form on their calculator and to give significant practice to reinforce the correct sequence.



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