OCR RECOGNISING ACHIEVEMENT

F

Wednesday 6 November 2013 – Morning

GCSE APPLICATIONS OF MATHEMATICS

A381/01 Applications of Mathematics 1 (Foundation Tier)

Candidates answer on the Question Paper.

OCR supplied materials:
None

Other materials required:
- Scientific or graphical calculator
- Geometrical instruments
- Tracing paper (optional)

Duration: 1 hour

A38101

A339340313

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Your answers should be supported with appropriate working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

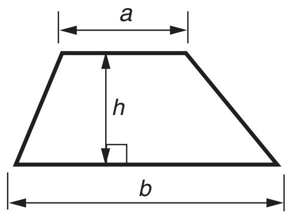
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- Your quality of written communication is assessed in questions marked with an asterisk (*).
- The total number of marks for this paper is **60**.
- This document consists of **20** pages. Any blank pages are indicated.

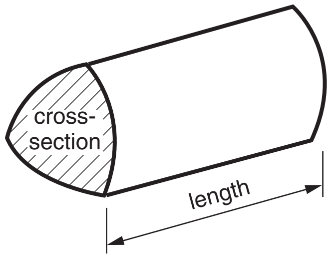
You are permitted to use a calculator for this paper

Formulae Sheet: Foundation Tier

Area of trapezium = $\frac{1}{2}(a + b)h$



Volume of prism = (area of cross-section) × length



PLEASE DO NOT WRITE ON THIS PAGE

Answer **all** the questions.

1 Prime numbers are used in codes for computer and mobile phone security.
They are used because they do not follow a pattern.

Here are all the prime numbers under 100.

2	3	5	7	11	13	17	19	23
29	31	37	41	43	47	53	59	61
67	71	73	79	83	89	97		

(a) What percentage of the 100 whole numbers from 1 to 100 are prime numbers?

(a) ______________________ % **[1]**

(b) Only 6% of the 100 whole numbers from 1 000 001 to 1 000 100 are prime numbers.

How many numbers is this?

(b) ______________________ **[1]**

(c) One fifth of the first 350 whole numbers are prime numbers.

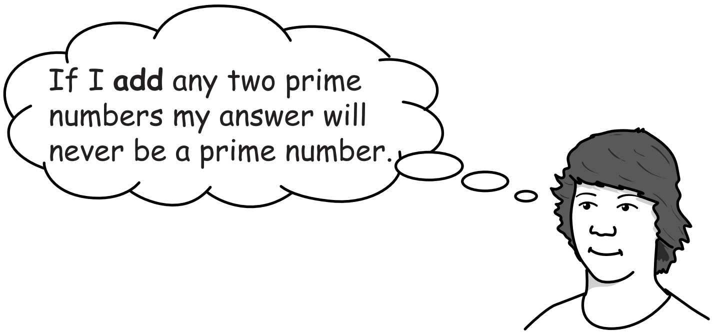
How many of the first 350 numbers are prime numbers?

(c) ______________________ **[1]**

(d) Kevin reads this in a book.

Multiply any two prime numbers and your answer will never be a prime number.

He thinks



Show that Kevin is wrong.

____

____ **[2]**

(e) The number 1 000 033 is a prime number.

Write 1 000 033 in words.

(e) ____ **[1]**

(f) Up to the middle of the year 2012 the largest known prime number had 12 978 189 digits!
The next largest prime number had 12 837 064 digits.

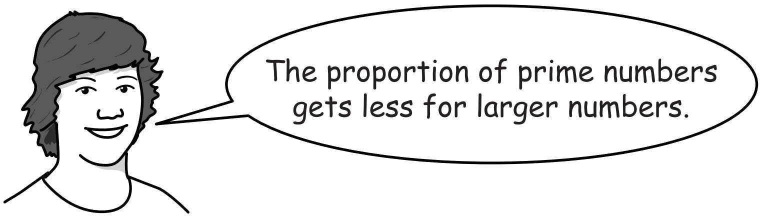
How many more digits did the largest prime have than the next largest prime?

(f) ______________________ **[1]**

(g)* This table shows how many prime numbers there are in different number ranges.

Number range	Number of whole numbers in this range	Number of prime numbers in this range
1 to 200	200	46
201 to 600	400	63
601 to 1400	800	113
1401 to 3000	1600	208
3001 to 6200	3200	376

Kevin says



Use the information in the table to find out if Kevin is correct.
Show the calculations you use to make your decision.

[4]

Most modern codes involve splitting a large number into two prime factors.
For example, the code breaker may know the code involves

$$6\ 236\ 173$$

but they will need to split this into two prime factors to break the code.

(h) The number 6 236 173 is the product of two prime numbers.
One of the prime numbers is 9883.

What is the other prime number?

(h) ______________________________ **[1]**

(i) Factorising very large numbers can take a long time even using a computer.
It has been estimated that to factorise a 600 digit number using a desktop computer would take 1 000 000 000 000 000 years!

Write this number of years as a power of 10.

(i) $1\,000\,000\,000\,000\,000 = 10^{\_\_\_\_}$ **[1]**

This table shows how long on average it takes a computer to split a number into its two prime factors.

Number of digits in the number tested	**Time to find two prime factors**
25	0.1 sec
100	10 min
150	30 min
200	1 year
400	Thousands of years

Modern codes use numbers which may be a thousand digits long!

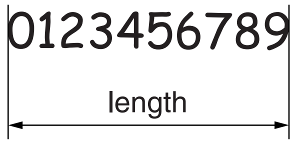
(j) Is the time taken to find the two prime factors of a number directly proportional to the number of digits in the number?
Support your answer with at least two examples.

____

____

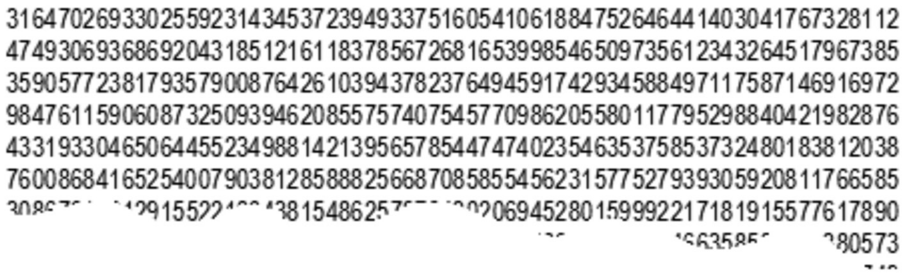
____ **[3]**

(k) **(i)** Measure the total length of these ten digits.
Give your answer correct to the nearest centimetre.



(k)(i) ______________________ cm **[1]**

(ii) It is expected that by 2015 high speed supercomputers will have found a prime number with 100 000 000 digits!



Imagine a number with 100 000 000 digits written down in a single line with the digits the same size as those in part **(k)(i)** and with no spaces between digits.

How many **kilometres** long would it be?

To change centimetres into kilometres divide by 100 000.

(ii) ______________________ kilometres **[3]**

(l) Many prime numbers are given by

$$6n + 1$$

where n is a whole number.

For example:

$n = 1$

the prime number 7 is $(6 \times 1) + 1$,

and

$n = 2$

the prime number 13 is $(6 \times 2) + 1$

(i) Complete this table.

n	$P = 6n + 1$	Is P prime? (yes/no)
1	7	Yes
2	13	Yes
3	19	Yes
4	25	No
5		

[2]

(ii) By looking at prime numbers greater than 50, find a value for n where the formula $P = 6n + 1$ produces a prime number.

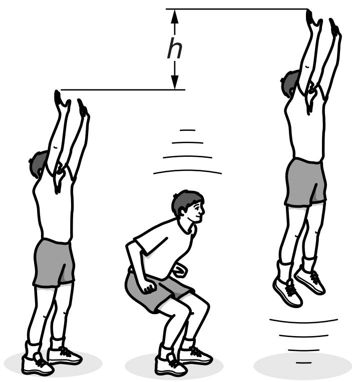
Don't forget the list on page 3!

Show all the steps in your working.

(l)(ii) ______________________________ **[3]**

2 Most sports involve explosive leg power, for example judo, sprinting and basketball.

An athlete's vertical standing jump height, h cm, is used to calculate their leg power.



There are several formulae that can be used to calculate the leg power, P watts.
These are shown below.

Lewis Formula

$P = 2.2 \times$ (person's weight in kg) $\times$ (square root of person's jump height in cm)

Harman Formula

$P = 21 \times$ (person's jump height in cm) $+ 23 \times$ (person's weight in kg) $- 1393$

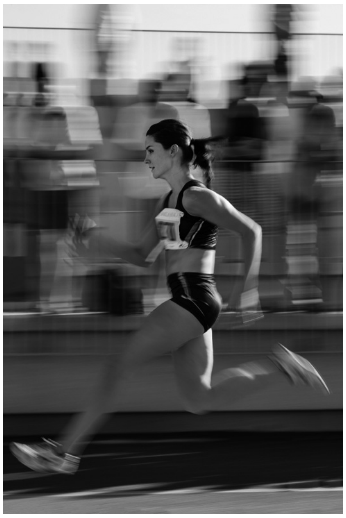
Johnson and Bahamonde Formula

$P = 44h + 33w + 430 - 17p$

h is person's jump height in cm
w is person's weight in kg
p is person's height in cm

Choy is a sprinter, here are her details:

- jump height = 48 cm
- weight = 60 kg
- height = 170 cm.



(a) Use each formula to calculate Choy's leg power.

(i) Lewis Formula

(a)(i) ______________________ watts **[1]**

(ii) Harman Formula

(ii) ______________________ watts **[1]**

(iii) Johnson and Bahamonde Formula

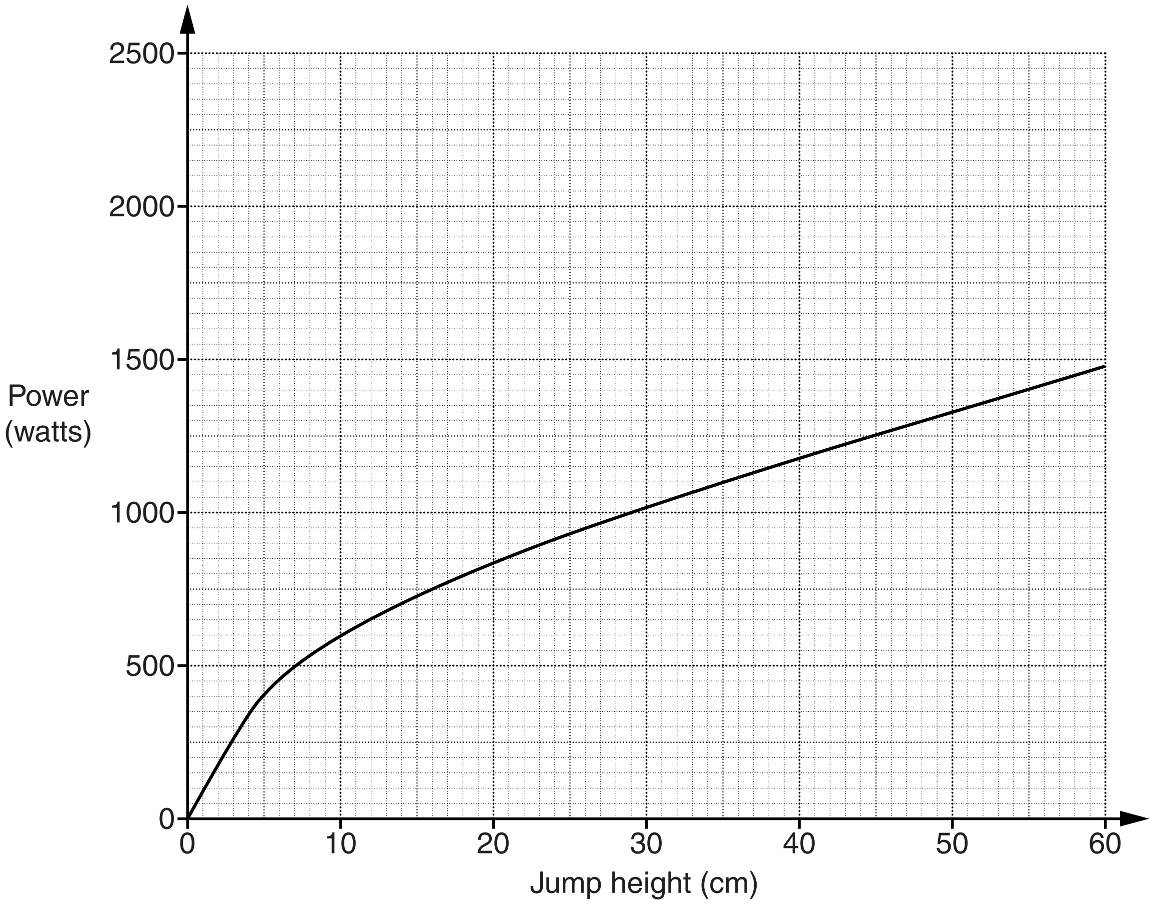
(iii) ______________________ watts **[2]**

(iv) Make one comment about your **three** figures for Choy's leg power.

____

____ **[1]**

(b) Frank plays basketball.
He uses this graph of the Lewis Formula to estimate his leg power.



Use the graph to estimate Frank's leg power if his jump height is 26 cm.

(b) ______________________ watts **[1]**

3 Prices increase over time.

(a) In the year 1400 the price of a loaf of bread was equivalent to 0.5p.
The price of a loaf of bread is now about 120p.

This number machine shows the multiplier for the price in 1400 to the price today.

Price in 1400 → $\times$ 240 → Price today

Use the multiplier to answer these questions.

(i) The price of a pair of shoes in 1400 was 8p.
What would this price be today?

(a)(i) ______________________ p **[1]**

(ii) The price today of a saucepan is £21.
What would its price have been in 1400?
Give your answer correct to the nearest penny.

(ii) ______________________ p **[2]**

(b) In 1400 the average wage was equivalent to 8p a week.
Today the average wage is about £400 a week.

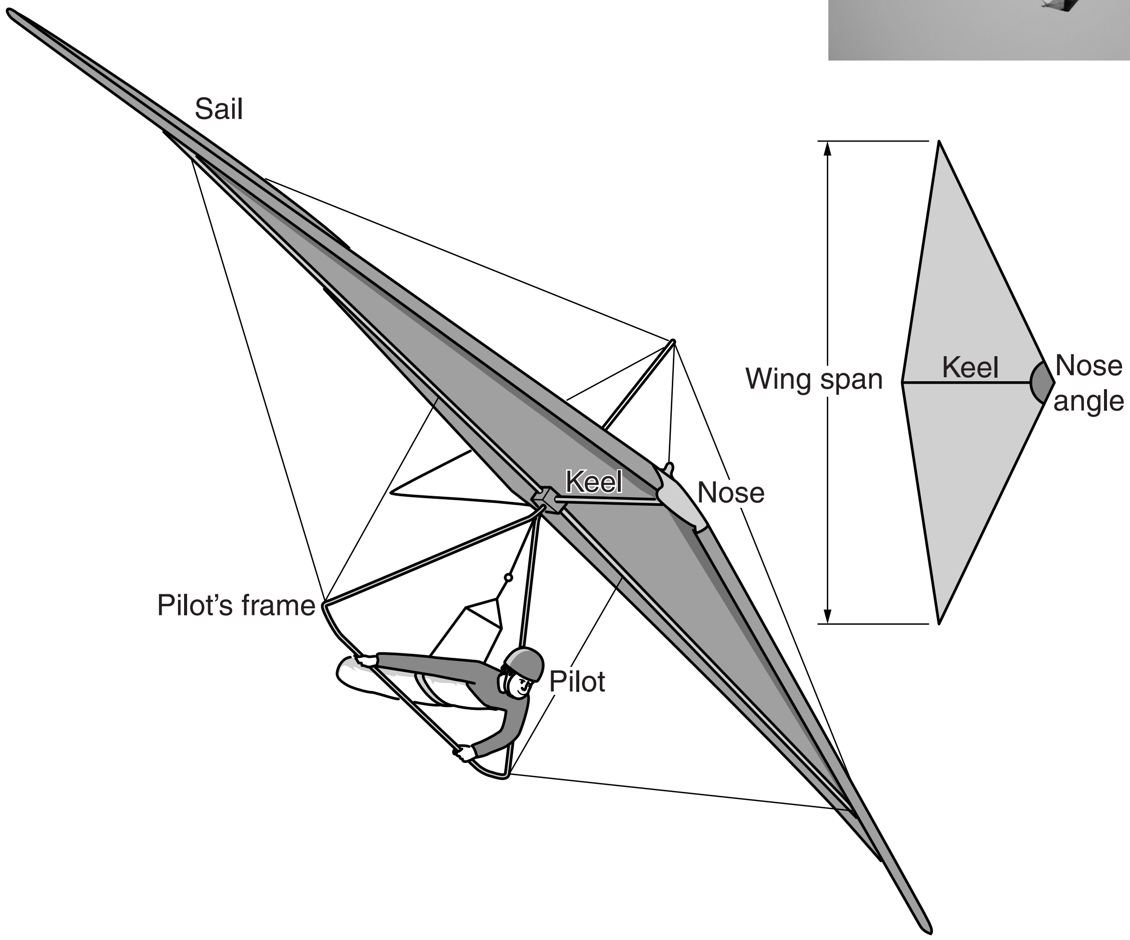
Complete this number machine to show the multiplier for the wages.

Wage in 1400 → $\times$............ → Wage today

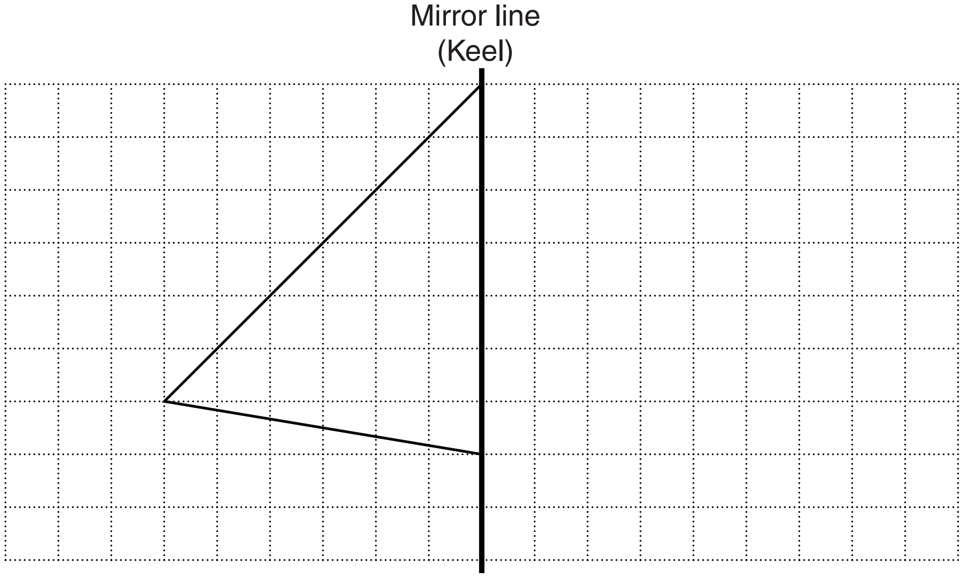
[2]

4 An important part of a hang glider is the sail.
The pilot is attached to this by a triangular frame.
The pilot controls the hang glider using wires attached to the sail.

The sketches show the names for parts of a sail.

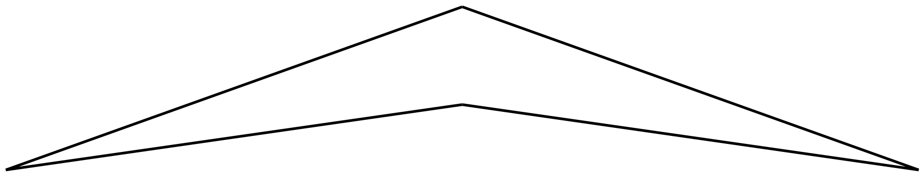


(a) The keel is a mirror line for the sail.
Complete this design for a sail.



[2]

(b) Here is the design for a different sail.



On the design

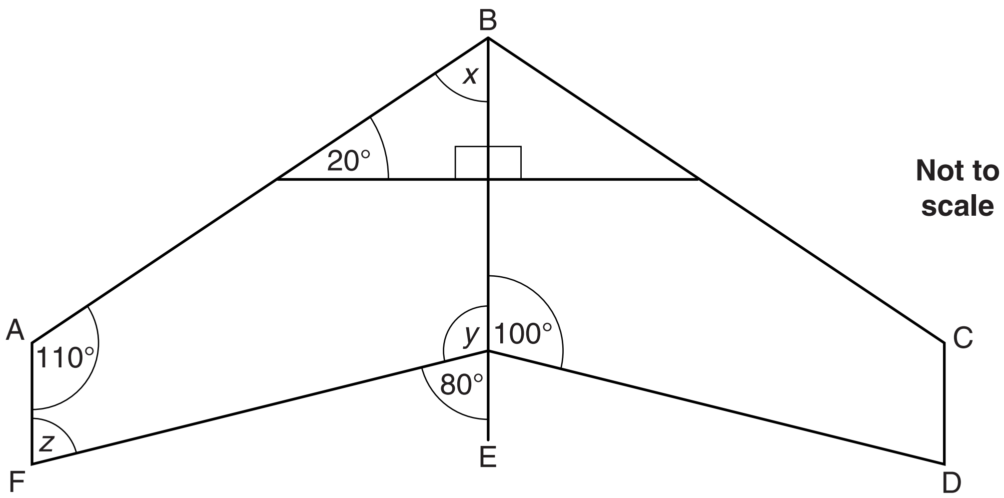
- mark and clearly label with an A an acute angle
- mark and clearly label with an O an obtuse angle
- mark and clearly label with an R a reflex angle.

[3]

(c) This diagram shows another sail.

- BE is the keel.
- AF, BE and CD are parallel.

Calculate the sizes of angles x, y and z.



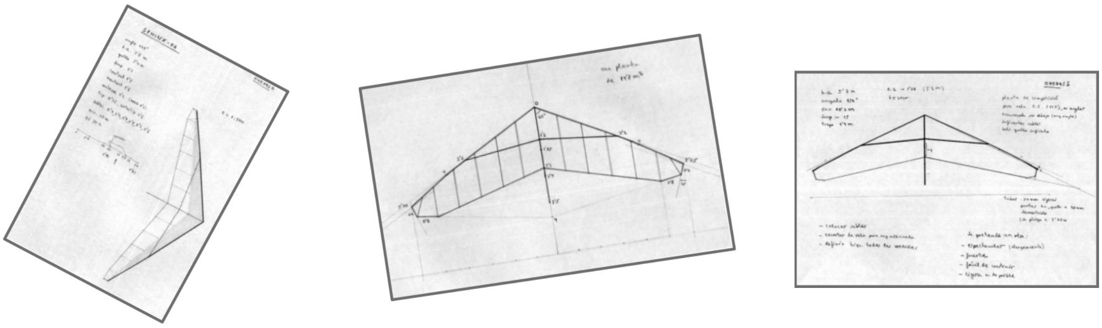
x = ____________________ °

y = ____________________ °

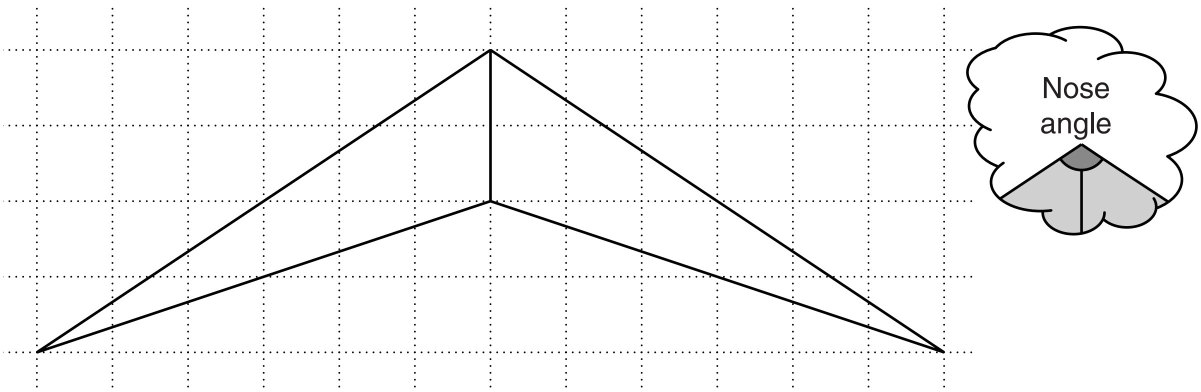
z = ____________________ °

[3]

Sasha draws several different designs for hang glider sails.



Her final design is shown below.
Each square represents an area of 1 square metre.



(d) Using Sasha's scale diagram,

(i) measure the nose angle,

(d)(i) ______________________ ° **[1]**

(ii) calculate the area of the sail.
Show your working.

(ii) ______________________ m^2 **[3]**

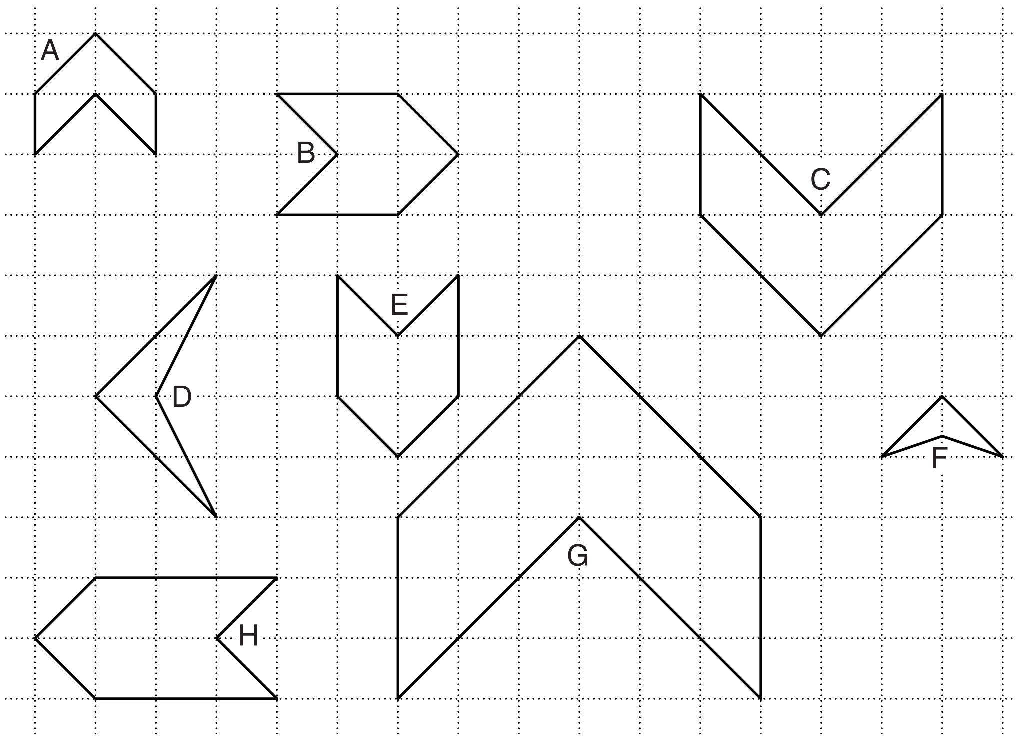
(e) An important measurement is the aspect ratio of a sail.
Complete this calculation to work out the aspect ratio of a sail.

$$\frac{9.7^2}{(14.5 + 2.1)}$$

=

[2]

(f) Sasha draws some more designs for hang glider sails.



(i) Which of the designs are mathematically similar to design A?

(f)(i) ______________________________ **[1]**

(ii) Which two of the designs are congruent to each other?

(ii) ______________ and ____________ **[1]**

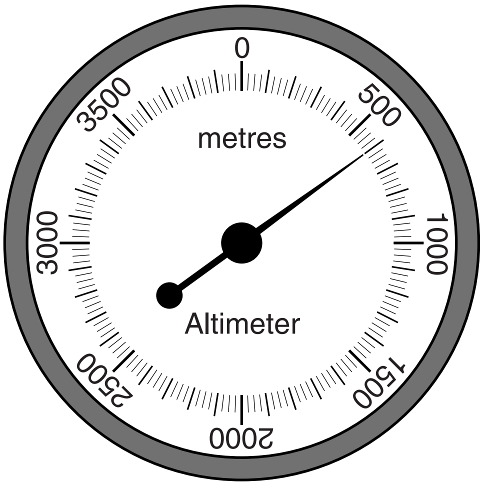
(g) **(i)** The world record height for a hang glider flight is 9100 m.

What is this height in kilometres?

(g)(i) ____________________ km **[1]**

(ii) Altimeters show the flying height of a hang glider.

What is the height shown on this altimeter?



(ii) ____________________ m **[1]**

(iii)*

Sasha took this photo while flying in a hang glider at a height of about 100 m.
It shows a train, marked by an arrow, waiting in a station.
There are some cars in the station car park.

Use the length of a car to estimate the length of the train.

____

____

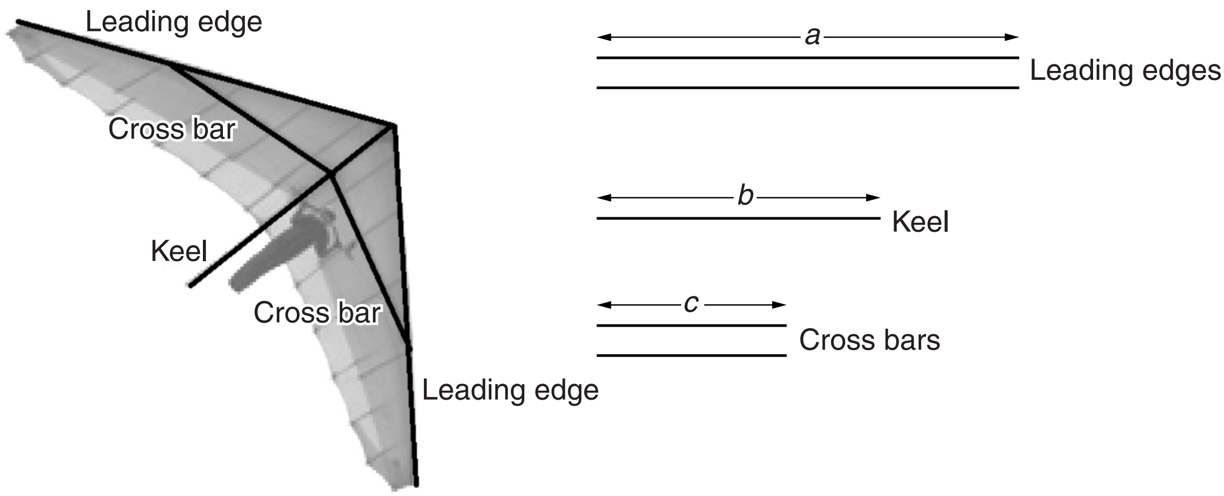
____ **[3]**

(h) Hang glider sails are kept in shape with carbon fibre rods.

This sail needs 5 rods:

- 2 for the leading edges, each of length *a*
- 1 for the keel, length *b*
- 2 for the cross bars, each of length *c*.

All lengths are in metres.



Write down the formula for the total length of the rods, *L*, used in the sail above.

(h) ______________________________ **[3]**

END OF QUESTION PAPER