































## Additional Pure

### Vector product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\hat{\mathbf{n}}$ , where  $\mathbf{a}, \mathbf{b}, \hat{\mathbf{n}}$ , in that order form a right-handed triple.

### Surfaces

For 3-D surfaces given in the form  $z = f(x, y)$ , the Hessian Matrix is given by  $H = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$ .

At a stationary point of the surface:

1. if  $|H| > 0$  and  $f_{xx} > 0$ , there is a (local) minimum;
2. if  $|H| > 0$  and  $f_{xx} < 0$ , there is a (local) maximum;
3. if  $|H| < 0$  there is a saddle-point;
4. if  $|H| = 0$  then the nature of the stationary point cannot be determined by this test.

The equation of a tangent plane to the curve at a given point  $(x, y, z) = (a, b, f(a, b))$  is

$$z = f(a, b) + (x - a)f_x(a, b) + (y - b)f_y(a, b).$$

### Calculus

Arc length  $s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

$$s = \int_a^b \sqrt{x^2 + y^2} dx$$

Surface area of revolution  $S_x = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$   $S_y = 2\pi \int_c^d x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$

$$S_x = 2\pi \int_a^b y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$S_y = 2\pi \int_c^d x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$