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LEVEL 3 CERTIFICATE

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For first teaching in 2015

Topic Exploration Pack

Probability

Version 1

Mathematics Education Innovation

www.ocr.org.uk/maths

Contents

This Topic Exploration Pack should accompany the OCR resource 'Probability' learner activities, which you can download from the OCR website.

Introduction

Probability is very often a difficult topic to teach. There are many reasons for this. Students are very used to using informal probability in their everyday life. They are used to making snap judgements on the likelihood of something happening whether that is correct or not. If they play games of chance involving dice then they may have an idea of how probability works but this doesn't mean that they understand it correctly. Indeed one of the most difficult jobs of a teacher teaching probability is to dispel a lot of the myths and misconceptions that students may have on probability. They will have studied all of these concepts at GCSE but the misconceptions unfortunately may remain (indicating the power of intuition in the mind). Classic misconceptions students have are:

- Knowing when events are independent. For example not rolling a six on 35 successive occasions does not mean that there is more of a chance rolling a 6 on the 36th time.
- The concept of randomness. Unfortunately in recent years the word 'Random' has come into the vernacular as meaning something strange or unexpected has happened. Actually the idea of randomness is very subtle and should be associated with unpredictability rather than something rare or unusual occurring (in fact in actual life we should actually be talking about pseudo-randomness as there are very few real life processes that replicate pure unpredictability – a roll of a die could be predicted given we know all of the variables and use Newton's laws).
- Experimental probability/Relative Frequency. Whilst a simple concept to calculate, the difference between the experimental and theoretical probability can be blurred. Students need to be aware that actually experimental probability is probably better stated as estimated probability; it is a best estimate for a probability that cannot be evaluated theoretically. They also need to be aware that the more trials they do the closer the estimated probability will be to the actual probability.
- Some students will intuitively understand Probability, while other may find the more difficult concepts very challenging. This doesn't mean all hope is lost for the latter students but it will require careful planning of activities to ensure they do end up understanding.

A good order of progression in teaching this topic is to spend a lesson dispelling all of the myths so that you can gauge where the individual students are at and introduce the main vocabulary, a nice activity is presented in **Activity 1** for this. After this looking at listing outcomes and two-way tables is a good way to continue. See **Activity 2** for something on this. Before looking at tree-diagrams it is then useful to study experimental probability and **Activity 3** has an example of an unusual experiment (not involving the loud dice!) for the students to try. Finally the teaching of formal tree diagrams is supported with **Activity 4**.

Activity 1 – Probability presentation

Resources – Probability True or False PowerPoint

This is a nice introductory activity that is best done at the beginning of the topic as it can dispel a number of different misconceptions. Each slide on the PowerPoint has a misconception. The idea is to use this as a starter activity. Play the presentation and ask the students to write down whether each slide is true or false (you can make the slides automatic with a set time period if you prefer not to change the slide). Afterwards the answers can be discussed and any misconceptions can be discovered and acted upon.

Answers

Slide 1 – False! Events are independent of each other. One coin flick doesn't affect the next flick!

Slide 2 – False! Same reason as before.

Slide 3 – False! The probability is actually calculated from

$$\text{P(Event A)} = \frac{\text{No. of ways of A occurring}}{\text{Total possible outcomes}} = \frac{3}{8}$$

Slide 4 – False! It is more complicated than that! Weather is not a random event. There are ways we can predict what weather can occur. It may be that the weather today affects the weather tomorrow and hence there is dependence.

Slide 5 – False! Each number is equally as likely; $\frac{1}{6}$, because 6 is the largest this has no effect on the probability.

Slide 6 – False! Same reason as above. Each combination of 6 is equally as likely as others. In fact this particular combination is a good combination to have because it doesn't decrease your chances and if they do come up you will probably have fewer people to share the jackpot with!

Slide 7 – False! The 3rd spinner has a probability of $\frac{3}{4}$ for getting a black.

Activity 2 – Two-way Tables – Games

Resources – Activity Sheet 2

In this activity students are to practise listing outcomes and using probability notation. Students are to have a copy of Activity sheet 2 and answer the questions. This is a basic exercise but will help consolidate their knowledge of listing outcomes and calculating probabilities.

Answers

1) Single experiment

I roll the dodecahedral (12 sided, numbers 1 – 12) die once. Work out these probabilities:

a) P(Even) = Possibilities are 2,4,6 hence $\frac{6}{12} = \frac{1}{2} = 50\%$

b) P(Even $\cup$ Multiple of 5) = Possibilities are 2,4,5,6,8,10,12 hence $\frac{7}{12}$

c) P(Even $\cup$ Prime) = Possibilities are 2,3,4,5,6,7,8,10,11,12 hence $\frac{10}{12}$

d) P(Square Number) = Possibilities are 1,4,9 hence $\frac{3}{12} = \frac{1}{4} = 25\%$

e) P(Even $\cap$ Prime) = Possibilities are 2 hence $\frac{1}{12}$

2) Double experiment

I roll the octahedral (8-sided, numbers 1 - 8) die and the tetrahedral (4-sided, numbers 1- 4) die. I add up the scores on each die.
List all the outcomes and draw a two way table for the outcomes:

Add +	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12

Work out these probabilities:

a) P(Prime): Possibilities are 2,3,3,5,5,5,5,7,7,7,7,11,11 hence $\frac{13}{32}$

b) P(Multiple of 3): Possibilities are 3,3,6,6,6,6,9,9,9,9,12 and hence $\frac{11}{32}$

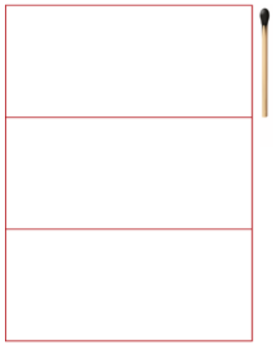
c) P(Numbers less than 6): Possibilities are 2,3,3,4,4,4,5,5,5,5 and hence $\frac{10}{32} = \frac{5}{16}$

d) P(Prime $\cap$ Numbers less than 6 $\cap$ Multiples of 3): Possibilities are 3,3 and hence $\frac{2}{32} = \frac{1}{16}$

e) P(Prime $\cup$ Numbers less than 6 $\cup$ Multiples of 3): Possibilities are same as c) + 6,6,6,6,7,7,7,7,9,9,9,9,11,11,12 and hence $\frac{25}{32}$

Activity 3 – Buffon's Needle

Resources – A box of burnt out matches, blank A4 paper, calculator, Activity Sheet 3



This is a classical problem in probability. The history of the problem and further technical details can be found here http://en.wikipedia.org/wiki/Buffon's_needle. Hand out a match to each student (make sure they have already been burnt out!). Students are to measure the match and then on a blank piece of A4 paper that is held in a portrait orientation draw horizontal lines across the paper which have a spacing which is the same as the length of the match like below. Students are then to drop the match from a decent height (30cm is usually good enough) and record whether the match touches the line (win) or not (lose). They are to record their results in a table like below (see Activity Sheet 3):

	Tally	**Relative Frequency**
Win	57	
Lose	35	
Total	92	

Give the students a time limit of between 5 and 10 minutes (any longer and they will get bored). At the end students are to calculate the relative frequency:

	Tally	**Relative Frequency**
Win	57	$\frac{57}{92} = 0.6196$
Lose	35	$\frac{35}{92} = 0.3804$
Total	92	1

The reason for doing this is that it can be proved analytically (see website for an advanced proof) that the probability of a match hitting the line is theoretically $\frac{2}{\pi}$. This may be surprising for the students but what this experiment does do is give students a way of estimating the value of π using a random method (Monte Carlo Method). For the numbers stated above we can find an approximation of π by calculating

$$\pi \approx \frac{2}{\text{Relative Frequency of Winning}} = \frac{2}{0.6196} = 3.2288$$

At the end of the activity the results as a class can be combined and (hopefully) a better approximation of π can be made. This gives students the notion that by increasing the number of trials the closer the relative frequency approximates the theoretical probability.

Activity 4 – Tree Diagrams

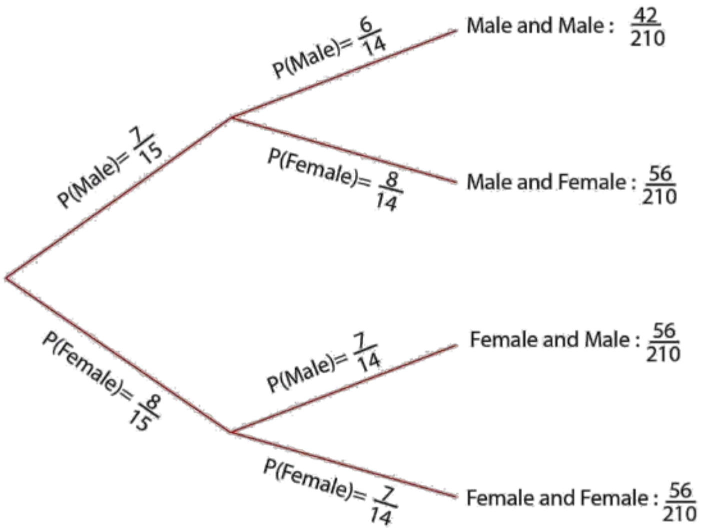
Resources – Activity Sheet 4

This activity helps students practise and consolidate probability tree diagrams. The main misconception students have when constructing these diagrams is that they have to choose events that are mutually exclusive (can't happen at the same time) and exhaustive (all possible options) to come from a particular point in the diagram. Students are to multiply probabilities along the branches and then add the probabilities up at the end. This is a rare case when students shouldn't simplify the fractions at the end because it makes it easier to add up at the end. Students should always add the probabilities at the end to make sure they add to 1 as a simple check whether they have done it correctly or not.

Answers

1) James decides to buy 2 monkeys from a monkey shop. The shop has 7 males and 8 females. James picks one at random. After James decides on one monkey he chooses another one at random from the remaining ones. You can assume James cannot tell the difference between the sexes.

a) Draw a probability tree diagram displaying the information.
b) Calculate the probabilities along the branches and at the end of the branches.



c) Use these probabilities to calculate the probability that he picks 2 males.

P(2 males) = $\frac{42}{210}$

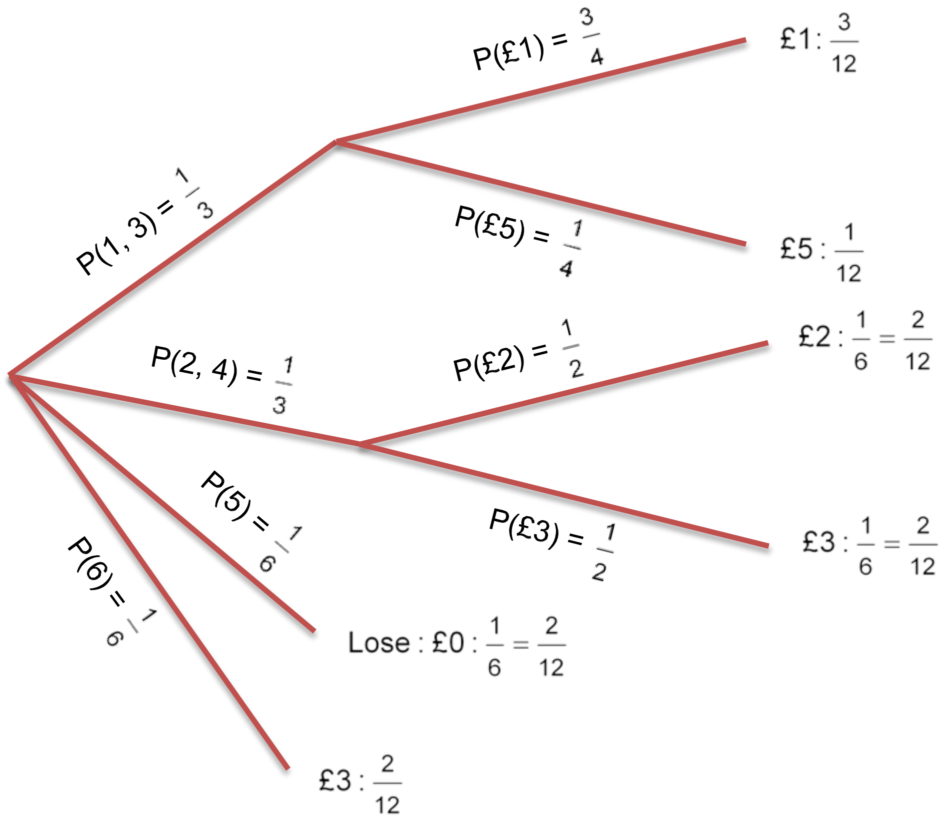
d) Use these probabilities to calculate the probability the he picks 1 male and 1 female monkey.

P(1 male + 1 female) = $\frac{56}{210}+\frac{56}{210}=\frac{112}{210}$

2) Heather decides to play a game of chance involving a roll of a die and a spinner at a school fete. It costs £2 to play. If she rolls a 2 or a 4 then she has to spin Spinner A. If she rolls a 1 or 3 then she has to spin Spinner B. If she rolls a 5 then she loses and if she rolls a 6 then she wins £3.

Draw a probability tree diagram with the roll of the die as the first event and the spinner the second event.

Calculate all the probabilities along the branches and at the end.



a) What is the probability of winning £1?

$$P(£1) = \frac{3}{12} = \frac{1}{4}$$

b) What is the probability of winning £3?

$$P(£3) = \frac{2}{12} + \frac{2}{12} = \frac{4}{12} = \frac{1}{3}$$

c) What is the probability of winning at least £3?

P(at least £3) = P(£3 from spinner B) + P(£5 from spinner A) + P(£3 from rolling a 6) =

$$\frac{2}{12} + \frac{1}{12} + \frac{2}{12} = \frac{5}{12}$$

d) What is the expected amount you expect to win if you play the game?

$$\text{Expected amount} = \sum(\text{outcome} \times \text{probability})$$
$$= \left(£1 \times \frac{3}{12}\right) + \left(£2 \times \frac{2}{12}\right) + \left(£3 \times \frac{4}{12}\right) + \left(£5 \times \frac{1}{12}\right) = \frac{24}{12} = £2$$

which is exactly the same as the stake. It is therefore not worth playing this game.

The small print

OCR Resources: *the small print*

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