Accredited

GCSE (9–1)

Transition Guide

J560

MATHEMATICS

Theme: Trigonometry

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Welcome

Welcome to the KS4–KS5 transition guide for **Maths.**

Key Stage 4 to 5 Transition guides focus on how a particular topic is covered at the different key stages and provide information on:

- Differences in the demand and approach at the different levels;
- Useful ways to think about the content at Key Stage 4 which will help prepare students for progression to Key Stage 5;
- Common student misconceptions in this topic.

Transition guides also contain links to a range of teaching activities that can be used to deliver the content at Key Stage 4 and 5 and are designed to be of use to teachers of both key stages. Central to the transition guide is a Checkpoint task which is specifically designed to help teachers determine whether students have developed deep conceptual understanding of the topic at Key Stage 4 and assess their 'readiness for progression' to Key Stage 5 content on this topic. This checkpoint task can be used as a summative assessment at the end of Key Stage 4 teaching of the topic or by Key Stage 5 teachers to establish their students' conceptual starting point.

Key Stage 4 to 5 Transition Guides are written by experts with experience of teaching at both key stages.

Go to topic comparison

Topic: Trigonometry

Key Stage 4 Content

GCSE (9-1) content Ref.	Subject content	Initial learning for this qualification will enable learners to..	Foundation tier learners should also be able to...	Higher tier learners should additionally be able to...	DfE Ref.
6.02	**Algebraic formulae**				
6.02d	Recall and use standard formulae	Recall and use: Circumference of a circle $2\pi r = \pi d$ Area of a circle πr^2	Recall and use: Pythagoras' theorem $a^2 + b^2 = c^2$ Trigonometry formulae $\sin\theta = \frac{o}{h}$, $\cos\theta = \frac{a}{h}$, $\tan\theta = \frac{o}{a}$	Recall and use: The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$ Area of a triangle $\frac{1}{2}ab\sin C$	A2, A3, A5
7.01	**Graphs of equations and functions**				
7.01e	Trigonometric functions			Recognise and sketch the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$.	A12
10.05	**Triangle mensuration**				
10.05b	Trigonometry in right-angled triangles		Know and apply the trigonometric ratios, $\sin\theta$, $\cos\theta$ and $\tan\theta$ and apply them to find angles and lengths in right-angled triangles in 2D figures.	Apply the trigonometry of right-angled triangles in more complex figures, including 3D figures.	R12, G20
10.05c	Exact trigonometric ratios		Know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0°, 30°, 45°, 60°$ and $90°$. Know the exact value of $\tan\theta$ for $\theta = 0°, 30°, 45°$ and $60°$.		R12, G21
10.05d	Sine rule			Know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, to find lengths and angles.	G22
10.05e	Cosine rule			Know and apply the cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find lengths and angles.	G22

Go to Key Stage 5 content

http://ocr.org.uk/Images/168982-specification-gcse-mathematics-j560.pdf

Topic: Trigonometry

Key Stage 5 Content

From the content for OCR Core 2 (4722)*

Candidates should be able to:

(a) use the sine and cosine rules in the solution of triangles (excluding the ambiguous case of the sine rule);

(b) use the area formula $\Delta = \frac{1}{2}ab\sin C$;

(c) understand the definition of a radian, and use the relationship between degrees and radians;

(d) use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ for the arc length and sector area of a circle;

(e) relate the periodicity and symmetries of the sine, cosine and tangent functions to the form of their graphs;

(f) use the identities $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ and $\cos^2\theta + \sin^2\theta \equiv 1$;

(g) use the exact values of the sine, cosine and tangent of 30°, 45°, 60° e.g. $\cos 30° = \frac{1}{2}\sqrt{3}$;

(h) find all the solutions, within a specified interval, of the equations $\sin(kx) = c$, $\cos(kx) = c$, $\tan(kx) = c$, and of equations (for example, a quadratic in $\sin x$) which are easily reducible to these forms.

*http://www.ocr.org.uk/Images/67746-specification.pdf

Key Stage 5 Content

From the content for OCR Core 3 (4723)*

Candidates should be able to:

(a) use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations, and relate their graphs (for the appropriate domains) to those of sine, cosine and tangent;

(b) understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude;

(c) use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations within a specified interval, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of

(i) $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$,

(ii) the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$,

(iii) the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$,

(iv) the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.

*http://www.ocr.org.uk/Images/67746-specification.pdf

Key Stage 4 Content

Explore the Guide

Topic: Trigonometry

Comment

The emphasis at GCSE is generally on the geometrical aspects of trigonometry, such as finding sides and angles in triangles through the use of trigonometric ratios in right-angled triangles and in the use of the sine and cosine rules in general triangles. There is only a very brief introduction to transforming graphs of functions and drawing trigonometric graphs. Many centres will allocate their learning time in favour of solving triangles; however in high achieving groups, especially where most will go on to study maths at A Level, it will be beneficial to spend more time on drawing, using and transforming graphs, which will all become more important at A Level.

Therefore the emphasis of teaching at GCSE should be on establishing the ratios and treating them as equations rather than teaching algorithms. Students need to learn to rearrange these equations so that any of the three elements could be the subject of the equation. They should spend more time on drawing trigonometric graphs and using angles greater than 90°, to make the move away from the use in triangles easier at A level and on transforming functions, because this topic forms a larger proportion of the A Level knowledge pool.

Trigonometry is a topic that lends itself to exploration by students and this needs to be encouraged as a learning method; there are some suggested lessons included here to enable this approach. The first lessons in trigonometry are important because if the ratios are immediately introduced, students miss out on a deeper understanding of the topic. Algebraic manipulation is also important because at A Level there is a much higher demand in this skill, such as solving equations and integration using substitution with one of the many trigonometric identities. Trigonometry also needs to be taught with 'similar triangles', as it is a good application of that topic too.

The emphasis at A Level is on algebraic skills rather than geometrical ones, such as solving trigonometric equations, a more detailed exploration of transformations and some uses of trigonometry when modelling in mechanics and in calculus. It therefore would benefit students to deepen their understanding, at GCSE, of drawing graphs and applying transformations to functions. They also need to learn to manipulate trigonometric expressions and equations. While drawing graphs, it will be helpful to use the graphs to solve simple equations so that it can be clearly seen that there is often more than one possible solution to a trigonometric equation.

Next

Comment

There is a need to attempt more problems in three dimensions; trigonometry questions at A Level are commonly set in 3D situations and it is often a feature of GCSE questions. It is necessary to see the two dimensional triangles within the three dimensional shape and this is still required at A Level, although alternative methods such as vectors may be used.

At GCSE there are two main misconceptions. It is common to see unstructured algebra at GCSE, especially by omitting the equals sign in equations, and this continues in trigonometry too. In calculating a side we often see just the calculation such as $4.5 \times \sin 36°$ or, for an angle, $\tan^{-1}(8 \div 10)$. The other main misconception is the failure to understand when to apply the formulae. A common method in right-angled triangles is to use the sine rule, and less commonly the cosine rule, which, although are not wrong, can often lead to errors. We also see the trigonometric ratios used in non-right-angled triangles, which is an incorrect use of that technique.

Previous

Possible Teaching Activities (GCSE focus)

Click here to view page

GeoGebra is a dynamic geometry package that is free to use. You can use spreadsheets here to do the calculations for you.

Resources: http://www.geogebra.org/cms/en/

Click here to view page

A series of practical lessons to establish understanding of trigonometric ratios. It starts with 'Building Bridges', a practical activity to construct a bridge out of triangles or quadrilaterals. The second activity is 'What is the problem?' and this challenges students to try to estimate a height using triangles. The final activity, 'Same shape triangles', allows students to measure sides of triangles and work out the ratio of the sides, to see that they are in the same ratio for the same angle. This is a pen and paper activity.

Resources: http://www.curriculumsupport.education.nsw.gov.au/secondary/mathematics/years7_10/teaching/trig.htm

Click here to view page

This resource allows students to use a trigonometric protractor to explore trigonometric ratios and see that they stay the same for equal angles. This follows on from the previous teaching activity because the triangles and the measuring can be done on a computer screen. The student constructs a right-angled triangle and uses the protractor to measure the sides and the ratios; by scaling the protractor students can see that the ratios stay fixed for the same angle. This needs a little practice to master the program and a demonstration to the students is a good idea.

Resources: http://nrich.maths.org/5601

Next

Possible Teaching Activities (GCSE focus)

Click here to view page

This approach uses a dynamic geometry package. This is an investigation where students start with pencil and paper to understand similar triangles; they then generate right-angled triangles using the computer package and measure their lengths and work out ratios of lengths. They discover that the sin, cos and tan buttons on their calculator can generate these ratios and then compare them with their calculated ones.

Resources: http://www.teachmathematics.net/page/7791/discovering-sohcahtoa

Click here to view page

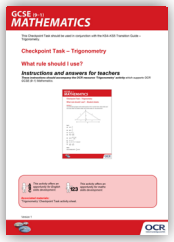
The first activity is 'Making statements'. This has three triangles and students are given an open-ended task to write down as many trigonometric equations as they can. This is a big leap from the other tasks and is similar to the checkpoint task below. The second task is 'Rearranging SOHCAHTOA', a PowerPoint presentation that can be used as a class activity. Students make correct trigonometric statements, and then learn how to rearrange them to find the information they need.

Resources: http://www.teachmathematics.net/page/12158/re-arranging-sohcahtoa

Previous

Topic: Trigonometry

Checkpoint Tasks



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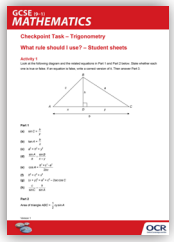

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Which rule should I use?

These activities can be done on paper or they can be displayed on a screen; students can work individually, in small groups or in large groups.

Topic: Trigonometry

Possible Teaching Activities (KS5 focus)

In this resource students learn how waves are seen in everyday use and how to draw trigonometric graphs using a circle. This approach is very similar to using the unit circle to draw the sine and cosine graphs and once the lengths representing sine and cosine have been identified, it can then be used to find the approximate values of sine and cosine for angles greater than 90°.

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Resources: http://www.teachmathematics.net/page/10977/sine-cosine-triangle-circle-wave

In this task students make a theodolite and then use it to look at surveying problems that use the sine rule to find lengths. They can find two angles in a triangle and one side, before using the sine rule to find the other sides.

Click here to view page

Resources: http://www.teachmathematics.net/page/15892/sine-rule-using-a-theodolite-

This is an investigation into the effect of changing variables in a general trigonometric expression. Through looking at the maximum and minimum values of these curves and the points they go through the *x*-axis, students are able to identify the transformations and sketch the resulting graphs.

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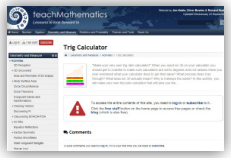
Resources: http://www.teachmathematics.net/page/10984/sine-cosine-transformations

Possible Extension Activities (KS5 focus)



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In this task students discuss and build a trigonometric ratio calculator using dynamic geometry and then use it to discover properties of trigonometric ratios, such as special values (0°, 30°, etc) and symmetry. If you do not have a dynamic geometry package then one of these is available free from http://www.geogebra.org/cms/en/ .

Resources: http://www.teachmathematics.net/page/11533/trig-calculator

Click here to view page

In this practical activity students either make or borrow from their Science Department a water-powered rocket and launch it. They calculate the height reached by the water-powered rocket using right-angled trigonometry.

Resources: http://www.teachmathematics.net/page/7353/3-2-1-blast-off

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This problem involves the use of trigonometric ratios to find out a rule about the angles in a 3 by 1 rectangle divided into triangles. There are many ways of solving this problem.

Resources: http://nrich.maths.org/1955

Resources, Links and Support

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Pythagoras

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Standard Form

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Topic: Trigonometry


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