

## A Level Further Mathematics B (MEI)

### Y434 Numerical Methods

#### Sample Question Paper

Version 3.0

## Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

**You must have:**

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

**You may use:**

- a scientific or graphical calculator



### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- In each question you must show sufficient detail of the method(s) which you are using.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

- 1 (i) Solve the following simultaneous equations.

[2]

- (ii) The coefficient 0.99 is correct to two decimal places. All other coefficients in the equations are exact. With the aid of suitable calculations, explain why your answer to part (i) is unreliable. [3]

- 2 The following spreadsheet printout shows the bisection method being applied to the equation  $f(x) = 0$ , where  $f(x) = e^x - x^2 - 2$ . Some values of  $f(x)$  are shown in columns B and D.

	A	B	C	D	E	F	G
1	$a$	$f(a)$	$b$	$f(b)$	$(a+b)/2$	$f((a+b)/2)$	mpe
2	1	-0.28172	2	1.389056	1.5	0.231689	0.5
3	1	-0.28172	1.5	0.231689	1.25	-0.072157	0.25
4	1.25	-0.07216	1.5	0.231689	1.375	0.064452	0.125
5	1.25	-0.07216	1.375	0.064452	1.3125	-0.007206	0.0625
6	1.3125	-0.00721	1.375	0.064452	1.34375	0.027728	0.03125

- (i) The formula in cell A3 is  $\text{=IF}(F2>0, A2, E2)$ . State the purpose of this formula. [1]

- (ii) The formula in cell C3 is  $\text{=IF}(F2>0, \dots, \dots)$ . What are the missing cell references? [1]

- (iii) In which row is the magnitude of the maximum possible error (mpe) less than  $5 \times 10^{-7}$  for the first time? [2]

- 3 The equation  $\sinh x + x^2 - 1 = 0$  has a root,  $\alpha$ , such that  $0 < \alpha < 1$ .

- (i) Verify that the iteration  $x_{r+1} = \frac{1 - \sinh x_r}{x_r}$  with  $x_0 = 1$  fails to converge to this root. [2]

- (ii) Use the relaxed iteration  $x_{r+1} = (1 - \lambda)x_r + \lambda \left( \frac{1 - \sinh x_r}{x_r} \right)$  with  $\lambda = \frac{1}{4}$  and  $x_0 = 1$  to find  $\alpha$  correct to 6 decimal places. [2]

4 The table below gives values of a function  $y = f(x)$ .

$x$	0.2	0.3	0.35	0.4	0.45	0.5	0.6
$f(x)$	0.789 922	0.754 628	0.749 199	0.749 997	0.756 257	0.767 523	0.804 299

(i) Calculate three estimates of  $\frac{dy}{dx}$  at  $x = 0.4$  using the central difference method. [4]

(ii) State the value of  $\frac{dy}{dx}$  at  $x = 0.4$  to an appropriate degree of accuracy. Justify your answer. [2]

5 A vehicle is moving in a straight line. Its velocity at different times is recorded and shown below. The velocities are recorded to 5 significant figures and the times may be assumed to be exact.

Time ( $t$ seconds)	5	10	12	15
Velocity ( $v$ metres per second)	5.1250	11.000	14.000	18.375

It is suggested initially that a quadratic model may be appropriate for this situation.

(i) Given that the vehicle is modelled as a particle with constant mass, what assumption about the net force acting on the vehicle leads to a quadratic model? [1]

(ii) Find Newton's interpolating polynomial of degree 2 to model this situation. Write your answer in the form  $v = at^2 + bt + c$ . [4]

(iii) Comment on whether this model appears to be appropriate. [2]

(iv) Use this model to find an approximation to the distance travelled over the interval  $5 \leq t \leq 15$ . [2]

Further investigation suggests that a cubic model may be more appropriate.

(v) What technique would you use to fit a cubic model to the data in the table? [1]

6 The secant method is to be used to solve the equation  $x - \ln(\cos x) - 1 = 0$ .

- (i) Show that starting with  $x_0 = -1$  and  $x_1 = 0$  leads to the method failing to find the root between  $x = 0$  and  $x = 1$ . [3]

The spreadsheet printout shows the application of the secant method starting with  $x_0 = 0$  and  $x_1 = 1$ . Successive approximations to the root are in column E.

	A	B	C	D	E
1	$x_n$	$f(x_n)$	$x_{n+1}$	$f(x_{n+1})$	$x_{n+2}$
2	0	-1	1	0.6156265	0.6189549
3	1	0.6156265	0.6189549	-0.175846	0.7036139
4	0.6189549	-0.1758461	0.7036139	-0.025245	0.7178053
5	0.7036139	-0.0252451	0.7178053	0.0011619	0.7171808
6	0.7178053	0.0011619	0.7171808	-7.4E-06	0.7171848
7	0.7171808	-7.402E-06	0.7171848	-2.16E-09	0.7171848
8	0.7171848	-2.16E-09	0.7171848	3.997E-15	0.7171848

- (ii) What feature of column B shows that this application of the secant method has been successful? [1]
- (iii) Write down a suitable spreadsheet formula to obtain the value in cell E2. [2]

Some analysis of convergence is carried out, and the following spreadsheet output is obtained.

	A	B	C	D	E	F	G	H
1	$x_n$	$f(x_n)$	$x_{n+1}$	$f(x_{n+1})$	$x_{n+2}$			
2	0	-1	1	0.6156265	0.6189549	0.084659	0.167629	1.980053
3	1	0.6156265	0.6189549	-0.175846	0.7036139	0.0141913	-0.044	-3.10054
4	0.6189549	-0.1758461	0.7036139	-0.025245	0.7178053	-0.0006244	-0.00633	10.13727
5	0.7036139	-0.0252451	0.7178053	0.0011619	0.7171808	3.953E-06	0.000292	73.83899
6	0.7178053	0.0011619	0.7171808	-7.4E-06	0.7171848	1.154E-09	-1.8E-06	
7	0.7171808	-7.402E-06	0.7171848	-2.16E-09	0.7171848	-2.109E-15		
8	0.7171848	-2.16E-09	0.7171848	3.997E-15	0.7171848			

The formula in cell F2 is  $=E3 - E2$ . The formula in cell G2 is  $=F3/F2$ . The formula in cell H2 is  $=F3/(F2^2)$ .

- (iv) (A) Explain the purpose of each of these three formulae. [3]
- (B) Explain the significance of the values in columns G and H in terms of the rate of convergence of the secant method. [2]
- (v) Explain why the values in cells F6 and F7 are not 0. [1]

[Question 7 is printed overleaf.]

7 Fig. 7 shows the graph of  $y = f(x)$  for values of  $x$  from 0 to 1.

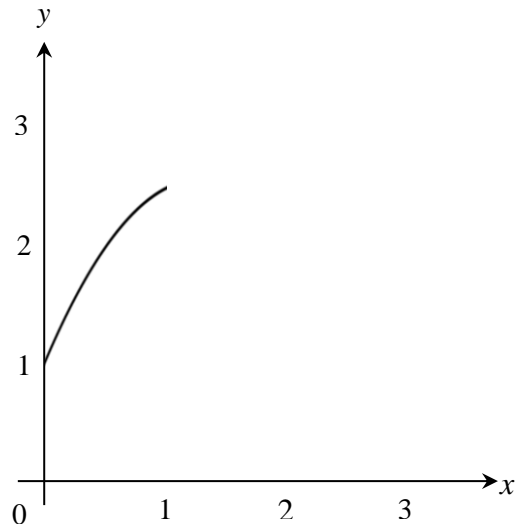


Fig. 7

The following spreadsheet printout shows estimates of  $\int_0^1 f(x)dx$  found using the midpoint and trapezium rules for different values of  $h$ , the strip width.

	A	B	C
1	$h$	Midpoint	Trapezium
2	1	1.99851742	1.751283839
3	0.5	1.9638591	1.874900631
4	0.25	1.95135259	1.919379864
5	0.125	1.94682102	1.935366229

- (i) Without doing any further calculation, write down the smallest possible interval which contains the value of the integral. Justify your answer. [2]
- (ii) (A) • Calculate the ratio of differences,  $r$ , for the sequence of estimates calculated using the trapezium rule.
- Hence suggest a value for  $r$  correct to 2 significant figures.
  - Comment on your suggested value for  $r$ . [3]
- (B) • Use extrapolation to find an improved approximation to the value of the integral.
- State the value of the integral to two decimal places.
  - Explain why this precision is secure. [4]

Using a similar approach with the sequence of estimates calculated using the midpoint rule, the approximation to the integral from extrapolation was found to be 1.944 27 correct to 5 decimal places.

(iii) Andrea uses the extrapolated midpoint rule value and the value found in part (ii) (B) to write down an interval which contains the value of the integral. Comment on the validity of Andrea's method. [2]

(iv) Use the values from the spreadsheet output to calculate an approximation to the integral using Simpson's rule with  $h = 0.125$ . Give your answer to 5 decimal places. [2]

Approximations to the integral using Simpson's rule are given in the spreadsheet output below. The number of applications of Simpson's rule is given in column N.

N	O	P	Q
$n$	Simpson	differences	ratio
1	1.91610623	0.01810005	0.3584931
2	1.93420628	0.00648874	0.3556525
4	1.94069502	0.00230774	0.3544828
8	1.94300275	0.00081805	0.3539885
16	1.94382081	0.00028958	0.3537638
32	1.94411039	0.00010244	0.3536568
64	1.94421283	3.623E-05	
128	1.94424906		

(v) Use the spreadsheet output to find the value of the integral as accurately as possible. Justify the precision quoted. [6]

**END OF QUESTION PAPER**

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**...day June 20XX – Morning/Afternoon**

**A Level Further Mathematics B (MEI)**

**Y434 Numerical Methods**

**SAMPLE MARK SCHEME**

**Duration:** 1 hour 15 minutes

**MAXIMUM MARK    60**



**This document consists of 12 pages**

## Text Instructions

## 1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction <b>In this question you must show detailed reasoning</b> appears in the question.

## 2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

### M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

### A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

### B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team

Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.). When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.
- k Anything in the mark scheme which is in square brackets [...] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like.

Question		Answer	Marks	AOs	Guidance	
1	(i)	$x = 101$ $y = -100$	<b>B1</b> <b>B1</b> [2]	<b>1.1</b> <b>1.1</b>		
1	(ii)	Substitution of 0.985 and 0.995 to obtain $-200 < y < -66.67$  $67.67 < x < 201$  E.g. calculation involves subtraction of nearly equal numbers	<b>M1</b>  <b>M1</b>  <b>A1</b>  [3]	<b>1.1</b>  <b>1.1</b>  <b>2.4</b>	For either condition  For either condition  E.g. equations represent lines which are almost parallel, so a small change in gradient affects the point of intersection dramatically	Or maximum possible error is 100% Or full argument based on determinants or gradients Do not allow general statements such as “the problem is ill-conditioned”
2	(i)	To determine whether 1 is replaced by 1.5	<b>B1</b>  [1]	<b>2.4</b>	Accept explanation in terms of method	
2	(ii)	E2, C2	<b>B1</b>  [1]	<b>2.2a</b>	In correct order	The resulting formula must be fully correct, so that it could be copied to produce the results in column C.
2	(iii)	$0.5 \times 0.5^n < 5 \times 10^{-7}$ soi  Row 22	<b>M1</b>  <b>A1</b>  [2]	<b>3.1a</b>  <b>1.1</b>	So $n > 19.93$ i.e. 20 cells further down	NB $\ln 10^{-6} \div \ln 0.5$  If <b>M0</b> , <b>B2</b> for correct answer www

Question		Answer	Marks	AOs	Guidance								
3	(i)	$\frac{1 - \sinh 1}{1}$ soi - 0.175..., - 6.7128..., - 61.442... so fails	<b>M1</b>  <b>A1</b>  <b>[2]</b>	<b>1.1</b>  <b>2.2a</b>	At least 3 iterates BC  Alternative Method $\alpha \approx 1$ and substitution in $g'(x) = \frac{-x \cosh x - 1 + \sinh x}{x^2}$ $ g'(x)  \approx 1.368 > 1$ so iteration fails								
3	(ii)	0.706199, 0.612353, 0.601606, 0.601403, 0.601402,.. 0.601402 cao	<b>M1</b> <b>A1</b> <b>[2]</b>	<b>1.1</b> <b>1.1</b>	Must see at least 3 iterates BC Answer only does not score								
4	(i)	Evidence of correct use of formula  <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th><math>h</math></th> <th>0.2</th> <th>0.1</th> <th>0.05</th> </tr> </thead> <tbody> <tr> <td><math>\frac{dy}{dx}</math></td> <td>0.03594..</td> <td>0.06447..</td> <td>0.07058</td> </tr> </tbody> </table>	$h$	0.2	0.1	0.05	$\frac{dy}{dx}$	0.03594..	0.06447..	0.07058	<b>M1</b>  <b>A1</b> <b>A1</b> <b>A1</b> <b>[4]</b>	<b>1.1</b>  <b>1.1</b> <b>1.1</b>	Answers to 3 or more significant figures
$h$	0.2	0.1	0.05										
$\frac{dy}{dx}$	0.03594..	0.06447..	0.07058										
4	(ii)	0.07 is reasonable  Accept any correct answer based on differences	<b>B1</b>  <b>B1</b>  <b>[2]</b>	<b>1.1</b>  <b>2.4</b>	E.g. The difference between successive approximations has reduced by roughly one fifth, so further reduction in $h$ is unlikely to generate an answer larger than 0.075								

Question		Answer	Marks	AOs	Guidance
5	(i)	[net] force is a linear function [of time]	<b>B1</b> [1]	<b>2.4</b>	
5	(ii)	Uses values at 5, 10, 15  First differences: 5.875, 7.375 Second difference: 1.5 $5.125 + \frac{(t-5)}{5} \times 5.875 + \frac{(t-5)(t-10)}{5^2 \times 2!} \times 1.5$ $v = 0.03t^2 + 0.725t + 0.75$	<b>M1</b> <b>M1</b> <b>M1</b> <b>A1</b> [4]	<b>3.3</b> <b>1.2</b> <b>1.1</b> <b>3.3</b>	soi  FT differences
5	(iii)	f(12) = 13.77  Quite close to 14.000, so the model may be a reasonable approximation	<b>B1</b> <b>B1</b> [2]	<b>1.1</b> <b>3.5a</b>	
5	(iv)	$\int_5^{15} v dt = \left[ \frac{0.03t^3}{3} + \frac{0.725t^2}{2} + 0.75t \right]_5^{15}$ 112.5	<b>M1</b> <b>A1</b> [2]	<b>3.4</b> <b>1.1</b>	Attempt at integration must be seen.
5	(v)	Lagrange's form of the interpolating polynomial	<b>B1</b>  [1]	<b>3.5c</b>	accept $\frac{(x-5)(x-10)(x-15)}{(12-5)(12-10)(12-15)} \times 14.000$

Question		Answer	Marks	AOs	Guidance	
6	(i)	$\alpha = \frac{-1 \times -1 - 0 \times -1.38347}{-1 - -1.38347}$ $= 2.601636$ $\cos 2.601636 = -0.8577\dots$ and $\ln(-0.8577\dots)$ is undefined so the method breaks down	<b>M1</b>  <b>A1</b> <b>A1</b>  <b>[3]</b>	<b>1.1</b>  <b>1.1</b> <b>2.4</b>		
6	(ii)	The values are approaching zero	<b>B1</b> <b>[1]</b>	<b>2.3</b>		“getting smaller” is insufficient
6	(iii)	$= (A2 * D2 - C2 * B2) / (D2 - B2)$ oe	<b>B2</b>  <b>[2]</b>	<b>3.1a</b> <b>1.1</b>	<b>B1</b> if * omitted	must show application of secant method
6	(iv)	(A) <ul style="list-style-type: none"> <li>= E2 – E1 finds the differences between successive estimates of the root</li> <li>= F2/F1 finds the ratio of differences of these estimates</li> <li>= F2/F1<sup>2</sup> finds the ratio of differences of each estimate to the square of the previous estimate</li> </ul>	<b>B1</b>  <b>B1</b> <b>B1</b>  <b>[3]</b>	<b>1.2</b>  <b>2.5</b> <b>2.4</b>		
6	(iv)	(B) <ul style="list-style-type: none"> <li>The values in column G suggest that convergence [of secant method] is faster than 1<sup>st</sup> order</li> <li>The values in column H suggest that the convergence [of secant method] is slower than 2<sup>nd</sup> order</li> </ul>	<b>B1</b>  <b>B1</b>  <b>[2]</b>	<b>2.2b</b>  <b>2.2b</b>	If <b>B0B0</b> , <b>SC1</b> for “neither 1 <sup>st</sup> order nor 2 <sup>nd</sup> order convergence”	
6	(v)	The values in E6, E7 and E8 are stored to greater accuracy than is displayed oe	<b>B1</b>  <b>[1]</b>	<b>2.4</b>		



Question		Answer	Marks	AOs	Guidance									
7	(i)	$1.9353662285 \leq I < 1.946821025$ The curve is concave down over this interval, so the Trapezium Rule generates an underestimate, and the Midpoint Rule generates an over-estimate	<b>B1</b>  <b>B1</b>  <b>[2]</b>	<b>2.2a</b>  <b>2.4</b>										
7	(ii)	(A) <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;">diffs</td> <td style="width: 15%;">ratio</td> </tr> <tr> <td>0.12361679</td> <td>0.359815</td> </tr> <tr> <td>0.04447923</td> <td>0.359412</td> </tr> <tr> <td>0.01598637</td> <td></td> </tr> </table> $r \approx 0.36$ Generally $r$ should be close to 0.25, as the Trapezium Rule is a second order method	diffs	ratio	0.12361679	0.359815	0.04447923	0.359412	0.01598637		<b>M1</b>      <b>A1</b> <b>A1</b>  <b>[3]</b>	<b>1.1</b>      <b>1.1</b> <b>2.4</b>	Differences and attempt to find ratio	Condone minor arithmetic slips
diffs	ratio													
0.12361679	0.359815													
0.04447923	0.359412													
0.01598637														
7	(ii)	(B) $1.93536623 + 0.01598637 \times \frac{0.36}{0.64} = 1.9443(58563)$ 1.94 This agrees with $T_8$ to two decimal places	<b>M1</b> <b>A1</b> <b>A1</b> <b>A1</b>  <b>[4]</b>	<b>1.1</b> <b>1.1</b> <b>1.1</b> <b>3.2b</b>	A0 for incomplete extrapolation									
7	(iii)	It is not appropriate. Extrapolation is an approximate method (in this case it has led to the values “swapping places”.)	<b>B1</b> <b>B1</b>  <b>[2]</b>	<b>2.3</b> <b>2.4</b>										
7	(iv)	$\frac{2 \times 1.95135259 + 1.91937986}{3} = 1.940695$	<b>M1</b> <b>A1</b>  <b>[2]</b>	<b>1.1</b> <b>1.1</b>										

Question		Answer	Marks	AOs	Guidance
7	(v)	$1.94424906 + 0.00003623 \times \frac{0.354}{1 - 0.354}$ <p>= 1.944268919 to 1.944268568            1.9443 looks secure, 1.94427 or 1.944269 is possible.            Sensible reasoning consistent with their answer.</p>	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>[6]</b></p>	<p><b>3.1a</b></p> <p><b>1.1</b></p> <p><b>2.1</b></p> <p><b>1.1</b></p> <p><b>3.2b</b></p> <p><b>2.4</b></p>	<p>Extrapolation using most accurate Simpson value</p> <p>Use of <math>r = 0.354</math> or <math>0.35</math></p> <p>Extrapolation to infinity</p> <p>Accept argument based on comparison with 1.94424906 for 4 sf</p> <p>or on further improvement due to satisfactory demonstration of convergence of <math>r</math> for 5 or 6 sf</p>

Question	AO1	AO2	AO3(PS)	AO3(M)	Totals
<b>1i</b>	2	0	0	0	2
<b>1ii</b>	2	1	0	0	3
<b>2i</b>	0	1	0	0	1
<b>2ii</b>	0	1	0	0	1
<b>2iii</b>	2	0	0	0	2
<b>3i</b>	1	1	0	0	2
<b>3ii</b>	2	0	0	0	2
<b>4i</b>	4	0	0	0	4
<b>4ii</b>	0	2	0	0	2
<b>5i</b>	0	1	0	0	1
<b>5ii</b>	2	0	0	2	4
<b>5iii</b>	1	0	0	1	2
<b>5iv</b>	1	0	0	1	2
<b>5v</b>	0	0	0	1	1
<b>6i</b>	2	1	0	0	3
<b>6ii</b>	0	1	0	0	1
<b>6iii</b>	1	1	0	0	2
<b>6ivA</b>	1	2	0	0	3
<b>6ivB</b>	0	2	0	0	2
<b>6v</b>	0	1	0	0	1
<b>7i</b>	0	2	0	0	2
<b>7iiA</b>	2	1	0	0	3
<b>7iiB</b>	3	0	1	0	4
<b>7iii</b>	0	2	0	0	2
<b>7iv</b>	2	0	0	0	2
<b>7v</b>	2	2	2	0	6
<b>Totals</b>	30	22	3	5	60

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates
January 2025	3.0	From 2025 E1/G1 marks will be annotated using M1, A1, or B1. The previous E1/G1 annotation awarding criteria has always existed as a sub-section within M (method), A (accuracy), or B (independent). Removal of E1/G1 annotation will increase marking consistency and teacher comprehension of our approach to marking.

## A Level Further Mathematics B (MEI)

### Y434 Numerical Methods

Printed Answer Booklet

Version 2

## Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

**You must have:**

- Question Paper Y434 (inserted)
- Formulae Further Mathematics B (MEI)

**You may use:**

- a scientific or graphical calculator



First name

Last name

Centre  
number

Candidate  
number

### INSTRUCTIONS

- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- In each question you must show sufficient detail of the method(s) which you are using.
- Final answers should be given to a degree of accuracy appropriate to the context.

### INFORMATION

- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

<b>1 (i)</b>	
<b>1 (ii)</b>	

**2 (i)****2 (ii)****2 (iii)** =IF(2>7, ... , ... )

<b>3 (i)</b>	
<b>3 (ii)</b>	





<b>5 (i)</b>	
<b>5 (ii)</b>	

<b>5 (iii)</b>	
<b>5 (iv)</b>	
<b>5 (v)</b>	

<b>6 (i)</b>	
<b>6 (ii)</b>	
<b>6 (iii)</b>	

<b>6 (iv) (A)</b>	
<b>6 (iv) (B)</b>	
<b>6 (v)</b>	

<b>7 (i)</b>	
<b>7 (ii)</b> <i>(A)</i>	
<b>7 (ii)</b> <i>(B)</i>	



