Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Advanced GCE A2 7895-8

Advanced Subsidiary GCE AS 3895-8

OCR Report to Centres June 2017

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About this Examiner Report to Centres

This report on the 2017 Summer assessments aims to highlight:

- areas where students were more successful
- main areas where students may need additional support and some reflection
- points of advice for future examinations

It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

The report also includes:

- An invitation to get involved in Cambridge Assessment's research into **how current reforms are affecting schools and colleges**

- Links to important documents such as **grade boundaries**

- A reminder of our **post-results services** including Enquiries About Results

- **Further support that you can expect from OCR**, such as our Active Results service and CPD programme

- A link to our handy Teacher Guide on **Supporting the move to linear assessment** to support you with the ongoing transition

Understanding how current reforms are affecting schools and colleges

Researchers at Cambridge Assessment[1] are undertaking a research study to better understand how the current reforms to AS and A levels are affecting schools and colleges.
If you are a Head of Department (including deputy and acting Heads), then we would be very grateful if you would take part in this research by completing their survey. If you have already completed the survey this spring/summer then you do not need to complete it again.
The questionnaire will take approximately 15 minutes and all responses will be anonymous.
To take part, please click on this link: https://www.surveymonkey.co.uk/r/KP96LWB

[1] Cambridge Assessment is a not-for-profit non-teaching department of the University of Cambridge, and the parent organisation of OCR, Cambridge International Examinations and Cambridge English Language Assessment.

Grade boundaries

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Enquiry About Results

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It allows you to:

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September 2017 sees the introduction of linear assessment for AS and A level mathematics and further mathematics. To help you navigate the changes and to support you with areas of difficulty, download our helpful Teacher guide: http://www.ocr.org.uk/Images/338121-moving-from-modular-to-linear-qualifications-teachers-guide.pdf

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OCR REPORT TO CENTRES

4751 Introduction to Advanced Mathematics (C1)

General Comments:

In general candidates were confidently applying the basic techniques required, with many candidates gaining most of the marks available in section A.

All questions were found to be accessible, with candidates rarely omitting to answer a question or part question.

It is pleasing to note that there were fewer errors in arithmetic this year, but as usual, questions involving fractions (such as question 4) saw more than occasional errors, whilst manipulating surds (such as in question 7) was another topic where candidates failed to gain all the available marks. Examiners were surprised how many candidates struggled to simplify correctly the numerical values in question 2(ii).

Comments on Individual Questions:

Section A

Question No. 1
Most candidates coped well with this question, usually scoring full marks. Errors included drawing a line with the wrong gradient, ie +2 or - 1/2 and marks were also dropped due to inaccurately drawn lines based upon plotting one point and roughly estimating where a gradient of -2 would be. It would have been much more accurate for those candidates to plot at least two points when drawing a straight line. If plotting points it would be wise for candidates to only mark a small point, not a large circular blob encompassing a complete square as some candidates did. It was sad to see some candidates not using a ruler and therefore drawing wobbly freehand lines.

Question No. 2
Not many candidates dropped marks in the first part. Those who did usually lost out due to their inability to convert a mixed number into an improper fraction, preventing them from scoring any of the marks. Candidates scoring 0 often seemed to have little idea with indices, but these were a minority. Some candidates reached $\dfrac{1}{\left(1\frac{7}{9}\right)^{\frac{1}{2}}}$, gaining a mark for this, but then did not know how to proceed with their triple-decker fraction. In the second part, the vast majority of candidates coped well, the main mistakes were usually due to the misapplication of the rules of indices, adding when the powers should be multiplied. What was concerning was the minority of candidates who could not multiply or divide the numerical values forming the coefficient.

Question No. 3
This was a straight-forward inequality with very few mistakes made. The most common mistake seen involved mistakenly multiplying out the bracket to give $5x-3$ rather than $5x-15$. Generally, candidates worked very well with the inequality sign in this question and most, if the need arose, remembered to change the sign of the inequality when dividing or multiplying by a negative value.

Question No. 4
Candidates coped very well with equation and fraction manipulation. Both method marks were nearly always earned. A variety of methods were used with the substitution of $x=2y+4$ into the first equation being the most common. Some multiplied both equations, in order to be able to use elimination. Where the second equation was multiplied by 2 there were some errors in subtracting the equations. Rearranging both equations to get $x=...$ or $y=...$ and then equating the results was

also fairly common and, even though this resulted in fractions, was usually successful. However, handling the signs when rearranging the second equation was a source of error. A minority of candidates stopped after finding one of the values (usually *y*) and failed to find the coordinates, as requested in the question. It should be noted that very few candidates checked their answers and it is advisable to do so in questions of this nature.

Question No. 5
There were a small number of candidates who incorrectly worked out the centre of the circle, usually giving the centre with incorrect signs. A number of candidates gave the radius as either 5 or 25 but most candidates scored full marks here. The vast majority of candidates found the correct equation of the required line, with those who dropped marks usually because they chose 5 as the gradient and not -5. Some of these candidates clearly misunderstood the concept of $y = mx + c$ as they clearly believed that the coefficient of *x* was the gradient no matter what side of the equation the *x* term appeared on. Only a small minority used a gradient based on the negative reciprocal.

Question No. 6
There were many candidates who found the new subject both efficiently and accurately. It was rare to find a candidate who didn't know to square both sides straight away but there were a very small minority who went off the rails at that point, not coping with the *a* + *b* as a denominator. A small minority of candidates solved for *a* instead of *b*. There were a handful of candidates who insisted on using a diagonal fraction line instead of a horizontal one and this lead to algebraic missteps when manipulating the algebra. A common error was to take the correct answer of $b = \frac{V - r^2 a}{r^2}$ and cancel this incorrectly to $b = V - a$. .

Question No. 7
In the first part, the vast majority of candidates understood the need to multiply the numerator and denominator by $\left(3 - \sqrt{7}\right)$, however a few tried to multiply both parts of the fraction by $\sqrt{7}$, or by $\left(3 + \sqrt{7}\right)$, or to 'cancel' the $\sqrt{7}$ in the numerator and denominator. The most common error was in determining $-2\sqrt{7} \times \sqrt{7}$ which commonly retained a multiple of $\sqrt{7}$. In the second part, most candidates could simplify $\sqrt{98}$ to $7\sqrt{2}$ (so scoring at least one mark) but many had difficulties with $\frac{12}{\sqrt{2}}$ with some multiplying the $\sqrt{2}$ by the $\sqrt{98}$ or leaving their answer as $\frac{26}{\sqrt{2}}$.

Question No. 8
This problem-solving binomial expansion question discriminated extremely well. Some candidates misunderstood the concept of the constant term being 32 and this was then applied incorrectly in a variety of ways, either being assigned to the value of *a* or to $^5C_3$. Another common error was to work with the term bx^3 rather than $(bx)^3$ sometimes leading to an answer $b = 27$ or -27. Having *x*'s on only one side of an equation and then ignoring them until the last statement was also common, as was a correct $b^3 = -27$ followed by the loss of the negative sign, leading to $b = 3$. Candidates' trialling factors of -108 (with no consideration of the 32) often reached correct values for a and b but were not awarded full marks since this went against the rubric on the front cover which requires that candidates show sufficient detail of the working to indicate that a correct method has been used. However even the poorest candidates usually gained a mark for identifying the binomial coefficient 10.

Question No. 9
The majority of candidates that attempted this standard proof question gained full marks, showing the needed interim step(s) to obtain the corresponding accuracy marks. A minority chose wrong expressions for the three integers (e.g. *n*, 2*n*, 3*n*). Unfortunately candidates missing the middle term of 4*n* when squaring the (*n* + 2) term was seen quite often. Some candidates considered the

first term squared minus the last term squared and then conveniently ignored the negative signs. A handful of candidates attempted an entirely numerical approach.

Section B

Question No. 10
(i) Many obtained two marks here without any difficulty. Candidates who used less formal notation often lost marks due to missing brackets or confusion about whether they were working with AB or AB^2. A few candidates confused lengths and gradients.

(ii) This was completed well by the majority of candidates. A few quoted the gradient formula incorrectly or had difficulty simplifying the gradient accurately, but were then able to find the associated perpendicular gradient and use the equation of a straight line well.

(iii) This presented a challenge to a significant number of candidates, with those who chose not to use a vector related method often getting bogged down with complicated algebra. A common error was to not appreciate the importance of the letter order ABCD, and instead give ACDB or ACBD, which earned partial credit but affected the difficulty of part (iv), so limiting the marks available there.

(iv) This part required candidates to apply some reason and insight rather than just applying well-drilled techniques. Candidates would have found it helpful to sketch a diagram with their D marked, to ensure that they were comparing E to the correct line (AD). Most started by finding the equation of AD and a good number successfully used this to decide whether E was above or below AD. Some who substituted $x = 8$ into AD found $y = 3\frac{5}{7}$ but did not prove that this is less than 3.8. Some compared with CD rather than AD. Some candidates used other methods, often efficiently, such as showing that the gradient of AE was greater than the gradient of AD.

Question No. 11
(i) This was completed accurately with many candidates able to sketch the graph with little preliminary working. The common errors were having a curve which stopped at the x-axis at one or both ends, or a curve which flicked out at an end towards a turning point, or either not marking the y intercept or calling it 15. A few candidates obtained y-intercepts of -30, or x-intercepts with incorrect signs/values. In a few cases this led to a negative cubic rather than a positive cubic. Very few failed to gain any marks. Some curves were a poor shape because candidates tried to make the scales on both axes the same.

(ii) Some candidates weren't sure which way to go so attempted pages of different combinations of brackets. Most who did know what to do got the full marks. Those who put $(x+3)$ into the expanded $f(x)$ gave themselves more long-winded expansions to do but most got there.

(iii) Where candidates set out a well-organised solution, they were able to progress directly to the fully factorised expression (with $g(x) = 6$ rather than 0 providing extra challenge). The majority were able to find the correct quadratic factor following division by $(x + 2)$, with a few using synthetic division and a sizeable minority finding the solution by inspection. At this stage most then found the correct final solution, although some had difficulty squaring 17, and a few stopped at attempts to factorise. Most earned the mark for showing that $g(-2) = 6$ by substituting $x = -2$ into the equation; those who relied on their division by $(x + 2)$ often failed to say what this showed.

Question No. 12
(i) Most completed the square correctly. Some candidates did not take notice of the 'hence' in the question and used the discriminant, which did not gain the final mark.

(ii) Finding the coordinates of the points of intersection of the two curves was done well. A few forgot to work out both coordinates, and some, having found x to be 6 or -2, put (6, 0) and (-2, 0).

(iii) This question posed problems for some candidates who were unsure of an appropriate strategy. However, a good number of the candidates found the set of values of k successfully. A significant minority, while knowing that they needed to use the discriminant, made arithmetical and/or algebraic errors, often caused by poor use of brackets. Few used the completing the square method.

4752 Concepts for Advanced Mathematics (C2)

General Comments:

The paper was accessible to most candidates, but the questions contained enough stretch and challenge material to discriminate across the full ability range. Some candidates demonstrated a good understanding of the syllabus material and proficiency in the appropriate techniques, but lost a significant number of marks through poor (GCSE level) algebra and arithmetical slips.

A number of candidates still lose marks through working with prematurely rounded values, and then over-specifying the final result.

“Show that” requests are often not treated with sufficient rigour and a failure to show sufficient detail can often prove costly.

Most candidates presented their work neatly and clearly, but in a few cases work was very difficult to follow, with evidence of mistakes introduced when the candidate had misread their own work, perhaps because a minus sign was not clear or because a figure had been scribbled so casually as to be almost illegible. Candidates should understand the importance of presenting a clear mathematical argument, especially when there is a “show that” request in the question.

Comments on Individual Questions:

Question No. 1
Part (i)
This was very done well. A small minority of candidates failed to score, usually through misusing formulae associated with arithmetic or geometric progressions. A small number of candidates demonstrated the correct method, but slipped up with arithmetic.
Part (ii)
This was done very well, too. However, some candidates failed to appreciate that d had to be negative, and a few interchanged a and d.

Question 2
Part (i)
Most candidates successfully integrated and went on to obtain the correct answer. A few spoiled this by leaving “$+ c$” in the final answer, and a small number either differentiated or simply evaluated the integrand.
Part (ii) Nearly all candidates achieved the method mark by integrating, but a surprising number omitted the constant of integration thereby losing an easy mark.

Question 3
Part (i)
Most knew what to do, but many slipped up by making a sign error in the numerator or by working with a rounded or truncated value of $\log_{10}0.2$, thus losing the accuracy mark.
Part (ii)
Nearly all candidates correctly identified a suitable point on the curve. A few guessed wrongly and placed C to the right of B, and a very small number placed C off the curve altogether.

Question 4
Most candidates were familiar with this sort of question and obtained the first four marks without difficulty. A few slipped up with the arithmetic, and a similar number found the equation of the tangent. A very small number of candidates integrated or went straight to working with $y = mx + c$.

Question 5
Part (i)
This caused difficulties for many. Far too many candidates did not seem to be familiar with the correct terminology, and attempted to describe what was going on by using an equation or by a (usually long-winded) sentence. “Enlargement”, “transformation” and “translation” were often seen. Similarly, a significant number of candidates ignored the request for a single transformation and described two, usually a stretch and a translation.
Part (ii)
As with part (i), many candidates opted for more general explanations. Slightly more candidates were successful with part (ii) than part (i), but once again many candidates ignored the request for a single transformation.

Question 6
The vast majority of candidates tackled this question successfully. A few slipped up with the arithmetic in finding c, and a small minority worked with $y = mx + c$ with $m = 12x^3 – 7$ and failed to score.

Question 7
Part (i)
Most candidates scored full marks with this part of the question, although the quality of the sketches were variable. A few drew $y = 2x$ or $y = x^2$, and some candidates marked the y-intercept as (0, 2), losing an easy mark.
Part (ii)
Over half the candidates failed to score on this question, with difficulties seen by candidates attempting to combine the logarithms successfully. In attempting to make w the subject, candidates sometimes “divided by $\log_a$” or raised both sides to the power 10, and only a minority earning the method mark.

Question 8
This was done well by most candidates. A few slipped up with the first part, making sign or bracket errors, but most went on to find the correct values of $\sin x$. Nearly all worked with radians and found $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ successfully. Some gave the other two values in terms of π and lost accuracy, and a small number of candidates decided that the values associated with $\sin^{-1}(-⅓)$ had to be outside the range.

Question 9
Part (i)
Most candidates scored full marks here, but poor algebra let some candidates down. A wide variety of solutions were seen, some of which very elaborate.
Part (ii)
In spite of the correct expression being given in part (i), some candidates worked with an expression involving h, which inhibited much further progress. Some candidates worked with 800^{-r} and some disregarded π or treated it as a variable. The majority, however, differentiated successfully to obtain full marks.
Part (iii)
A sizeable minority of candidates failed to score any marks in this part, beginning with an inequality in the second derivative. A good number of candidates started on the right track by setting the first derivative to zero, but then failed to make progress. Only rarely did candidates successfully find r and A and then use the second derivative correctly to establish that they had indeed found the minimum surface area.

Question 10
Part (i)
This was very well done. A few candidates worked in radians and lost the accuracy mark. A small minority misquoted the Cosine Rule or mis-used Pythagoras.
Part (ii)

Over half of the candidates failed to score on this part. Most worked with a perpendicular from D to AE and presumed that by doing so they were either bisecting angle ADE or the length AE. Those who correctly worked with the Sine Rule to find angle DAE or angle DEA generally went on to score full marks, although a few found the base of their triangle instead of the height.

Part (iii)

Most candidates knew what to do here and successfully found the area of the triangle and the area of the sector. A minority left it at that or slipped up with the subtraction and lost an easy mark. A few candidates used $\theta = 116$ radians, thus losing the first two marks, or converted to radians and then worked with their rounded decimal value, thus losing the accuracy mark.

Part (iv)

A significant number of candidates were unable to marshal the information to form a coherent strategy for solving this problem, and thus failed to score.

A wide variety of approaches were seen, with many opting for convoluted methods which were often partially successful, but usually lost accuracy towards the end. Some candidates clearly knew that the best approach was to find the length BC, but even though this only involved GCSE level maths, were unable to do so.

Question 11

Part (i)

The majority of candidates gained full marks on this question. A few candidates listed all the terms and lost accuracy on the way, and a few misused the formulae.

Part (ii)

This part of the question was also very well done, but some candidates did not give enough detail to “show that” Arif and Bettina earned the same amount to the nearest £100. A common mistake was to write down Bettina’s earnings as £646 000 without showing the value before rounding.

Part (iii)

A minority of candidates presented clear, concise solutions to derive the inequality, and went on to obtain the correct value of *n*. Many candidates, however, did not attempt the derivation or started with the final statement. A few went on to obtain the correct value of *n,* although 25 was a common wrong answer.

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments:

The performance of candidates for this paper appears to have been broadly similar to recent years. Section A offered straightforward assessment of specification items, with the proof question found to be more accessible than in recent years, while the two section B questions contained more challenge.

The general standard of presentation of scripts was good, and there was little evidence that candidates had insufficient time to complete the examination. Pages 14-16 of the answer booklet were available for additional work, and used by a significant number of candidates, usually for multiple attempts at questions.

Comments on Individual Questions:

Section A

1 This was a straightforward test of the chain rule, in which over three quarters of the candidates scored full marks. Occasionally we saw a quotient rule used, which required to be simplified to gain full marks. Another occasional error was to get the wrong sign, e.g. $–12x^2(5 – 2x^3)^{–3}$.

2. Only a third of candidates scored all three marks here. The final mark required the domain of the graph to be correct – often the ‘v’ shape extended beyond $x = –1$ to 1. Other attempts bore no relation to the correct answer.

3(i) The first two marks for finding the inverse function were nearly always gained. However, accurate notation for the domain and the range was not often seen. Not many candidates scored full marks, with the domain proving particularly awkward to get right.

3(ii) Many candidates got all three marks here, though the structure of their ‘show’ was sometimes weak. Very occasionally we saw $\mathrm{fg}(x) = \ln(1 – x)^2$.

4(i) The implicit derivative here was a straightforward example, and virtually all the candidates got the derivative equation correct. However, simplifying the fractional expression to get the final mark was often missing or incorrect: in particular, many learners made mistakes when dividing 2 by 2/3.

4(ii) There were two easy marks here, and virtually all candidates achieved the ‘M’ mark for substituting for x and y in their derivative.

5(i) The initial temperature was almost always correct, but the boiling point was sometimes incorrect or missing, suggesting that the limit of $e^{–kt}$ as t tends to infinity was not known.

5(ii) Exponential growth and decay equations are usually well answered, and this was no exception, with most candidates scoring full marks.

5(iii) This question depended on the boiling point being correct, so the facility was lower. However, over half the candidates got full marks. The solution was made considerably harder if an inequality was used, as the working needed to show the reversing of the inequality signs.

6 Candidates scored full marks or zero marks in roughly equal numbers here. Most gave the first method shown in the mark scheme, namely solving $180(n – 2) = 155n$ to get $n = 14.4$, but we also

saw some examples of the second approach, finding the interior angles for 14 and 15 sides. By far the most common error was to solve $180(n - 2) = 155$, getting $n = 2.86$.

7(i) The majority of candidates scored both these marks. Occasionally they found dy/dx instead of dx/dy and lost a mark.

7(ii) Virtually all candidates wrote down a chain rule and scored 1 mark. Thereafter, many scored all the remaining marks. Errors were caused by muddling derivatives like dy/dx and dx/dy. Occasionally candidates attempted to use the derivative of arcsin x, though this was often incorrect through missing out the '½' factor.

Section B

8 Some candidates lost marks here from working in degrees rather than radians.

8(i) It is important that candidates state both the coordinates the right way round, so 'A = $\pi/2$ ' and 'B = ½ ' scored zero.

8(ii) Over half got full marks here. The quotient rule was well answered, and the subsequent simplification using $\sin^2 x + \cos^2 x = 1$ was good. The most common error was in the sign of the derivatives of sin x and cos x, which could fortuitously lead to the correct turning point – the final 'A' marks here being withheld in this case.

8(iii)(A) Most candidates used a substitution $u = 2 - \sin x$. Errors thereafter were $\mathrm{d}u/\mathrm{d}x = \cos x$, or getting the limits the wrong way round, perhaps under the misconception that the larger number must be the upper limit of the integral).

8(iii)(B) This question was the most demanding in the paper, with nearly half the candidates scoring zero marks. Often the problem seemed to lie with getting expressions consistent with the limits of the integral, either in terms of x or u.

9(i) Most candidates scored 2 or 3 here. We required to see $\mathrm{f}(-x) = (-x)^3 \exp(-x)^2$ in the proof that $\mathrm{f}(x)$ was an odd function, with the brackets correctly placed. For the 'B' mark describing the property of the graph, we needed to see reference to 'symmetry', 'half-turn,180° or order 2', and 'about the origin'.

9(ii) The main problem with the product rule here was to get the correct derivative of $\exp(-x^2)$. A common mistake was to think this is $\exp(-x^2)$. Having found the correct derivative and equated it to zero, the next issue was dividing through by, or factorising, $\exp(-x^2)$. After this, not many candidates got all three turning points, either omitting the origin or (–1.22, –0.41) or both. Also, evaluating the y-coordinates was sometimes done incorrectly. Where these issues were overcome, half of the candidates scored 6 or over; of these, half scored full marks.

9(iii) Very few candidates scored both marks here. Many omitted the inflection at the origin, and the graphs were often lacking the point symmetry stated in part (i).

9(iv)(A) Half the candidates scored these two marks. Using a substitution in this context was perhaps unexpected.

9(iv)(B) They could get three out of the four marks with a missing, or incorrect, value for k, but not many succeeded with this.

4754 Applications of Advanced Mathematics (C4)

General Comments:

On the whole candidates found Paper A this year slightly less demanding than last year and the standard of work in the majority of cases was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Paper B, the comprehension, was well understood and most candidates scored good marks here.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Questions 1, 2, 4, 7(ii) and 7(iii))
- Failure to include a constant of integration (Question 7(i))
- Failure to give clear descriptions in the comprehension paper (Questions 2 and 3)
- Inappropriate accuracy, for example in Question 3(ii), candidates either gave insufficient accuracy (answers to 1 decimal place) or they gave too much accuracy (answers to 3 or more decimal places). Candidates are reminded to give answers to 1 decimal place for questions involving trigonometry (Questions 2(ii) and 5)
- Failure to give exact answers when required (Question 7)
- Failure to give sufficient detail when verifying given results (Questions 6(ii), 7(i) and 7(iii))

Quite a number of candidates did not attempt some parts but there did not appear to be a shortage of time for either Paper.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper. Furthermore, centres are requested that Papers A and B are not attached to each other and are sent separately for marking.

Comments on Individual Questions:

Paper A

Question 1

Part (i) was answered extremely well with the vast majority of candidates correctly expressing $\frac{5-x}{(2-x)(1+x)}$ in partial fractions.

In part (ii) most candidates used their answer to part (i) in their attempt to find the binomial expansion of $\frac{5-x}{(2-x)(1+x)}$ although some candidates did (with varying degrees of success) attempt to expand $(5-x)(2-x)^{-1}(1+x)^{-1}$ directly. Whilst the majority of candidates correctly dealt with the expansion of $\frac{2}{1+x}$ (and so scored at least two marks in this part) it was surprising how many candidates (at this level) struggled in re-writing $\frac{1}{2-x}$ as $\frac{1}{2}\left(1-\frac{1}{2}x\right)^{-1}$. In some cases it was clear that candidates either did not realise or even recognise that the 2 inside the bracket had to be removed before this term could be binomially expanded. Those candidates who expanded both terms correctly usually went on to score full marks.

Question 2

Part (i) was answered extremely well with the vast majority of candidates correctly substituting the parametric form of the equation of the line into the equation of the plane to find the value of the parameter at the point of intersection (which if done correctly gave $\lambda = -3$). This value was then substituted back into the equation of the line to find the coordinates of the point of intersection. Although many candidates gave their answer as a position vector (rather than as coordinates) this was condoned by examiners.

The main stumbling block in part (ii) was not the application of the scalar product to find the angle between the line and the normal to the plane but it was in the choosing of the correct two direction vectors; in many cases candidates incorrectly used either $\begin{pmatrix} -7 \\ 9 \\ -2 \end{pmatrix}$, or even more surprisingly $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. It was pleasing to note that in most cases, where candidates found the correct obtuse angle of 111.5°, they then correctly subtracted this value from 180° to obtain the correct acute angle.

Question 3

The first mark in part (i) was awarded to the vast majority of candidates for correctly stating that T_4 was less than T_2 although some candidates did not make it explicitly clear which value of the two was the least. Candidates found the second mark a lot harder to come by as it was not sufficient to simply state that the approximations given by the trapezium rule were an over-estimate. Candidates needed to make it clear that these approximations were an over-estimate because the tops of the trapezia are above the curve which would then, in turn, mean that the error (in using a trapezium rule approximation for the value of the integral) would become less when the number of strips increases.

Part (ii) was answered extremely well with the vast majority of candidates giving the correct answer of 3.25. When errors occurred it was usually due to an incorrect value for the width of the strips or with the omission of a value. It was very rare for candidates to use the x values or to not give the answer to the required 3 significant figures.

Question 4

On the whole, candidates made an impressive start to this unstructured vector question and the majority of candidates correctly achieved $\sqrt{a^2+b^2+25} = \sqrt{27}$ and $a-7b-10=0.$ It was therefore extremely disappointing that at this level so many candidates failed to solve this pair of simultaneous equations accurately. Too many candidates simplified $\sqrt{a^2+b^2+25} = \sqrt{27}$ to $a+b+5=27$ or re-wrote $a-7b-10=0$ as $a^2-49b^2-100=0$ and so therefore did not obtain a three-term quadratic equation in either a or $b.$ Of those that did achieve a correct equation, most notably $(7b+10)^2+b^2=2,$ it was disappointing how many then expanded this as $49b^2+100+b^2=2$ and still arrived at a real solution for the value of b from the equation $b^2=-\dfrac{49}{25}.$ For those candidates who did expand correctly most achieved the correct values for both a and $b.$

Question 5

It was pleasing to note that most candidates used the correct double angle formulae for $\tan 2\theta$ to obtain a correct equation in terms of $\tan\theta.$ However, some candidates over complicated the problem by re-writing tan in terms of sin and cos and in these cases it was extremely rare for candidates to make any real significant progress. Of those that correctly re-arranged

$4\tan\theta\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)=1$ to $\tan^2\theta=\frac{1}{9}$ it was disappointing that so many candidates then only considered the solutions of the equation $\tan\theta=\frac{1}{3}$ and ignored any possible solutions that would have come from the equation $\tan\theta=-\frac{1}{3}$.

Question 6

In part (i) most candidates correctly wrote down the differential equation relating P, the time t, and the constant k. The most common errors in this part were those candidates who wrote $\frac{\mathrm{d}t}{\mathrm{d}P}=k\sqrt{P}$ or $\frac{\mathrm{d}P}{\mathrm{d}t}=kP^2$.

Even though in part (ii) the question asked for candidates to verify that $P=(A+Bt)^2$ was a solution to the differential equation many decided instead to solve the differential equation by separating the variables and integrating. It was disappointing how many candidates gave the final answer in this part as $B=\frac{k}{2}$ even though the question specifically asked for k in terms of B.

Question 7

Part (i) was answered extremely well by the vast majority of candidates with nearly all scoring the first three marks for separating the variables and correctly integrating both the x and y terms. The main issue for candidates in this part came in the last two marks as many did not consider the constant of integration. It was disappointing that a small number of candidates either did not state the equation of the curve correctly or did not show sufficient working in verifying this given result.

In part (ii) the vast majority of candidates considered both the correct integral (with correct limits) for the volume of revolution generated by rotating the given curve about the y-axis and went on to integrate correctly. A number of candidates, however, misread the question and instead tried to calculate the volume of revolution generated by rotating the curve about the x-axis. It was also relatively common to see a value of –2 being used as a lower limit on the integral. More worryingly was how many candidates incorrectly re-arranged their expression for x^2 to $\frac{1}{9}\left(4y^2-24y\right)$ which even though it led to a negative volume did not seem to concern many of these candidates. Finally, it was the mention of a rotation of 180° that seemed to concern many candidates and a significant number divided the correct answer of $\frac{176}{27}\pi$ by 2.

The responses to (iii)(*A*) were extremely mixed with many candidates making errors in the substitution or expansion of $9(2\tan\theta)^2-4\left(3(\sec\theta-1)\right)^2-24\left(3(\sec\theta-1)\right)$. Furthermore, many candidates assumed that this expression was equal to zero and then thought it was mathematically correct to divide through by 36. This highlighted a key misunderstanding even at this level between an expression and an equation. Most candidates correctly used a Pythagorean identity in the simplification of this expression although it was rare for candidates to accurately verify the given equation and therefore score all four marks in this first part.

Candidates' attempts at (iii)(*B*) were significantly more encouraging but it was slightly disappointing the number of candidates who showed no working in going directly from $\frac{18\tan\theta}{12\sec\theta}$ to $\frac{3}{2}\sin\theta$. While it could easily be argued that a well-prepared candidate could complete the required steps to obtain this final answer without the need to show any working, as the question paper required candidates to obtain their answer in the form $k\sin\theta$ the required step of $\frac{18\left(\frac{\sin\theta}{\cos\theta}\right)}{12\left(\frac{1}{\cos\theta}\right)}$ or equivalent had to be shown. Those candidates that did not show at least this key step of working failed to understand the demands of this type of question and therefore lost a significant number of marks. Those candidates who had been successful up to this part usually went on to find the gradient of the curve at the point with an x-coordinate of 2 correctly.

Question 8

In part (i) most candidates obtained the correct x-coordinate of point B but it was then surprising how many of these candidates incorrectly stated that at this point $\theta = \pi$ even though this leads to a clearly incorrect value for the y-coordinate. In fact, a number of candidates incorrectly used this value of theta to obtain an answer of $y = -1$ and then attempted to argue (via symmetry) that $y = 1$.

In part (ii), the majority of candidates correctly calculated the values of R and α although some lost the first method mark by not including R in the expanded trigonometric statements $R\cos\alpha = 1$ and $R\sin\alpha = 2$. Some candidates failed to give α in radians and a small minority stated R as 5 rather than the correct $\sqrt{5}$.

Part (iii) was answered well with many candidates correctly deriving the coordinates of points A and C from their answer to part (ii). The main issue seemed to be in not giving answers exactly; in a number of cases the x-coordinates of these two points were given as $\pm 0.600087...$

While the majority of candidates scored the first two marks in part (iv) for correctly stating $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ (although there were the usual sign errors and errors in differentiating the $\cos 2\theta$ term) very few candidates correctly substituted the value of θ, corresponding to point A, into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ to obtain the magnitude of either tangent gradient at A. To find β all candidates then had to do was to equate this value to $\tan\left(\frac{1}{2}\beta\right)$ and solve to obtain the correct answer of 1.90.

Paper B

Question 1

Both parts of this first question on paper B proved to be surprisingly demanding for many candidates with only a minority correctly stating that in part (i) $0 < x_0 < 1$ and that in part (ii) $x_0 = 0$ or $x_0 = 1$.

Question 2

Even though in part (i) the question specifically asked for candidates to find the equilibrium point by using the algebraic method indicated in the text, many candidates incorrectly used an iterative method. For those candidates that stated $x = 1.6x(1-x)$ most went on to solve this equation correctly.

In part (ii) whilst many candidates correctly stated that at the point of equilibrium $x = x^2 + 2$ many did not give a mathematically sound reason for why this equation does not have any real solutions. It was insufficient to use an iterative approach and it was simply incorrect to state that the quadratic equation does not factorise. However, many candidates did re-arrange the equation correctly and then proceeded to show that the discriminant of the resulting three-term quadratic was negative.

Question 3

This question was answered extremely well with many candidates scoring all four marks in part (i). The vast majority of candidates scored the three marks for filling in the table correctly for $k = 2,3$ and 5 but for the case when $k = 4$ a number of candidates did not complete a sufficient number of empty cells to clearly establish the outcome for this value.

Most candidates in part (ii) commented extremely well on what the values in the table implied about the four values of k; the main error being in the use of incorrect phrases or terminology (e.g. using ‘fluctuating’ to describe the behaviour when $k = 4$).

Question 4

Candidates seemed to be evenly split in part (i) in either making no real progress when attempting to find the values of k (for the next two points of bifurcation) or they correctly found both. Where issues occurred it was mainly due to accuracy.

In part (ii)(*A*) it was expected that candidates would use the sum to infinity formula for a geometric series and not use the less mathematically sound approach of simply adding up the first few terms on their calculator. Furthermore, it was surprising in this part how many candidates either struggled with finding the common ratio (many claiming that $r = 4.6692$ rather than the correct $r = \frac{1}{4.6692}$) or gave a final answer to a degree of accuracy which was not appropriate to the given context.

Finally, in part (ii)(*B*), while many candidates left this part blank or simply worked out the value of k it was encouraging that many candidates correctly stated that at this value chaos would be occurring.

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments:

Many good scripts were presented but by question 8 it was clear that some candidates were pressed for time. Many candidates struggled to demonstrate mathematical precision with basic algebraic manipulation, even in the standard simplifications involved in questions 1, 2 and 3. A somewhat reckless disregard for brackets and for correct notation often made many answers technically incorrect and sometimes led to errors by the candidate in following work. Candidates need to understand what it means to show method; to show the mathematical reasoning behind the calculation, rather than just the result from the calculator, and to show logical steps one after another. Incorrect manipulation of negative numbers was particularly noticeable. One of the requirements at this level is a correct understanding of mathematical terminology, notation and language coupled with and an ability to write with precision.

Comments on Individual Questions:

Question 1
1(i) A quite surprising number of errors were seen in evaluating 2**A**. Sometimes this was not shown and could not be inferred from a wrong final result. A few candidates calculated **A** x **A**. The biggest problem was incorrect arithmetic. A considerable proportion of candidates made errors with the addition and subtraction of negative numbers and consequently failed to calculate correctly all 4 elements of the matrix.

1(ii) Nearly all candidates earned full marks here.

1(iii) This was also done well though some demonstrated an inability to solve a simple quadratic equation, merely substituting the value a = 4.
A significant number of candidates did not know what a singular matrix was.
Some produced the matrix $\begin{pmatrix} 4 & 4 \\ 7 & 7 \end{pmatrix}$ with no indication of how it came about.

Question 2
2(i) The value of $|z|$ was nearly always correct. Most candidates gave fully correct answers, but arg(z) was quite often written as positive. Some expressed it in degrees or as a reflex angle, neither earning the mark.

2(ii) Finding the value of *a* was completed in most cases but there were a lot of arithmetical errors in finding the value of *b*, demonstrating an inability to manipulate negative numbers. A surprising number of candidates went from the correct
$-b = 10.5$ to then write $b = 10.5$. Many failed to evaluate $-6\times-\frac{1}{4}$ correctly, or wrote $2a\times-3\mathrm{j}$ as $-6\mathrm{j}$, or wrote $3\times3\mathrm{j}$ as $6\mathrm{j}$. Candidates who carefully wrote out the equations for the real parts and the imaginary parts were least likely to make mistakes.

Question3
3(i) This question was mostly well done, with many candidates showing good algebraic skills, although there were some very clumsy final expressions. The aim should be to clear fractions from factors, obtaining a single numerical factor. The initial split was achieved apart from a few who failed to expand the square correctly. A fairly frequent mistake was to leave $\sum 1$ as 1, a costly error as factorisation was impossible. Another more common error was to leave the result as $\frac{1}{3}n(4n^2-1)$ where the difference of two squares was not noticed. Candidates should write n

firmly as a multiplier of a fraction or in the numerator because, for example, $\frac{1}{3}n$ was often written to look like $\dfrac{1}{3n}$.

3(ii) “Hence” in the question required that the summation be carried out using the previous result. Work which did not show this explicitly earned no marks. Several made the mistake of subtracting the sum of 25 terms from the sum of 75 terms.

Question 4
4(i) The solutions to this question were very mixed with many candidates unable to write the equations of the circle and the line in complex number form. The circle was more often given correctly but we often saw $|C-3-4\mathrm{j}|$ or (z – 3 – 4j) or $|3+4\mathrm{j}|=5$.. In dealing with l, ‘arg’ was often omitted or replaced by modulus, and $\arg(l-1-3\mathrm{j})=-\frac{\pi}{4}$ was also seen. Some candidates anticipated part (ii) and introduced an inequality. Some gave a reflex angle, not always accurately.

4(ii) The question clearly stated that the boundaries were included but many candidates used strict inequalities. Upper and lower bounds of the argument written $-\frac{\pi}{4}\le\ldots\le-\frac{\pi}{2}$ were not infrequently seen.

Question 5
A substantial minority demonstrated an inability to cope with indices. This meant that, for many, this question was not about demonstrating the concept of proof by induction, but about index manipulation.
The algebra proved too difficult for most. In particular, a minus sign outside the bracket was easily the most common error here.
Bad handwriting meant that indices often got confused, e.g. the index numbers were written in line with everything else and then became coefficients instead of indices. In the denominator for example, 2^{k+1} often became 2^k+1 or $2(k+1)$ or even $2k+1$ in later work.
Some multiplied out and created very complicated expressions, often carrying through terms such as 1^{-2k} for several lines seemingly unaware that it could be simplified. Rough working was interspersed between lines of their deductive steps without making it clear what they were doing. We often saw a whole line multiplied by 2^{k+1} and later if we were lucky, divided by 2^{k+1} with no explanation given. The most successful were those who started by creating two fractions with clear denominators of 2^k and 2^{k+1}, and developed the work from there.
Many candidates write the last statements down even if they have not done previous work. They need to be aware that this wastes their time because of the way this question is marked, where previous steps need to be correct.
Apart from these points, it was good to see that a great many candidates had learned the appropriate introductions to the proof, and where they were successful in the algebraic argument, followed up with the crucial inductive argument, earning full marks. There were still a few candidates who insisted that their result showed that “n = k+1 is true”.

Question 6
6(i) There were few problems here, but some write simply $y=2$ instead of $(0,2)$ or $x=0, y=2$.

6(ii) In this part the most frequent error was in not writing the asymptote as an equation so “asymptote = –3/4” was often seen.

6(iii) Similarly, here, “asymptote = 1/2” lost a mark.

6(iv) A common error in drawing the graph was to have the right hand part of the curve approach the $y = ½$ asymptote from below.
Many candidates lost a mark because they did not clearly demonstrate that they knew how the right hand part of the curve behaved. They tried to cross the asymptote and at the same time move parallel to it so that there was no indication of a turning point followed by a point of inflection. Too many candidates believe that a "sketch" can be as roughly drawn as they like. It would be good to see a ruler used for the axes. The curve and the scales cannot be exact, which is why all the intercepts must be labelled, likewise the asymptotes. Some candidates omitted some or all of these, in particular the labelling at ½ on the y-axis, where the line itself was not labelled. That said, many excellent sketches were seen with all salient points and asymptotes clearly shown.

6(v) Most candidates answered this part well. There were the usual errors with inequality signs the wrong way round, using '<' instead of '$\leq$' or using 'y' as the variable instead of x.
Trying to save time or space by writing $-\sqrt{6} \geq x \geq \sqrt{6}$ resulted in a careless loss of 2 marks.

Question 7
7(a) This part was answered very well by those who used the substitution 'x = w/3' with most going on to gain all 5 marks. Marks were usually lost either by forgetting to multiply the numerical term by 27 or by forgetting to equate the expression to zero.
The candidates who used the sums and products of roots fared less well, one of the reasons being that there was more opportunity to go wrong. Sign errors were common and so "sum of roots = -1/2" was often seen as was "product of roots = 1". Also some who correctly had the values 1/2 , 2 and –1 then went on to multiply all of these by 3 rather than by 3, 9 and 27 respectively. The signs had again to be carefully considered when transferring the new coefficients into the new equation.

7 (b) (i) Nearly all candidates were able to gain 2 marks by writing '$3a = -p$' or an equivalent. Very rarely the sum of the roots was equated to p not to $-p$.
A variety of methods were used. Those that went on with the simpler substitution 'x = –p/3' usually gained the next 2 marks with little trouble.
Many did not realise that this was the simplest method and instead derived expressions for q and r in terms of a and λ. . They then substituted these values into the given expression and showed that all of the terms cancelled out. There was more scope for error using this method and it was more time consuming. Again the sign errors cropped up in the expression for r with many writing $r = a^3 - a\lambda^2$ instead of the correct version $-r = a^3 - a\lambda^2$. After substitution, another source of error was over hasty simplification of the expression, with several multiplications of minus signs required. Very often the errors cancelled out to obtain a "correct" result, but the explanation mark was given for a solution "without wrong working".
It is worth pointing out that while the first method of substituting a root into an equation gives an expression equal to zero straight away, in the second method, using the given expression as the starting point, this equality to zero should not be claimed before the analysis demonstrates that zero results.

7 (b) (ii) In a few cases it appeared that candidates believed that the cubic equation in part (a) was relevant in part (b). Throughout part (b) there was a commonly seen change of notation with α appearing consistently instead of a. Indeed a was here sometimes confused with the coefficient of x^3, giving $a = 1$. Most candidates used $p = -6$ to obtain a. They then obtained a quadratic equation for λ using either the value of q and the sum of the root products in pairs, or using the product of the roots after calculating r. When solving $\lambda^2 = -25$, most candidates gave $5\mathrm{j}$ as their solution rather than $\pm 5\mathrm{j}$. Fortunately both were then used in giving the complex roots. Many candidates could not believe that the other two roots would involve complex numbers and ignored their negative λ^2. Some of the solutions offered found r from the earlier result and produced a cubic equation. It was expected that candidates would demonstrate their ability to solve this, rather than resorting to a calculator.

Question 8

8(i) Plenty of correct answers were seen. Most recognised that the matrix represented a rotation. There was confusion over the amount. Many candidates described the rotation as 60^o anticlockwise, while several thought that the angle was 30^o. A few chose 45^o or even 90^o. Many unfortunately failed to specify the centre of the rotation.

8(ii) Many candidates seemed unaware that in order to prove that matrices are not equal it is sufficient to demonstrate that two corresponding elements are not equal. They wasted valuable time completely evaluating both products **PR** and **RP**, which was by no means straightforward. Incorrect matrix products were common, but the marking for this was generous.
A common error was to calculate 2 corresponding elements and then state that because these two elements were different then the matrix multiplication was not commutative. 'Unequal elements, so not commutative' was insufficient. 'Unequal elements, so that **PR** is not the same as **RP'** on the other hand shows the meaning of "multiplication of **P** and **R** is not commutative". The candidates who had found the full matrix product in each case usually achieved this explanation mark, a trade-off for the time spent!

8(iii) Most candidates knew that **R** was equal to QP. About half the candidates then aimed to use $\mathbf{P}^{-1}$ and the other half set up equations involving the unknown elements of **Q**. The former group very often went on to calculate $\mathbf{P}^{-1}\mathbf{R}$ instead of $\mathbf{RP}^{-1}$. A method mark was available for the multiplication either way, but not for some candidates who thought that $\mathbf{R}^{-1}$ was involved.
The equations set up by the second group were usually solved successfully. Many candidates did not realise that the rational and irrational parts could in this instance be easily separated.

8(iv) Many candidates knew that the determinant supplied the scale factor for the area. Candidates who clearly stated a value for their determinant and used this as the multiplier for the area 4 earned at least one mark. If their matrix **Q** was correct, and their new area correct, they earned both the marks. If the determinant was not clearly stated, a correct **Q** and correct new area earned both the marks, but anything else did not.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments:

Most candidates were able to demonstrate a good understanding of the topics being examined. The marks were generally high with a fair number achieving full marks. Q.1 (on calculus) and Q.3 (on matrices) were answered somewhat better than Q.2 (on complex numbers) and Q.4 (on hyperbolic functions).

Comments on Individual Questions:

Q.1(a)(i) Most candidates differentiated the equation correctly. Relating dy/dx to $1/(a^2 + x^2)$ caused some difficulty, and the factor a was often missing.

Q.1(a)(ii) Most candidates substituted $x = r\cos\theta$ and $y = r\sin\theta$ and obtained $r^2 = 36/(9\cos^2\theta + 4\sin^2\theta)$. However, many then just put this equal to the given answer without explanation, and this lost a mark. As the answer is given on the question paper, each step should be fully justified.

Q.1(a)(iii) Most candidates wrote down a correct integral expression for the required area; the factor ½ was sometimes omitted, and a square root sometimes introduced. The procedure for integration by substitution was well understood and the given result was very often obtained convincingly.

Q.1(b) Candidates who differentiated f(x) twice usually obtained the correct series. Many started by replacing x with $(1 + x)$ in the standard series for arctan(x), but this did not achieve anything useful.

Q.2(a) Almost all candidates recognised that C + jS was a geometric series and could write down its sum to infinity. This earned 4 marks out of the 9, however many candidates made no progress beyond this. Those who understood how to realise the denominator were usually able to complete this part.

Q.2(b)(i) Most of the Argand diagrams were correct. Reasons for not gaining full credit included not labelling the points, or showing symmetry in the imaginary axis instead of the real axis.

Q.2(b)(ii) This was generally well done, using either the polar form or the Cartesian form for z_1 and z_2. Some unfortunately used $a \pm$ jb and then became confused over the meaning of a.

Q.2(b)(iii) The value of γ was usually given correctly.

Q.2(b)(iv) The argument was usually given correctly, but the modulus was quite often given as 1 or 1/3 instead of 3. This part was omitted by about 10% of the candidates.

Q.2(b)(v) This part caused quite a lot of difficulty, and was omitted by about 15% of candidates. It was necessary to use the fact that triangle OAB was equilateral to obtain the argument of z_1 as $\pi/6$, but many candidates gave $\pi/3$ instead, and some tried to answer the question without finding a value for the argument of z_1.

Q.3(a)(i) Almost all candidates successfully found the value of k making the determinant equal to zero.

Q.3(a)(ii) The method for finding the inverse of a 3 x 3 matrix was very well understood, although there were quite a few careless slips.

Q.3(b)(i) Almost all candidates knew how to find the eigenvalues and corresponding eigenvectors, and the work was usually completed accurately.

Q.3(b)(ii) Almost all candidates understood how **P** and **D** are related to the eigenvectors and eigenvalues.

Q.3(b)(iii) Most candidates understood how to find $\mathbf{Q}^n$, and the given result was usually obtained convincingly.

Q.4(i) This was generally well answered.

Q.4(ii) The logarithmic form for artanh(x) was very often derived confidently by converting $x = \tanh(y)$ into exponential form. Some candidates used the quadratic formula to obtain e^y from the equation $(1 - x)e^{2y} - (1 + x) = 0$ resulting in unnecessarily complicated working. The condition $|x| < 1$ was quite often omitted, or given wrongly, despite being printed in the formula book.

Q.4(iii) The usual approach was to use the identity in part (i) to form a quadratic equation for tanh(x), then use the result of part (ii) to convert the two values of x into logarithmic form. This was very often completed accurately. Another approach was to convert the equation into exponential form, which gave a quadratic equation for e^{2x}; but this tended to be less successful.

Q.4(iv) About 10% of the candidates omitted this part, and many others could not make significant progress. Those who wrote the expression to be integrated as $\cosh(x)/(\sinh(x) - 1)$ usually carried on to obtain the correct answer.

4757 Further Applications of Advanced Mathematics (FP3)

General Comments:

The work on this paper was generally of a high standard, with most candidates producing substantial attempts at all three of their chosen questions. Q.1 (on vectors) was the most popular question, chosen by over three quarters of the candidates. Q.2 (on multi-variable calculus) and Q.4 (on groups) were each chosen by about 70% of candidates; and Q.5 (on Markov chains) was chosen by about 60% of the candidates. The least popular question was Q.3 (on differential geometry), which was chosen by fewer than a quarter of the candidates.

Comments on Individual Questions:

Q.1(i) This was very well answered, with most candidates using the standard formula involving the magnitude of a vector product. Some used the alternative method of finding when the vector from A to a general point on CD was perpendicular to CD.

Q.1(ii) Most candidates knew how to find the shortest distance between the two lines, almost always using a scalar triple product.

Q.1(iii) Candidates used a variety of methods to find the points where the shortest distance occurred. Some applied scalar products of the general chord with the directions of the two lines to obtain two simultaneous equations. Some put the general chord parallel to the common perpendicular, which had already been found in part (ii). Another approach was to take the general point on one line and put its shortest distance from the other line equal to $\sqrt{26}$, applying the formula from part (i). The proportion of candidates who obtained the correct two points P and Q was very high. Some candidates tried putting the length of the general chord equal to $\sqrt{26}$, but the resulting equation proved much too difficult to solve.

Q.1(iv) Most candidates were able to explain this satisfactorily, for example by stating that A and P are different points, so the distance from A is not the shortest distance.

Q.1(v) This was very well answered. Most candidates gave the Cartesian equation of the plane, but of course any standard equation (such as the vector equation involving two parameters) was equally acceptable.

Q.1(vi) Almost all candidates quoted a correct formula for the volume of a tetrahedron, as one sixth of the scalar triple product of three edge vectors, and this was very often evaluated accurately. Candidates who had not found P in part (iii) usually gave up without doing any calculations, although they could have earned 2 marks by finding the vector product of any two edges; there was sufficient information in the question to find the volume without knowing the coordinates of P (the answer is the same wherever P is on the line AB).

Q.2(i)(A) The partial derivatives were almost always found correctly, and shown to be zero at the origin. Many candidates omitted to verify that the origin was on the surface.

Q.2(i)(B) Here candidates were expected to explain why $(x^2 + y^2)(x + 1)$ is positive for all sufficiently small values of x and y, and deduce that the origin is a minimum point on the surface. Many considered only the sections given by $x = 0$ and $y = 0$, which is not enough to establish that it is a minimum. Some simply substituted in a few numerical values.

Q.2(ii)(A) This was well answered, with most candidates finding the additional stationary point and showing that there are no others.

Q.2(ii)(B) There were very many good solutions here, in which the sections corresponding to $x = -2/3$ and $y = 0$ were each found and suitably investigated. Some candidates used other valid methods (such as numerical investigation) to show that the point is neither a maximum nor a minimum, but unless these referred explicitly to sections of the surface they did not earn any marks.

Q.2(iii) Most candidates answered this correctly.

Q.2(iv) Most candidates found the equation of the tangent plane correctly; although some gave the equation of the normal line instead.

Q.3(i)(A) Almost every candidate showed that $\theta = 0$ gave the origin.

Q.3(i)(B) The formula for finding the arc length was well known, and the given result was very often obtained correctly and efficiently. However, a significant number did not make any attempt to use half-angle formulae to simplify the integral, and they were unable to earn more than one mark.

Q.3(ii) Most candidates wrote down a correct integral expression for the curved surface area. Further progress required the use of half-angle formulae to obtain an integrable form; this was often completed elegantly, but sometimes the integral was split into two terms, followed by very lengthy, though ultimately successful, manipulation.

Q.3(iii)(A) This was often answered correctly. Success again depended on the use of half-angle formulae, in this case to obtain the result $\mathrm{d}y/\mathrm{d}x = \tan(\frac{1}{2}\theta)$, leading to $\psi = \frac{1}{2}\theta$.

Q.3(iii)(B) The radius of curvature was usually found correctly, either by differentiating the intrinsic equation, or by using the formula involving first and second derivatives of the parametric equations. Finding the unit normal vector, and obtaining the centre of curvature, were also well understood and quite often completed accurately.

Q.4(a)(i) Almost every candidate gave the correct identity element.

Q.4(a)(ii) The orders of the elements were almost always given correctly.

Q.4(a)(iii) The inverses were usually given correctly, although some candidates stated that the identity element does not have an inverse.

Q.4(a)(iv) Most candidates could explain why the group G was cyclic.

Q.4(a)(v) Most candidates gave the two non-trivial subgroups correctly, although quite a few included an extra 'subgroup' consisting of e, b and d (where in fact b and d were generators of the whole group G). Some candidates just quoted Lagrange's theorem; this was not sufficient to earn the mark for a comment unless it was applied to the group G.

Q.4(a)(vi) Most candidates gave a correct isomorphism. Those who considered powers of generators of the two groups, or wrote out the composition table for F, could be confident that their answer was correct. Many candidates just considered the orders of the elements, but this by itself does not guarantee to pair up the elements correctly.

Q.4(b)(i) Most candidates correctly gave the order of the group H as 30. The most common incorrect answer given was 15.

Q.4(b)(ii) Most candidates considered powers of ab and correctly showed that this element had order 6. Some thought that it was sufficient just to show that $(ab)^6 = e$.

Q.4(b)(iii) A good proportion of candidates identified abc as an element of order 30, which therefore generated the cyclic group H.

Q.5(i) Most candidates explained how the 1/3 and 2/3 referred to the probabilities of losing or gaining a ball, but many did not explain what the zeros meant. Most assumed that the columns represented the states of having 0, 1, 2, 3 balls in A from left to right; and others assumed that it was 3, 2, 1, 0. Both these interpretations worked equally well.

Q.5(ii) The expected definition was a state from which the next state is certain (and not the same). Variations which applied to this example (such as a state from which the system always returns to its previous state) were also accepted. The states of having 0 balls and 3 balls were usually correctly identified as the reflecting barriers.

Q.5(iii) The two powers of **M** were almost always given correctly.

Q.5(iv) The limiting values were usually given correctly.

Q.5(v) A variety of methods were used to find the equilibrium probabilities. Some solved **Mp** = **p**, others applied the limiting matrices found in part (iv) to an initial condition in which all four states had probability 1/4, and others found the 'average' of the two limiting matrices. Some candidates gave probabilities (usually 0.25, 0.75, 0.75, 0.25) which did not add up to one.

Q.5(vi) Most candidates stated that it was impossible for the system to be in the same state. This was usually explained by the zeros in the main diagonal of $\mathbf{M}^3$, but some gave a convincing argument based on the context (such as whether the number of balls in A was odd or even).

Q.5(vii) This was well understood, with most candidates giving the correct transition matrix.

4758 Differential Equations (Written Examination)

General Comments:

Candidates performed well on this paper and the majority of the responses were of a high standard. The level of accuracy displayed by most candidates was commendable. The methods required to solve the second order differential equations in Questions 1 and 4 were known by almost all candidates and these two questions were attempted by the majority of the candidates. Question 3 was the least popular question. Very few candidates attempted more than three questions.

Comments on Individual Questions:

Question No. 1

Second order linear differential equations

(i) All candidates were familiar with the method of solution required in this part and there were very few arithmetical errors.

(ii) A common error in this part was to interpret the condition “the particle is initially at rest” as meaning that the particle was initially at the origin.

(iii) Almost all candidates noted that for large values of t the exponential term in the solution to part (ii) tended to zero, leaving a trigonometric expression.

(iv) Very few candidates spotted the connection between this part and the previous parts of the question. Most candidates worked from scratch, finding a new auxiliary equation and complementary function. The only errors that occurred were arithmetical or algebraic in solving the simultaneous equations involving k.

(v) A significant number of candidates did not provide a convincing explanation. It was necessary to highlight the coefficient, $3k + 6$, of the exponential term with a positive power and use the positivity of k to show that this coefficient was never zero.

Question No. 2

First order differential equations

(a) Almost all candidates recognised that the given differential equation required the application of the separation of variables method. A pleasing number went on to use partial fractions to enable integration and worked with accuracy to find v in terms of t.

(b)(i) Almost all candidates recognised that the given differential equation required the application of the integrating factor method and most began correctly by dividing through by x, the coefficient of $\frac{\mathrm{d}y}{\mathrm{d}x}$. Most candidates found the correct integrating factor and worked through accurately to find the particular solution.

(ii) This part was well-answered.

(iii) The majority of candidates stated the values that were requested and then used them to produce a good sketch.

Question No. 3

First order differential equations

This was the least popular choice of question.

(a)(i) This was well-answered.

(ii) Most candidates realised that the isoclines were circles, and were given credit in spite of the inaccuracy of their sketches. The tangent field sketches were less successful, often with only a couple of lines drawn on each of the circles. Candidates are advised to draw a good number of these lines, not least because it will help them in drawing solution curves.

(iii) The solution curves were usually drawn well by those candidates who used their tangent fields from part (ii). A minority of candidates drew curves that had no connection with what they had already done.

(iv) Almost all candidates obtained the correct estimate, and it was pleasing to see that their work was usually presented neatly in a table.

(v) This was almost always correctly answered.

(b)(i) Most candidates earned full marks in this part. The only errors were arithmetical slips in finding the particular integral.

(ii) This was well-answered.

(iii) This part discriminated well between the candidates. Some produced very convincing arguments for y being always positive, by expressing the quadratic term as a perfect square. Others looked at each term individually and rarely made any real progress. The sketch graphs were variable in quality. It is worth noting that in such requests for sketches candidates are not expected to do any extra calculations. They have been given or asked to find sufficient information for the sketch.

Question No. 4

Simultaneous second order linear differential equations

(i) There were many excellent responses to this part and the majority of the candidates scored full marks.

(ii) Almost all candidates gained the three method marks and the majority also gained the accuracy mark. The only errors were arithmetical.

(iii) There were many good accurate solutions to this part. Some candidates had difficulty in knowing how to apply the condition "the population of species Y is k times the population of species X when $t = 0$." Other candidates made arithmetical and algebraic errors when solving their simultaneous equations.

(iv) Most candidates realised that they needed to equate each of their expressions for x and y to zero and obtained at least the method marks.

(v) The model used in this question only applied while species X and Y were competing for survival. A comment that acknowledged that the model was no longer valid when one species had died was required here.

4761 Mechanics 1

General

This paper was answered with a pleasing level of confidence. Most candidates knew what techniques were appropriate to the various questions and were usually able to apply them. They had clearly been well prepared.

The majority of candidates were able to find questions that allowed them to show what they knew. There were also many candidates who scored highly across all the questions, even on the most demanding parts of the paper.

There was no evidence of candidates being under time pressure. Although there were some who did not complete the final question, they were almost entirely those who had not correctly completed all the previous questions.

Individual questions

Sections A

1. Equilibrium of a block on a rough slope

Part (i) of this question involved drawing a diagram for the forces acting on a block in equilibrium on a rough slope. While on the whole this was well answered, there were some surprising errors, such as showing the weight acting perpendicular to the slope or the normal reaction acting vertically. Some candidates showed the components of the weight as well as the weight itself; this is accepted if the components are presented differently from the other forces, for example using broken lines, but not if they all look the same.

In part (ii) candidates were required to use the given information to find the angle of the slope and the frictional force. Most did this successfully. However, the question asked for the angle to be given to the nearest degree and many lost a mark by giving it to some other level of accuracy.

2. Using vectors to describe the flight of a bird

This question was about vectors, using the context of the flight of a bird. The position vector of the bird was given in term of the time.

In part (i) candidates were asked about the velocity of the bird and this was well answered using vectors.

In part (ii), the time was to be found at which the bird had a given speed. This involved using a vector expression to form a scalar equation. Many candidates did not know how to go about this. Others obtained the right answer but their explanations were not always the most elegant. However there were very good answers written by some candidates.

In part (iii), candidates were asked to find the times when the bird was flying at an angle of 45^o to the horizontal. Correct answers to this were somewhat uncommon. One common mistake was to equate the components of the position vector rather than the velocity; most of those who did use the velocity considered only the case when the bird was flying above the horizon and not when it was flying below it.

3. Newton's 2nd law applied to the motion of a sledge

In part (i) the forces acting on a sledge and its acceleration were given and the question asked for the force of resistance. This was answered correctly by nearly everyone.

In part (ii) candidates were asked to find the new acceleration and the percentage increase when the forces were applied in a different manner. Almost all found the new acceleration correctly but there were quite a lot of errors working out the percentage, for example using the wrong denominator and forgetting to subtract the original value. Teachers using this question in the classroom may like to compare the percentage increases in the forward force on the sledge (6.4%) and the acceleration (67.3%) and consider why there is such a large difference between them.

4. Particles connected by a string passing over a pulley

Part (i) asked for force diagrams for the two connected particles, in this case blocks. A minority of candidates failed to get this right; the most common error was to mark different tensions for the different parts of the string.

Part (ii) followed on from part (i) with a request for the equations of motion of the two blocks. This was not universally well answered; sign errors were common.

In part (iii) the blocks were released and candidates were asked to find the time taken for one of them to reach the floor. This involved finding the acceleration of the system. Many, including those who has made mistakes in the earlier parts, did this using a whole system approach rather than working from the equations of motion.

5. Constant acceleration formulae based on the motion of two cars

This question involved the motion of two cars in parallel lanes on a road. One had constant acceleration and the other travelled at constant speed. One car was initially behind the other.

In part (i) candidates were asked to find the times when the cars were side by side. While there were many fully correct answers to this, there were also plenty of sign errors involving the 75 m difference in starting position. A few candidates did not realise that answering this question involved setting up an equation for the time *t*.

In part (ii) the question for the distance for which one particular car was ahead of the other. This was lower scoring than the other parts of the question.

In part (iii) they were asked to find the speed of one of the cars after it had travelled 400 m and this was very well answered, even by those who had made mistakes on earlier parts.

Section B

6. Modelling using the motion of a train

This was the first of the two long questions. It was based on two different models for the motion of a train from rest to maximum speed. It involved both constant and variable acceleration. On the whole this question was well answered with many high marks.

Part (i) was based on a constant acceleration model. It was very well answered with most candidates obtaining all the five available marks. However, many candidates lost one mark by giving the acceleration to only one significant figure.

The question then moved on to a model with variable acceleration. Part (ii) required candidates to recognise that when the train reached maximum speed its acceleration is zero and so obtain a given value for the time taken. While most candidates were successful in this, a substantial number did not recognise the significance of zero acceleration and tried other unsuccessful approaches, often involving a lot of fruitless work.

Part (iii) required candidates to integrate the acceleration to find the speed and then to integrate again to find the distance travelled by the train. Many candidates did not consider the constants of integration, or just declared them to be zero without any reason, and this was penalised.

In part (iv) candidates were expected to use the time given in part (ii) and their expression for the distance travelled from part (iii) to verify the distance the train had travelled in attaining maximum speed. Many knew just what to do and were successful. Some tried to do the question in reverse, substituting the distance and forming a quartic equation for the time; this was a viable approach and a few candidates realised that their equation could be written as a quadratic in t^2 and went on to solve it.
The question continued to ask for the maximum speed and this was well answered, even by those who had not been successful with the distance.

The final part (v) required the two models to be shown on a speed-time graph. This produced a wide spread of marks. Most candidates knew what they were trying to do but made errors. Some lost a mark by not showing the motion after the train had reached maximum speed and many others drew a straight line rather than a curve for the variable acceleration model.

7. Projectiles

This was the second of the two long questions. It was based on the context of tennis players serving. There were several points that candidates had to take into account: the ball was served from a given height; it had to pass over the net; it had to land in the service court. In addition the three players served with different speeds and at different angles to the horizontal. All of this meant that a significant amount of analysis was required and as a result some candidates were not successful on the later parts of the question.

In parts (i) to (iii) the serve was modelled as a projectile with given horizontal and vertical speeds. Most candidates were reasonably successful on all three parts: finding the speed and angle of projection in part (i), showing the ball passed over the net in part (ii) and finding out whether the ball landed in the service court in part (iii). The most common mistake was confusing horizontal and vertical components of the motion; there were also many sign errors.

In part (iv) a different player was serving, this time horizontally. The question asked candidates to find the range of possible values of the initial speed for the serve to land in the service court. This involved essentially the same work as parts (i) to (iii) although the situation was actually simpler with no vertical component of the initial velocity. However, no guidance was given and so candidates were required to analyse the situation; a substantial minority of candidates failed to do so and scored no marks. Among those candidates who did come to terms with the situation, some obtained both limits for the initial speed but many made a mistake with the lower limit, finding the minimum initial speed for the ball to reach the net without bouncing rather than to pass over the net.

In part (v) a third player served with initial direction below the horizontal. Only a minority of candidates scored any marks on this question and, among those who did, sign errors were quite common.

4762 Mechanics 2

General Comments:

The standard of the solutions presented by candidates was generally pleasing. Most candidates were able to make a reasonable attempt at most parts of the paper. There was some evidence that candidates felt rushed towards the end of the paper.

Candidates who were able to apply the appropriate mechanical principles to a problem were often hampered by their algebraic skills in being able to simplify and solve equations.

As always, candidates should be encouraged to draw clear and labelled diagrams and these are always appropriate when dealing with forces or velocities. A lot of potentially very good work was marred by sign errors that, perhaps, could have been avoided by having a clear diagram.

Comments on Individual Questions:

Question No. 1

Momentum and Impulse

(i) Most candidates indicated that the momenta of the discs before the collision were equal and opposite, with a sum of zero. Many then simply quoted the result in the question, without any reference to the fact that the total momentum had to be conserved.

(ii) Candidates were now on more familiar territory and they showed that they were able to write down equations using the principle of conservation of linear momentum and Newton's experimental law. A minority of candidates made sign errors in one or other of these equations.

Some candidates ignored the first part of the request, and used the information it conveyed about the speed of A to find the speed of B.

(iii) Candidates often seemed confused about directions and signs in their calculations. Drawing a diagram may have helped candidates avoid errors here.

(iv) Many candidates made a good attempt at this question which tested their ability to decide upon a strategy for solution. A number of different approaches were seen, some using the individual speeds, distances and times for each of the discs and some using relative speeds and distances.

(v) There were some neat and concise correct solutions to this problem. Most candidates were able to state that the angle between the new direction of motion and the barrier was $90° - \alpha$. The next step was to use Newton's experimental law and the principle of conservation of momentum to find the connection between the components of the speeds before and after the collision. This leads to another connection between the angles: $\tan\beta = \frac{1}{3}\tan\alpha$. Some candidates quoted this result without proof, and that was acceptable. The final two marks were awarded for combining these connections between the angles and finding α. A significant minority of candidates were not able to do this convincingly.

Question No. 2

Work, Energy and Power

(i) Most candidates obtained full marks through a correct application of the work-energy equation to this scenario. There were a few sign errors. A minority of candidates did not use the value of *g* as 10, given at the top of the question. They were not penalised in this part of the question.

(ii) Most candidates gained the majority of the marks in this question. A common error was to find the vertical distance travelled as $2\sin\alpha$ instead of $\frac{2}{\sin\alpha}$. Those candidates who insisted on taking the value of *g* as 9.8 were unable to derive the given result, but unfortunately this did not serve as a warning sign that something was wrong.

(iii) Explanations here were very poor. A common misconception was that a steeper slope meant that more work was done, and so the particle would come to rest below D. It was necessary to consider the effects of both reducing the friction, which was proportional to $\cos\alpha$, and the shortening of the ramp, to arrive at the conclusion that the work done by friction was reduced. An alternative approach was to use the result proved (given) in part (ii) to show that less work was done.

(iv) This was usually well-answered.

(v) Again, this was very well-answered.

(vi) The explanations offered by candidates were often lacking the required depth. Most candidates realised that a non-zero acceleration implies a non-constant velocity, but were unable to make any convincing further progress. The key was to use the fact that the force *F* was constant. Almost all candidates gained the final mark for stating that the use of *suvat* equations requires constant acceleration.

Question No. 3

Forces and Equilibrium

Candidates appeared to be confident with the content of this question and there were many very good, well-presented solutions.

(a) Candidates seemed confident when tackling this framework question. Errors were usually sign errors, and usually only occurred when the diagram given in the answer book had not been used to label the internal forces clearly. Some candidates did not realise that some of the external forces had not been given and assumed that there was no external force at L.

(b) There were many good solutions to this question. The given result for the force exerted by the peg on the ladder was usually obtained by writing down a correct moments equation. Some candidates did not realise that they were dealing with a 5, 12, 13 triangle and used inexact values for the trigonometrical ratios. Resolution of the forces on the ladder horizontally and vertically followed by use of $F \le \mu R$ was the most efficient way to find the range of possible values of the coefficient of friction. Some candidates confused themselves by labelling different forces as *F*. There was also a fairly commonly seen belief that the value of the coefficient of friction had to be less than one.

Question No. 4

Centre of mass

(a)(i) This was well-answered by almost all candidates. A minority used the area of the lamina as its mass, rather than its given mass of 2 kg.

(ii) The mechanics required for this part were clearly understood, but the algebra involved created a surprising amount of difficulty for candidates. Having taken moments for the new situation, candidates used the fact that $y = \frac{2}{3}x$ to form an equation in *X*. Those who noted the common factor of $12 + m$ reduced the amount of algebra that was required considerably, and they usually obtained the given result. Those candidates who multiplied everything out usually got lost in their algebra. Some candidates confused *X* and $\bar{x}$ and managed to find values for *m* and *X*.

(b)(i) Most candidates gained at least three of the four marks available in this part. The fourth mark required a statement that the centre of mass was on OB due to symmetry, or a calculation leading to this result.

(ii) This part required the solution of a fairly simple equilibrium problem and there were many routes to the solution, by resolution and/or moments equations. There were a pleasing number of neat and well-crafted solutions. Some candidates did not choose wisely the points about which to take moments, and involved themselves in complicated algebra. Other candidates chose to change the orientation of the shape, and again produced complicated equations. There was some evidence that candidates were running out of time.

4763 Mechanics 3

General Comments:

The marks on this paper were generally high, with an average mark of about 75%, and a fair number of candidates scored full marks. The majority of candidates appeared to have sufficient time to complete the paper, and were able to demonstrate a good understanding of most of the topics being examined. Questions 2 (on dimensional analysis and motion in a horizontal circle) and 4 (on centres of mass) were answered slightly better than questions 1 (on motion in a vertical circle) and 3 (on simple harmonic motion).

Comments on Individual Questions:

Q.1(i) The least possible speed was almost always derived correctly. As the answer was given it was of course necessary to give an adequate explanation, and some candidates failed to do this.

Q.1(ii) This was answered well, with the conservation of energy and the radial equation of motion being used to obtain the tension in terms of θ. There were sometimes errors in the potential energy term, and in the algebra. However, the most common error was to ignore energy altogether and assume that the motion was at constant speed $\sqrt{(ag)}$.

Q.1(iii) Most candidates found the minimum tension (at the top of the circle) correctly. It was then necessary to use the radial equation of motion and the conservation of energy, in a similar way to part (ii), to obtain the value of θ and hence the vertical height. Many candidates managed this successfully, although some assumed that the speed was constant. Another serious error was to use the formula for the tension obtained in part (ii), not realising that it was no longer valid as the value of V had changed.

Q.1(iv) Most candidates used the fact that the maximum tension occurred at the lowest point. Again several assumed that the speed was constant, but the majority used conservation of energy to obtain the upper limit $k \le 7$. However, the lower limit $1 \le k$, which follows from part (i), was usually omitted.

Q.2(a)(i) Almost every candidate gave the dimensions of force and density correctly.

Q.2(a)(ii) The method for finding powers in a formula by dimensional analysis was very well understood, and usually carried out accurately. There were a few careless arithmetic and algebraic errors. Some candidates took the dimensions of velocity to be MT^{-1}, presumably because the units of length are metres (m).

Q.2(b)(i) This part was quite well answered. The radial equation was usually written correctly; however, many resolved perpendicular to the track ($R = mg\cos\alpha$) instead of vertically ($R\cos\alpha = mg$). Because of the given answer, a full explanation was expected, and quoting a formula such as $\tan\alpha = v^2/(ag)$ did not earn any marks.

Q.2(b)(ii) Here it was necessary to consider friction (F) and the normal reaction (R), to resolve vertically and use the radial equation; and then use $F = \mu R$. Many candidates did this neatly and efficiently, and some eventually arrived at the correct answer after a very lengthy and complicated piece of algebra. A fairly common error was to assume that the value of R was the same as that found in part (i). Some candidates had the friction acting upwards instead of downwards.

Q.3(i) This was answered well, and the great majority of candidates obtained the given results convincingly.

Q.3(ii) Many candidates had difficulty expressing the tensions in terms of y, which should have been simply putting $x = 1.2 + y$ into the expressions used in part (i). The initial value of x (1.35) often featured in this, even though it is not relevant to finding the equation of motion. The principles behind forming the equation of motion were well understood and the correct result was often obtained. However, as this was given on the question paper, full marks could only be earned by a fully complete and accurate derivation free from any confusion over signs. The period was almost always given correctly.

Q.3(iii) Most candidates used the formula $v^2 = \omega^2(A^2 - y^2)$ to find the speed, although the values of A and y were often incorrect. Sometimes $\omega = 6.125$ was used instead of $\omega^2 = 6.125$.

Q.3(iv) Most candidates used a displacement-time equation to find a relevant time, with a few choosing instead to use velocity-time together with the speed from part (iii). Relating the time they had found to the required time when the particle was first moving upwards at C proved to be more challenging, but about half the candidates obtained the correct answer. This part was omitted altogether by about 15% of the candidates.

Q.4(i) The method for finding the centre of mass of a lamina by integration was very well understood, and most candidates obtained the correct coordinates.

Q.4(ii) Most candidates took the approach suggested in the question, considering a cone (for which standard results could be quoted) and a solid of revolution (for which the volume and centre of mass needed to be found by integration), then using the formula for the centre of mass of a composite body. This was very often completed efficiently and the given result obtained convincingly. Several candidates considered it instead as a single solid of revolution with the function defined piece-wise, often successfully. However, some continued to consider it as a lamina.

Q.4(iii) This part was omitted by about 10% of candidates. Most candidates recognised that the angle required was OAG, but the necessary trigonometry caused a lot of difficulty, with only about half the candidates obtaining the correct answer. The simplest method was $\tan^{-1}(½)+\tan^{-1}((x-1)/2)$ but many preferred to use the sine and cosine rules. A very common error was to assume that OGA was a right-angle.

4766 Statistics 1

General Comments:

As in the last two years, the majority of candidates were well prepared for this paper, with a large number scoring at least 60 marks out of 72. There was no evidence of candidates being unable to complete the paper in the allocated time. Most candidates had adequate space in the answer booklet without having to use additional sheets. Candidates who did need additional space usually used the last page of the answer book, but a few did not, presumably not realising that it was available, and instead used additional sheets. Losing a mark due to over-specification was mainly seen in question 1, although the majority of candidates realised that they should give their answers to 4 significant figures or less. Only a very few candidates lost a mark for giving probabilities to more than 5 significant figures.

Candidates usually scored very well on question 1 on mean and standard deviation, question 3ii on finding expectation and variance, question 4 on probability (including fairly well in part (iii) on conditional probability), and question 7 part (i) on the binomial distribution. Part (ii) of question 7, where candidates had to state hypotheses and define *p*, was well answered. This is very pleasing, as up until recently, this topic has caused problems for many candidates.

Question 6, which is largely covering work candidates will have met at GCSE was less well done than expected at this level, largely through poor drawing of the cumulative graph. There were many examples of poor choices of scales (and interpretation of scales by candidates), plotting at midpoints (and lower bounds) as well as omitting the point (40,0) when drawing their curve

Other questions on which candidates did not score so highly included question 2 part (iii) on probability and question 5 part (ii) again on probability.

The majority of candidates used correct notation. However poor interpretative skills let down many candidates. They have, in most cases, been well prepared for calculations required in the paper but struggled to gain full credit when required to analyse their findings. Literacy and handwriting were often not of a good standard.

Comments on Individual Questions:

Q1(i) Nearly all candidates worked out the mean correctly. Many candidates also found the standard deviation but some over-specified the answer thus losing a mark. A minority of candidates made an error in the formula for standard deviation or worked out the RMSD. It was encouraging to see most candidates recalling and using correct formulae.

Q1(ii) Most candidates used the transformation to obtain the correct mean. Many candidates also obtained the correct standard deviation, with a pleasingly small number mistakenly adding 7.2 to their final answer. Some candidates lost marks for over-specification, although there was only a penalty of 1 mark in the whole question for this error. It was encouraging to see most candidates giving answers to an appropriate degree of accuracy.

Q2(i) This part was answered well by the majority of the candidates. A small minority of the candidates used permutations instead of combinations and a very few simply gave an answer of 4! = 24.

Q2(ii) This was answered well by many candidates. However a common misconception was to assume ‘with replacement’ probability giving an answer of $0.34^4 = 0.0134$. A few candidates gave their answer in fractional form but failed to cancel, hence losing the accuracy mark.

Q2(iii) This question was found to be rather difficult. Many candidates wrote more than one page as they attempted to find all the probabilities of two or more using the same method, almost always without success. In fact around half of the candidature scored zero on this question part. A fair number of candidates did find the correct product and took their answer away from 1, but few found the correct multiplier of 4!, and thus only gained 2 marks.

Q3(i) Many candidates failed to provide a convincing argument for the value of $P(X = 1)$. Some candidates subtracted the sum of the other probabilities from 1. Many others found three possibilities but then stated that there were 8 possibilities in total, rather than 16. Mention of '2 ways round' or equivalent gained one mark. The best answers drew a sample space diagram and highlighted those differences equalling 1. A list of all six possibilities with sound workings was also a successful response to this part.

Q3(ii) This part was generally well-answered with around 90% of candidates getting full marks. Most correctly found $E(X)$ and, in many cases, $Var(X)$, although a number of candidates only found $E(X^2)$. A small number found $E(X^2) - E(X)$. Very few candidates attempted to find $E(X- \mu)^2$ and those who did were rarely successful.

Q4(i) Most candidates found the correct probability although a few did incorrectly find $0.4 \times 0.8 = 0.32$ or $0.4 \times 0.2 = 0.08$.

Q4(ii) This part was again very well answered. Many candidates gave the correct method and answer. A minority of the candidates gave an answer of 0.56 by adding two terms rather than three terms. These candidates missed out the 0.32 term.

Q4(iii) It is pleasing to report that this conditional probability question was well-answered by many candidates, two thirds whom gained full credit. Most recognised that they had to divide by the probability found in part (ii) although a few did not use the correct numerator. Those who found the correct probability sometimes lost the accuracy answer for poor rounding, giving, for example, a final answer of 0.366 rather than 0.364.

Q5(i) Around 95% of candidates answered this correctly. Most gave a decimal answer of 0.166 or 0.1664.

Q5(ii) This caused numerous problems for many candidates – time was spent drawing tree diagrams and trying to list all combinations. Most candidates found the probability of Emily winning in 3 games, 0.45^3, and thus gained a mark. A large number also found the probability of her winning in 4 games (in some order) and in 5 games (in some order). The number of valid combinations was frequently wrong - sometimes failing to find them all systematically and often using ${}^4C_3$ and ${}^5C_3$. Some candidates used 1-P(Sakura wins) and these candidates achieved mostly between 1 and 3 marks with mistakes with the coefficients resulting in not gaining full credit. An elegant solution seen was from a candidate who stated that Emily must win the last game and so worked on the possible ways of Emily winning two of the previous matches.

Q6(i) The majority of candidates made a good attempt at the graph. Very few failed to recognise that cumulative frequency was required, with only an occasional histogram or frequency graph seen. The values for the cumulative frequency were on the whole correctly calculated but a few tried to make them up to 365, failing to read the question correctly. The scales were usually linear but some chose difficult intervals, especially on the vertical scale, for example intervals of 24. Labelling was not as successful; missing labels or labelling the vertical scale as frequency was common. A number of candidates used mid-points rather than upper boundaries for plotting and a few used lower boundaries. Even if correct boundaries were used, the point (40,0) was often omitted with candidates either not joining their graph to the axis or joining it to (0,0). Just one third of candidates scored full marks in this question.

Q6(ii) Candidates were fairly even spread between reading off the graph or using linear interpolation to find the cumulative frequency for $x = 200$. A sufficiently accurate value was usually obtained from the graph, but many responses stopped short of even writing it as a fraction over 358, let alone converting this value into a proportion (decimal or percentage).

Q6(iii) This was a generally well done. The main problems were caused by unhelpful scales chosen in part (i), which candidates then interpreted wrongly in this part. Some candidates used cumulative frequencies of 100, 200 and 300, rather than the correct values to find the median and quartiles.

Q6(iv) This was again generally well done with most candidates correctly calculating the outlier limits. Most responses correctly stated that there were no outliers at the lower end but some stated that there were definitely outliers at the upper end rather than that there may be some. A number of candidates used the median instead of the lower and upper quartiles to find the limits and others used 2 × IQR, rather than 1.5 × IQR. A very few candidates found the mean and standard deviation and then using these, found the limits correctly.

Q6(v) Although this part was generally answered well, a minority of candidates lost the final mark by not having the end of the whiskers plotted at 40 and/or 300, often plotting these at 29 and/or 358. Some candidates did not show a horizontal scale, making their response difficult to mark. Other candidates had trouble drawing the box and whisker diagram due the lack of a ruler.

Q6(vi) Many candidates struggled to answer the question which was asked. Often zero marks were scored as the candidate wrote a short essay with no mention of skewness. Being precise and talking about both locations generally gained the marks. Some candidates still referred to it as left and right skew or mixed up positive and negative. The question did ask for a comparison, which was generally missed.

Q7(i)A Around 90% of candidates gained full marks here, with most using the formula, rather than tables.

Q7(i)B Again this was well answered, usually by use of cumulative probability tables, although some candidates did calculate the four probabilities, usually summing them correctly. A few candidates forgot to subtract from 1, and a few just subtracted $P(X = 8)$ from 1 rather than $P(X \le 8)$.

Q7(ii) Candidates did well on this part, with over 80% gaining at least 3 marks out of 4. Most candidates scored the first two marks for the hypotheses, with many knowing that they needed to define p, thus scoring the third mark, although some definitions were wrong. For example 'p = the probability that dogs suffer from the allergy'. A valid explanation of the reason for the form of the alternative hypothesis was usually given, even if not always very well worded.

Q7(iii)A Approximately half of the candidates scored full marks in this part and also the final part. Most candidates started off correctly by using 0.1353, but there were still quite a few who used point probabilities, scoring zero. The use of 0.0395 was not uncommon, again scoring zero. There were a few candidates who did not compare their probability to the significance level and so could only be awarded one mark. Some candidates used the critical region method and in this part the two correct probabilities were used most of the time and compared with the significance level. The final mark was often lost due to failure to provide a statement, failure to include context or failure to include an element of doubt.

Q7(iii)B The majority of candidates used 0.0916 but there again there were quite a lot of candidates who used point probabilities. Some candidates used a critical region method but there were far too many who didn't use the correct two cumulative probabilities, but just 0.0916 and 0.0453, comparing both to 0.1. To score marks using the critical region method candidates needed to compare both 0.0916 and 0.1637 to 0.1 to justify the critical region.

4767 Statistics 2

General Comments:

The paper proved to be accessible yet provided suitably challenging questions to enable differentiation between those gaining high marks. Most candidates provided well-structured responses to the hypothesis test questions which included appropriately worded conclusions. Many candidates lost marks through over-specification of final answers. Candidates coped well with probability calculations with which most provided clear unambiguous working. Questions requiring interpretation of results or explanation of terms proved to be challenging for many; with such questions examiners look for concise, statistically focused comments.

Comments on Individual Questions:

Question 1.

(i) Most candidates identified independent and dependent variables correctly. Many commented upon the pre-determined nature of the independent variable whilst far fewer successful comments contained references to random variation.

(ii) There were many completely correct calculations of the regression line *s* on *l,* with the main errors being in over-specification of the values in the final equation or in the use of *x* and *y* instead of *l* and *s*.

(iii) Many candidates commented that an increase in *l* led to an increase in *s*, but far fewer understood that that the coefficient of *l* was the increase in *s* per unit length. A few candidates did not appear to understand the word coefficient, taking it to mean that they needed to rearrange their equation making *l* the subject.

(iv) The correct value of *s* was obtained in many cases. The subsequent subtraction to find the residual was carried out the wrong way round by many candidates leaving them with a positive rather than a negative residual. Over-specification of the final answer was penalised.

(v) Most candidates correctly calculated the required estimate, but many candidates did not understand the difference between interpolation and extrapolation. Over-specification of the estimate was penalised.

(vi) Most candidates managed to recalculate the estimate successfully using the new equation and provide a suitable comment in favour of one of their two estimates. Of the successful attempts, answers relating to the removal of the outlier were seen more frequently than answers relating to using all available information. Few candidates referred to both.

Question 2.

(i) Reasonably well answered though a few candidates neglected to answer in context.

(ii) Well answered. Some candidates neglected to provide a parameter for their distribution of X.

(iii) Well answered. A few candidates mistakenly thought that $P(X \geq 3)$ was equivalent to $1 - P(X \leq 2)$.

(iv) Well answered.

(v) Well answered.

(vi) In general this was well answered. Some candidates used incorrect parameters. Most applied the correct continuity correction with only a few inappropriate or missing ones seen. The structure of the probability calculation was mostly correct.

(vii) Many students commented that the existence of multiple births would make the assumption of independence invalid. Very few candidates who recognised that there could be multiple births commented further on their likelihood and how this might relate to the assumption of independence.

Question 3.

(i) Generally well done with most candidates obtaining the correct answer. A few used the wrong tail of the normal distribution. Few made the mistake of applying a 'continuity correction' of 49.5 or 50.5. The use of 0.72 as standard deviation instead of variance was not uncommon.

(ii)(*A*) Generally well done, although the use of a *z*-value leading to a value of μ below 50 was not uncommon.

(ii)(*B*) Generally well answered. Some candidates used +1.645 in place of –1.645 and were happy to work with a negative standard deviation. A few candidates did not finish off by squaring their value of σ and so missed the final mark.

(iii) Despite the relative complexity of this part of the question, there were many good solutions. The question required z values of -2.326 and -0.6745 and this was interpreted well by successful candidates. Some candidates used wrong signs with their z-values, thus forming equations leading to a negative standard deviation. Premature rounding leading to an inaccurate final answer was fairly common.

(iv) This question proved to be too difficult for the majority of candidates. Of the successful attempts, those using $1 - \mathrm{P}(X = 0)$ were more successful than those attempting $\mathrm{P}(X = 1) + \mathrm{P}(X = 2) + \mathrm{P}(X = 3)$.

Question 4.

a(i) Well answered.

a(ii) Well answered.

a(iii) Well answered.

a(iv) Well answered. A few candidates misinterpreted the result as not significant. A small number of candidates did not state the number of degrees of freedom.

a(v) Some successful, concise but sufficiently detailed answers were seen for this part of the question. Many candidates made no reference to contributions to the test statistic or simply quoted values without interpreting their magnitude. Some candidates were unclear as to whether their comments related to "yes" or "no" answers. Some candidates recognised that the small contributions indicate that the results are as expected but then go on to say "slightly more than expected" or "slightly fewer than expected".

(b) Generally well answered. Often one or two marks were lost through incomplete/poorly worded conclusions or incomplete definitions of μ. Some candidates attempted a one-tailed test and were penalised. Some candidates calculated a positive test statistic despite the observed value lying below the mean and did not make it clear why they were doing this; a diagram showing the correct test statistic with its symmetrical equivalent or statement that absolute values are being used would have sufficed. A few provided an incorrect critical value, often -1.282, or made inappropriate comparisons such as 0.029 < 0.1 for the probability method.

4768 Statistics 3

General Comments:

The standard of candidates on this paper was again very high. Almost all candidates attempted every question and it was clear that they were well prepared for this unit. Statistical tests and calculations were generally performed accurately and presented with sufficient detail and clarity. Candidates should be reminded to round their answers to an appropriate degree of accuracy; in particular, in most situations, an accuracy of more than four significant figures is unlikely to be justified.

In hypothesis tests most candidates are writing their hypotheses and conclusions correctly. The most common error is to forget to make it clear that they are referring to a *population* parameter. A more subtle mistake arises in conclusions, which should state whether there is evidence to reject H_0, rather than evidence to accept it (for example, 'There is insufficient evidence that the mean exceeds 30mg' rather than 'There is evidence that the mean does not exceed 30mg'). A good guiding principle is to use the wording of the question when writing down the conclusion.

Although mathematical presentation and communication were generally very good, there was often lack of clarity in notation for random variables. In particular, when working with combinations of random variables, candidates should be careful to distinguish between, for example, $2T$ and $T_1 + T_2$. Not only would this improve the clarity of their presentation, but it would also help them find the correct variance in questions such as Q.4iii and Q.4iv.

Several of the questions on this paper tested candidates' understanding of conditions and assumptions used in their calculations. The assumptions required for the use of various statistical tests were generally well known, although not always clearly expressed. Many candidates were able to reproduce standard statements accurately, for example to explain the meaning of a confidence interval. The most challenging concept seems to be the distribution of the sample mean. Many candidates seem to be forgetting that, if the underlying distribution is Normal, then the sample mean will also be Normally distributed, regardless of the sample size (and hence the Central Limit Theorem would not be required). In general, candidates seem very confident in stating the required assumptions for the application of various theorems or test, but not so good at recognising when those assumptions are not met. They could benefit from seeing more situations in which those theorems do not apply.

Many candidates are using calculators effectively and appropriately to find values from statistical distributions. For those who are confident with it, technology could be utilised further in teaching to develop deeper understanding of the concepts such as the distribution of the sample mean and confidence intervals.

Comments on Individual Questions:

Question No. 1

This question required using the t-distribution to conduct a hypothesis test and construct a confidence interval.

Part (i)A asked why a test based on Normal distribution might not be appropriate in this case. Many candidates seem to associate a t-test with small samples, forgetting that, if the underlying distribution is Normal and the population variance is known, the sample mean would follow a Normal distribution even for a small sample.

The t-test in part (i)(B) was generally done well, with the hypotheses and conclusions clearly stated in a majority of cases. The most common mistake was to use 2.131 as the critical value (which is for the 2-tail test at the 5% significance level).

In part (ii) candidates were generally able to construct the confidence interval correctly, with a very small number using 1.753 (which is for a 90% confidence interval) or 1.96 (from Normal distribution). However, a significant number lost a mark for giving the final answer to 5 or 6 significant figures; this level of accuracy is unjustified.

Part (iii) asked for an interpretation of a 95% confidence interval. Many candidates were able to give a correct interpretation, but a significantly smaller number managed to explain clearly why the proposed statement was incorrect. Many seemed to be saying that 95% of samples would have a mean in “this interval” (seemingly referring to the particular interval calculated in part (ii)). Candidates should be encouraged to understand that the population mean is fixed and the confidence interval changes with each sample.

Question No. 2

Part (i) of this question involved a Wilcoxon test. Candidates were first asked why a t-test was not appropriate. Many knew that this was to do with the distribution of the underlying population, but few seemed aware that only the differences, rather than the scores themselves, needed to come from a Normal distribution. The test itself was generally conducted accurately. However, many marks were lost for the hypotheses and the conclusion, which did not always clearly refer to the population median difference (some candidates talked about the difference between the scores, which is not precise enough).

Part (ii) required a chi squared test and was generally very well done. The simplicity of the expected frequencies meant that there were very few accuracy errors, and a vast majority used the correct critical values. The main loss of marks came from the conclusions, which were not always given in context. (‘The model of equal probability is not appropriate’ was not considered sufficient context.) The hypotheses were sometimes stated in a generic form (‘The model fits the data’); this was not penalised, although a more specific statement of what “the model” is would be preferable. Candidates should be reminded that ‘Data fits the model’ is not a correct statement of the null hypothesis.

Question No. 3

The topic of continuous probability distributions seems to be very well understood and this question produced many correct answers. The sketches in part (i) did not always clearly show the zero gradient at the origin and so many candidates scored only 1 out of the 2 marks.

In part (ii) most scored full marks. A minority thought that the mode was f(4) rather than 4. Calculator solutions needed to clearly refer to the maximum point on the graph rather than just giving the answer; candidates should be reminded that calculator answers are inexact and should therefore be given to an appropriate degree of accuracy (in this case, *not* to the nearest integer).

Part (iii) was probably the best done question on the whole paper. There was plenty of detail shown and there were very few mistakes.

In part (iv) there were very many correct calculations, the most common mistake being the confusion between the standard deviation and the variance. The reasons for requiring the Central Limit Theorem did not seem to be very well understood: many candidates stated that it was needed ‘because the sample was large’. Another common misconception is that, for a large sample, the population distribution becomes Normal. Some also seem to believe that the CLT states that the variance of the sample mean needs to be divided by n; this is in fact the case regardless of the distribution and the sample size.

Question No. 4

This question was about linear combinations of Normal random variables. The first two parts were generally very well done.

Part (iii) proved more challenging; many candidates considered $M + 2T$ instead of $M + T_1 + T_2$. (Some wrote $M + 2T$ but then produced the correct variance; they were able to score full marks.) Candidates should be advised to write out their combinations of random variables carefully.

In part (iv)A many candidates seemed to confuse the discounted price with the discount, resulting in incorrect answers; they were still able to score 3 out of the 4 marks in part (iv)B. Many also lost one mark for writing down the variance instead of the standard deviation. In part (iv)B common mistakes were to use the wrong tail of the distribution (for example, using 1.282 instead of -1.282), and to give answers to an inappropriate degree of accuracy (anything other than 2 decimal places, as the value represented the amount of money).

4771 Decision Mathematics 1

General Comments:

There were a number of candidates for this paper who exhibited very poor communication skills. Candidates who did not take the time to read the questions carefully, and who did not express their answers clearly, could not do well on this paper.

Many of the general comments from last year's report resonate with this year's scripts.

There were very significant difficulties with respect to accurate and concise communication which are affecting the quality of the mathematics presented by some candidates.

Having said that, there were cases in which candidates moved from very poor communication in, for instance, the explanation required in 3(ii), to excellent answers to subsequent algorithmic work. Candidates would benefit from more emphasis on the "explain" element, an emphasis which should not be restricted to preparing for examinations in decision mathematics.

Comments on Individual Questions:

Question No.1

Many candidates had difficulties with applying the algorithm in part (i). It was very common to see FI selected at the end instead of FE. This proved to be expensive in terms of marks lost, in considering the context of the question it should be obvious when drawing the network that it is better to connect in F directly to E rather than to I.

A minor mistake, often seen, was to fail to number the columns following the inclusion of nodes.

There were many cases where candidates who made a good attempt at part (i) struggled with part (ii). A common minor error was to connect A to B instead of to W.

The last mark was more difficult. We have been trying to discourage fanciful answers to these interpretational parts, but we still see them. It may well be the case, for instance, that a rare spider is nesting on the direct route from A to C, diverting both arachnophiles and arachnophobes, but no marks will be awarded for supposing thus. The mark is for interpretational work on the maths, and not for creative thinking. In this case the preferred answer was to increase the resilience of the system to pipe bursts.

> [For the probabilists, and certainly not required, if we assume a constant risk of breakage along the entire length of pipe, then the expected number of plants downstream from a burst is $5\frac{1}{15}$ in the answer to (i) and $2\frac{9}{14}$ in the answer to (ii).]

Many candidates offered "shorter pipe runs", and this was accepted as a proxy for the above. Others offered answers rooted in physics such as higher pressures or faster delivery of water, which were ultra vires.

Question 2

Candidates were very good indeed in following this algorithm, but not so good in the mechanics of answering the question. Many answers to part (i) were very long indeed, with much writing, often on several continuation sheets. A table of values does the job quickly and efficiently.

Most candidates scored 2 out of 3 in part (ii) because they did not know the convention of labelling in tens so that other statements can be inserted. Some resorted to writing out a whole new algorithm, incorporating their insertions and still labelling in tens.

A large proportion of candidates were not awarded the mark for part (iii) because they did not answer the question. Some wrote mini essays about proffering money to pay the price and collecting change, but failed to link P to price, M to money and C to change.

Question No. 3

There were many very good answers to this question, except for part (v), and except for candidates who had forgotten about bipartite graphs. In part (iii) the instruction to start from $K_{2,3}$ was helpful, and most candidates who did that were able to put together a convincing argument, which was pleasing.

The drawing required in part (iv) was intricate, and it was pleasing to see so many good attempts.

In contrast very few creditable answers were seen to part (v). This question did not require an extensive knowledge of electronics, there was enough in the question to deduce what was needed. If it can't be done in the plane then either go into 3-D (layers) or insulate your crossings – which is the same thing really. Instead most candidates accepted planarity as a constraint, and wrote about what could not be connected together on a (printed) circuit board.

Question No. 4

The LP was really well done, particularly the graph. Candidates did well in extracting the information from the text, although there were one or two "pinch points".

Part (i) was meant as a helpful hint, but in the event there were many candidates who correctly computed £9 and £6 for the profits in part (i) and then proceeded to use a profit function of 25x + 22y in the rest of the question.

Some candidates failed to label their axes. A few used non-uniform scales and a few used poor scales, e.g. 2cm representing 300. A few did not include the origin, using instead a false origin - but they invariably incorrectly used their lower and left boundary lines as if they were axes. A few failed accurately to compute $6\times433\frac{1}{3}$. However, in general, the graphical work was done with great competence.

The interpretational work in (iv) and (v) was challenging, and it was pleasing to see some good answers.

Question No. 5

The quality of answers to part (i) was mixed. We looked for 0000 and for 1111, and for a recognition that the equal probabilities for a head and a tail on the coin tossing leads, via the binary count, to an equal probability for each possible number. For instance, identifying 2^4 outcomes and 16 numbers earned the mark.
Despite the range of 0 to 15 being given in the question, quite a few candidates started counting at 1, often referring to probabilities of 1/15 in their explanations.

The modelling question in part (iii) prompted some essays and some misdirected consideration of the differences between game participants. All that was needed was the observation that the probability of a ball ending in a bottle depends on the bottle.

Some candidates failed to read "repeatedly thrown until …" in the question, including "miss" as a possibility in their modelling.

Many candidates seemed to think that “50-50” is a synonym for “of equal probability” in all contexts.

Most candidates handled part (iv) with aplomb, which was very pleasing. A small but significant number tried to answer a more interesting but more difficult question, to simulate whether a ball ends up in a corner bottle, an edge bottle or an inner bottle. It is more difficult because the probabilities have to be computed first from the information in the question. They are 1, 4 and 4 ninths respectively. Credit was allowed for those that did that.

Question No. 6

CPA modelling revolves around producing an activity graph, and that was tested in this question. That necessarily means that there have to be descriptions of the activities, and how they relate to each other. To compensate for this reading burden, the question was arranged so that the resulting graph was as simple as possible, within the constraints of testing the need for dummy activities, et al.

Candidates handled this well in parts (i) and (ii) of the question although, as always, some failed to give the duration and/or critical activities in part (ii).

The following stem, together with parts (iii) and (iv) proved to be a step too far for most candidates. Scheduling is always difficult but very many candidates struggled to demonstrate that they understood what was needed or how to present it. The mark scheme has a row for each task, with shading showing when and who attends to it. It is just as acceptable to have a row for each person, showing when and what they do.

Of course, the problem is that this is not tackled algorithmically, but rather by finding those combinations which use resources most efficiently, and that is not an easy task.

4772 Decision Mathematics 2

General Comments:

The candidature for this paper was much reduced this year. These candidates were clearly good mathematicians who were well-prepared for the paper. However there were some difficult parts to the paper, including two part questions on counting which proved to be very challenging.

Comments on Individual Questions:

Question No. 1

The first part of the logic question was very challenging, and few candidates scored all three marks. The question invited candidates to explore the apparent paradox of Epimenides. Few considered carefully what might be meant when someone is said to be a liar. Far too many seemed to think that the negation of "All Cretans are liars" is that "No Cretans are liars".

The rest of the question was solidly grounded in learned techniques, and most parts were answered very well. Not all candidates realised that the result in (c)(iii) was intended to provide the means for the proof asked for in (c)(iv).

Question No. 2

This question was very well answered. The majority of candidates scored full marks on parts (i) and (ii), which was very pleasing. Part (iii), utility, was also done well. Part (iv) was a little more problematic. Many failed to get a new EMV of £313. Of those that did, few then subtracted it from their EMV from part (ii). Not all attempted a subtraction and many who did, subtracted their new "Budget" EMV from their Budget EMV in part (ii), instead of from their overall EMV.

Question No. 3

Parts (a)(i) and (a)(ii) were appropriately challenging, and the majority of candidates could answer them. Parts (a)(iii) and (a)(iv) turned out to be too difficult for the majority of candidates.

In part (b) more information was given than the lengths of arcs. Candidates were also told that direct arcs were shortest distances, and were given other shortest distances where there were no direct connections. This caused some confusion in (b)(ii), where the nearest neighbour algorithm starting at C stalls, but where almost all candidates carried on. It also caused confusion for a few in (b)(iii), when they thought that indirect connections were not allowed in pairings. Apart from that, and from the difficulties which some candidates had in giving a route in (b)(iii), part (b) was done well.

Question No. 4

It is almost always the case that LP formulations should start with "Let … be the number of …". This question was an exception, and only a very few candidates saw that. Of those that did, some were confused between concentrations (of fat, salt and sugar) and proportions (of pasta, sauce, cheese and olive oil).

Parts (ii), (iii) and (v) were done superbly well … candidates were very well prepared.

Most answered part (iv), although not always very efficiently, but few thought to provide the same interpretation in part (vi).

4776 Numerical Methods (Written Examination)

General Comments:

Most of the candidates seemed well prepared for this paper. The routine numerical work was generally done well and the candidates performed well on most of the questions on this paper. Several of the questions had sections that required the candidates to comment on the results of their calculations. The interpretation of their results was often inadequate. The candidates can carry out the algorithms correctly, but are less sure of the theoretical conditions underlying these algorithms.

In two questions there were sequences of values that were converging on an approximation to either a derivative or an integral. The candidates were asked to comment on the justification of the accuracy of their answer. There was a tendency to choose the final term of the sequence as the most accurate. The candidates should make a judgement based upon examining all the terms of the sequence. This approach usually results in an estimate that results in a value whose accuracy has fewer significant figures that the final term of the sequence.

Comments on Individual Questions:

Question No. 1.

This question involved relative errors and was generally well answered. A significant number were penalised for giving negative numbers for parts (ii) and (iii). In part (i) nearly all the candidates correctly evaluated the relative error in the approximation for π. A few candidates miscopied the $\frac{355}{113}$ as $\frac{355}{133}$ and so could only gain credit for the method. The answers to part (ii) were more varied. The elegant way was to add the relative error in part (i) to the relative error obtained by approximating the diameter of the circle from 226.3 cm to 226 cm as indicated in the question. Most candidates used an alternative method of finding the relative error by using both the approximations for π and the diameter in a single rational expression. A common error was to ignore the instruction in the question to round the value of the diameter and use the approximation to π. Naturally, this approach lead to the same answer as part (i). These comments for part (ii) can be applied to part (iii) where the area instead of the circumference was required.

Question No. 2

The finding of a Lagrange Interpolating polynomial was well understood and very well answered. Part (i) asked why a Newton forward difference interpolation formula could not be used to construct a second degree polynomial from the given data. The fact that it was not possible, since the data were not evenly spaced, was almost universally well-known and given as the reason. In part (ii) the initial formula for the Lagrange method was normally correctly stated. The subsequent algebra to produce the required quadratic polynomial was carried out successfully by most candidates. In part (iii) the answer to part (ii) was used to estimate the value of y when $x = 2.5$. This was usually straightforward, but some candidates with an incorrect polynomial just quoted a wrong value. They could have received some credit if they had shown some evidence of substituting of $x = 2.5$ into their trinomial.

Question No. 3

Most candidates found this question straightforward. Showing the location of the root in part (i) and performing the iteration to find the root to five decimal places in part (ii) presented no difficulties. Part (iii) was more of a challenge for the candidates. The question stated that there was another root $\beta \approx 1.39$ and the task was to determine whether the iteration in part **(**ii**)** could be used to find β to greater accuracy. Many candidates started with $x = 1.39$ and stopped after two or three iterations and stated that the iteration was diverging. This statement about divergence, although true, was based on insufficient evidence. More iterations were needed to conclusively demonstrate the divergence. The iteration formula was $x_{r+1} = \frac{x_r^5 - 1}{3}$ and the better candidates used a theoretical approach and showed that substituting the value of x = 1.39 into the derivative $\frac{5x^4}{3}$ of the right hand side gave a value greater than 1, guaranteeing divergence. A few candidates misunderstood the theory and differentiated the left hand side of the original equation $x^5 - 3x - 1 = 0$ to try and demonstrate divergence.

Question No. 4

In part (i) the candidates were given the formula $I = \frac{(Z_2 + Z_1)^2}{(Z_2 - Z_1)^2}$ and told that the values of Z_1 and Z_2 were correct to one decimal place. The question then asked for the range of possible values of *I*. The majority of candidates used the upper and lower bounds of Z_1 and Z_2 correctly and used them to calculate values for *I*. A significant number of attempts misunderstood this question and failed to realise that the values of the variables needed to be consistent in the calculation; many used different values for Z_1 and Z_2 in the numerator and denominator, leading to inaccurate limits for *I*. Even with incorrect values for the range it was still possible to attempt part (ii) and explain why the range was so large. The answers seen to part (ii) were often poor and usually not creditworthy.

Standard responses, such as “the problem was ill-conditioned” or “subtraction of unequal values” did not receive credit unless specific features of the calculation were used to illustrate the point. The fact that the difference between the numbers in the denominator is a small positive number was missed or not clearly stated by the majority of the candidates.

With the numbers in the denominator being very close together a small change leads to a large relative change in their difference and squaring this difference before division magnifies the effect.

Question No. 5

Almost all the candidates knew that you could not use the forward difference formula to estimate the derivative *f '(0.4)* since the value for *f(0.4)* was not given in the table.

The central difference method was used in finding the three estimates of *f '(0.4)* in part (ii) and was generally done well. Sometimes the differences in the denominator were increased by a factor of ten due to misplaced decimal points thus causing an error in the estimate. Stating the value of *f '(0.4)* to the accuracy which seemed justified was not done as well. Many candidates treated their estimate of -0.2872 for the derivative *f '(0.4)* as the best and quoted this to three or four decimal places. When a sequence of values is created, it is necessary to consider all three values before justifying the accuracy. The previous estimate for *f '(0.4)* was -0.2862 and since both these estimates round to -0.29 this seems to be the secure answer.

Having been given the value of *f(0.4)* in part (iii) then using the value of the gradient times 0.01, the difference between *f(0.4)* and *f(0.41),* was the easiest way to find the approximation for *f(0.41)*. A significant number of candidates chose to use various methods of linear interpolation to find the approximation. These methods were not always of sufficient accuracy and a common error was to use 0.1 instead of 0.01 as the difference between 0.4 and 0.41. Since the value of *f(0.4)* was given to five decimal places most answers for the value of *f(0.41)* were also recorded to five decimal places. Candidates often overlooked the fact that the value of the gradient was an approximation to two decimal places and the difference between 0.4 and 0.41 is also two decimal places. Hence, at most only four decimal places can be considered in justifying the accuracy and the answer should be quoted to three decimal places to be secure.

Question No. 6

Finding $x^{(x^2)}$ for x = 0.1, 0.01 and 0.001 was straightforward, except that some candidates did not use the full accuracy of their calculator for the 0.001 value.

The hint in part (i) was not fully understood by the candidates in part (ii). There were not very many convincing arguments for both questions in part (ii). A reasonable number of candidates said in one form or another that 0^0 was undefined, but fewer used part (i) to suggest that a value of 1 at the origin would enable the trapezium rule to be used.

The application of the midpoint rule in part (iii) was understood and done well. All three values should be considered before deciding on the accuracy of the integral. In this case both the certain answer of 0.9 and the probable answer of 0.94 were acceptable.

The candidates were given a table of values containing five estimates of the integral using the trapezium rule for use in part (iv). The task was to obtain four Simpson's Rule estimates of the integral and hence give the value of the integral to the accuracy that appeared justified. Those candidates who used $\frac{4T_{2n} - T_n}{3}$ normally produced the four required answers. Those who chose to use the weighted mean of $\frac{2M_n + T_n}{3}$ could only produce three answers before having to do further work. This either involved reverting to the trapezium formula or calculating another midpoint rule. The latter approach sometimes caused inaccuracies in the final Simpson's Rule estimate. Considering the sequence of four values, the answers of 0.940 being secure and 0.9403 as possible were acceptable.

In part (v) nearly all the candidates knew the 0.0625 theoretical value for the ratio of differences of a sequence of estimates to a definite integral using Simpson's Rule. Most used this value to try and improve their estimate for the integral, but not always successfully. The correct answer of $I \approx$ 0.94032 was seldom achieved. It is not clear that the candidates understood that each section of question 6 was guiding them through increasing improvements in the estimates of the $\int_0^1 x^{(x^2)}\ dx$.

Question No. 7

Part (i) was a routine exercise in showing that the equation $3x^5 - 5x^3 - 1 = 0$ had a root α, such that $1 < \alpha < 2$, and a root β such that $-1 < \beta < 0$. The vast majority correctly answered this exercise, but some candidates did not provide the values at the end points of the two intervals to justify their statements about a change of sign.

The requirement in part (ii) was to obtain the Newton-Raphson iteration formula for the above equation. The answer was displayed so most candidates had no problems in obtaining the correct answer. Since the answer was displayed, it is incumbent on the candidates to give their answer in the same form. Several candidates presented their result as an equation in x rather than x_r.

Part (iii) was a straightforward application of the Newton-Raphson method to find α and it was generally done well.

The following part (iv) asked for an explanation of why it was not possible to use the Newton-Raphson iteration formula with $x_0 = 0$ or $x_0 = -1$ to obtain a value for β. This part was not answered very well by most of the candidates and illustrated the difference between carrying out the algorithms and understanding the theory that underlines them. The usual answer was that at $x_0 = 0$ and $x_0 = -1$ the value of $f'(x_r)$ was 0 and this caused a math error and the iteration could not be performed. Very few candidates mentioned the fact that $f'(x_r) = 0$ showed that the gradient was zero and so the tangent was parallel to the x-axis and cut not intercept it to provide a new iterate.

Part (v) required another application of the Newton-Raphson iteration formula to show that starting with x_0 = -0.3, a value close to the root β, the iteration produced a new root γ = -1.2173. This application was quite slow to converge and took several iterations to reach the requested answer of five significant figures. There was confusion amongst some candidates about the distinction between significant figures and decimal places, which usually cost them the final answer mark.

The bisection method was used in part (vi) to estimate the value of β. The method was well understood and most candidates chose the correct intervals as the maximum possible error was reduced. The tables were not always set out in a readable format and occasionally the final answer was left as an interval containing β rather than the value of β itself.

The final part (vii) asked how many further applications of bisection were needed to obtain a value for β with a maximum possible error of less than 5×10^{-5}. This was only a 1 mark section and the answers varied from those who appeared to have guessed, those who just wrote down the correct answer of 9 and those who worked out the answer was 9. The majority of candidates did get the mark.

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