# OCR - Oxford Cambridge and RSATeacher Delivery Guide Pure Mathematics: Trigonometry

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: TRIGONOMETRY (1)** |
| Basic trigonometry | \* | Know how to solve right-angled triangles using trigonometry. |  |  |  |
| Trig. functions | Mt1 | Be able to use the definitions of sin *θ*, cos*θ* and tan*θ* for any angle. | By reference to the unit circle,  . |  |  |
| t2 | Know and use the graphs of sin*θ*, cos*θ* and tan*θ*for all values of *θ*, their symmetries and periodicities. | Stretches, translations and reflections of these graphs.Combinations of these transformations. | Period. |  |
| \* |  Know and be able to use the exact values of sin*θ* and cos*θ* for *θ* = 0, 30, 45 , 60 and 90 and the exact values of tan*θ* for *θ* = 0, 30, 45 and 60. |  |  |  |
| Area of triangle; sine and cosine rules | t3 | Know and be able to use the fact that the area of a triangle is given by ½. |  |  |  |
| t4 | Know and be able to use the sine and cosine rules. | Use of bearings may be required. |  |  |
| Identities | t5 | Understand and be able to use . | e.g. solve  for . |  |  |
| t6 | Understand and be able to use the identity . | e.g. solve  for . |  |  |
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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: TRIGONOMETRY (1)** |
| Equations | t7 | Be able to solve simple trigonometric equations in given intervals and know the principal values from | Equations | t7 | Be able to solve simple trigonometric equations in given intervals and know the principal values from |
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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: TRIGONOMETRY (2)** |
| Trig. functions | Mt8 | Know and be able to use exact values of for and multiples thereof and for and multiples thereof. |  |  |  |
| t9 | Understand and use the definitions of the functions arcsin, arccos and arctan, their relationship to sin, cos and tan, their graphs and their ranges and domains. |  |  |  |
| Radians | t10 | Understand and use the definition of a radian and be able to convert between radians and degrees. |  |  |  |
| t11 | Know and be able to find the arc length and area of a sector of a circle, when the angle is given in radians. | The results and where *θ* is measured in radians. |  |  |
| t12 | Understand and use the standard small angle approximations of sine, cosine and tangent. |  where is in radians. |  |  |
| **Radians** |
| Image of a radian*r**r**r* | A radian is the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle. |
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| **Specification** | **Ref**. | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **PURE MATHEMATICS: TRIGONOMETRY (2)** |
| Secant, cosecant and cotangent | t13 | Understand and use the definitions of the sec, cosec and cot functions. | Including knowledge of the angles for which they are undefined. |  |  |
| t14 | Understand relationships between the graphs of the sin, cos, tan, cosec, sec and cot functions. | Including domains and ranges. |  |  |
| t15 | Understand and use the relationships and *.* |  |  |  |
| Compound angle formulae | Mt16 | Understand and use the identities for , , . | Includes understanding geometric proofs. The starting point for the proof will be given. |  | Proofs using de Moivre’s theorem will not be accepted. |
| t17 | Know and use identities for , , *.* | Includes understanding derivations from , ,   |  |  |
| t18 | Understand and use expressions for  in the equivalent forms and. | Includes sketching the graph of the function, finding its maximum and minimum values and solving equations. |  |  |
| Equations | t19 | Use trigonometric identities, relationships and definitions in solving equations. |  |  |  |
| Proofs and problems | t20 | Construct proofs involving trigonometric functions and identities. |  |  |  |
| t21 | Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces. | The argument of the trigonometric functions is not restricted to angles. |  |  |

# Thinking Conceptually

### General approaches

The foundation to this topic is seeing it as a continuous flow rather than a series of disjointed formulae. At AS level the flow should be from the original formulae learned at GCSE and the link between  and to . This helps with the understanding of the exact values of the various trigonometric functions. The right-angled triangle can then be extended into working with the unit circle and how this relates to Pythagoras' Theorem leading to the first of the Pythagorean Identities. If this is shown as a plot then the link from here to the graphs becomes natural rather than forced. The open source software, Geogebra, is an excellent graph package that easily allows this demonstration. There are a number of pre-created tutorials on this topic which means no more than limited understanding of the package itself is needed. It also saves on time. For learners who value mobile phones above calculators there is an app from Google Play called trigonometry - unit circle which links all aspects of this process together.

At A-Level the same cohesion is again required. The various formulae are not to be treated as individual aspects but as part of a whole unit. In addition to that, the concept of proof always seems to breed terror. If the proof is supported visually then it will often seem less daunting and easier to recall. The MathPage has an extensive trigonometry section including all the necessary proofs. The Wolfram demonstration project has many of these proofs in an almost purely visual form. Cut the knot has the same thing but in static rather than interactive form.

For solving of equations, learners are usually adept at finding the first solution but struggle with the later ones. This is often down to a reluctance to draw the graphs as they feel this is a little simplistic. However, the graphs are always a key to getting the correct solutions.

### Common misconceptions

Firstly there tends to be a limited understanding that  ,  and  are both values and functions. This means that  is both a symbol and a process. Learners tend to grasp the former rather than the latter.

In light of this, understanding that  and  are inverse functions then increases the awareness that and  are inverse functions.

Also, further to this initial misconception, there is sometimes a desire on the part of the learners to separate  as being too distinct elements leaving the sin function on its own without an angle attached to it.

These misconceptions may well have been formed at GCSE level and so it is useful to break them early on.

Additionally understanding the difference between  ,and  or similarly and becomes essential to avoid mis-manipulation of the various functions.

The transfer to a new system of measuring angles can sometimes prove to be a barrier to development. Learners do not always find this transition easy to make and time will need to be invested early on rather than hoping that they will pick it up at some stage. Some of this can be put down to a lack of conceptual understanding regarding the need for a different system for recording angles. It is often only much later in the process, usually during the work on integration that this finally makes sense to them. It is a similar challenge faced as that of trying to teach learners to work in a different numerical base, eg binary. A careful and disciplined instilling of this will help to overcome the frustration that creeps in when they discover they have been using the wrong units.

When solving equations involving, for example , learners tend to add the other values on after dividing by two rather than beforehand. This is often due to not understanding that they are working with the graph of  not merely adapting the graph of  to fit their needs.

Fear is one of the largest barriers in this area. For some reason learners tend to be wary of this topic and this will often cause them to stumble before they have truly begun and significantly hamper progress.

A common problem learner’s experience is remembering to change the angle units on their calculator, either from degrees to radians, or vice versa.

### Conceptual links to other areas of the specification

**Prior Knowledge from GCSE (9 – 1) Mathematics**

A higher tier learner at GCSE should be able to:

recognise and sketch the graphs of ,  and  ,

know exact values of  and  for  = 0o, 30o, 45o, 60o, 90o,

know exact values of  for  = 0o, 30o, 45o, 60o

know and apply the sine rule to find lengths and angles, know and apply the cosine rule to find lengths and angles,

know and apply the formula  .

know and apply Pythagoras’ Theorem and can thus extend this to the trigonometric identities and should then understand why they are called the Pythagorean Identities.

**Links Across the A-Level Course**

Surds – the manipulation of surds is a basic skill that then allows the learner to use the values of sine, cosine and tangent in exact form, particularly when solving trigonometric equations.

Quadratic Equations – being able to comfortably solve quadratic equations specifically those in a function of the unknown like ,  or  is important when solving trigonometric equations.

Curve Sketching – although the initial skills are largely applied to polynomial equations the techniques developed here can easily be adapted and applied to the graphs of trigonometric functions.

Functions – The use of function notation, range and domain, and the relationship between  and  will assist in the graphing of the trigonometric functions and understanding their connection to each other.

Graph Transformations – as with curve sketching the initial work is done with polynomial functions, however exactly the same skill set applies to the transformation of trigonometric graphs.

Circles – There is a link to circles particularly in the creation of the Pythagorean Identities. That is why these functions are correctly named the Circular Trigonometric Functions.

Parametric Equations – A number of parametric equations are given in terms of trigonometric functions and so the understanding of how trigonometric functions work and link together becomes fundamental to this.

Differentiation – This requires the learner to be able to differentiate trigonometric functions from first principles.

Integration - the various techniques of integration will require a greater degree of manipulation of the trigonometric functions.

Vectors – There is a small and limited use of trigonometry in finding the direction of vectors.

Resolving Forces – The Mechanics strand involves using trigonometry to resolve forces horizontally and vertically, or parallel and perpendicular to an inclined plane.

# Thinking Contextually

In terms of the trigonometric graphs and their solutions, the modelling of tides is perhaps the most obvious use of this area in context. This will naturally extend to tidal wave energy although the equations for its full motion are a little beyond the A-Level course. In kinematics this can then be extended to the waves themselves and the sporting context to surfing. Passys World of Mathematics has a good number of examples to investigate.

Another form of wave energy that can be modelled in this way is that of earthquakes, however, these waves do not maintain the same amplitude throughout but dissipate.

Sound waves in music also conform to trigonometric curves but as with earthquakes they tend to dissipate rather than maintain the same volume. However, it is not just the sound levels but also the harmonies and harmonics that fit with this area. There is a nice pdf file by Mark Petersen on this topic.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
| --- | --- | --- | --- |
| [Trigonometry Unit Circle](https://play.google.com/store/apps/details?id=processing.test.trigonometrycircleandroid) | Amra Studio | Visual understanding and calculating sine, cosine, tangent, cotangent, secant and cosecant function, degrees and radians.Description of functions. | t1-t21 |
| [Trigonometric Functions](http://demonstrations.wolfram.com/topic.html?topic=Trigonometric+Functions&limit=20) | Wolfram demonstration project | A very visual and interactive series of demonstrations to help see the proofs rather than restricting it to algebra. | t1-t21 |
| [Sine, Cosine, and Ptolemy's Theorem](https://www.cut-the-knot.org/proofs/sine_cosine.shtml) | Cut the Knot | All the proofs but in a static visual form. | t1-t21 |
| [Dave's Short Trig Course](http://www2.clarku.edu/~djoyce/trig/index.html) | Clark University | An online walkthrough of all aspects of the trigonometry course providing all the key formula and a few more besides. | t1-t21 |
| [TheTopics in Trigonometry](http://www.themathpage.com/aTrig/trigonometry.htm)  | The Math Page | A concise and algebraic approach to trigonometry with examples alongside all the necessary proofs. | t1-t21 |
| [Have a sine](https://undergroundmathematics.org/trigonometry-triangles-to-functions/have-a-sine) | Underground Mathematics | This rich problem is accessible to any student with a basic understanding of trigonometry in right-angled triangles. The process of “chasing” the angles and side-lengths in this diagram is very instructive, not just for the “trigonometry” component but also for getting students to realise how little information about a diagram they require to be able to solve a problem. | t1 |
| [Trigonometric Identities](http://www.malinc.se/math/trigonometry/identitiesen.php) | Malin Christersson’s Math Site | Notes on the difference between an equation and an identity and leads into the application of identities in trigonometry. | t1 and t16 |
| [Trigonometry (AS)](http://mei.org.uk/files/sow/05-trigonometry.pdf) | MEI | Scheme of work planning notes from MEI, teaching ideas, sample resources and common misconceptions. | t1-t7 |
| [Exact Angles](http://www.malinc.se/math/trigonometry/exactanglesen.php) | Malin Christersson’s Math Site | Introduces the idea of combining known angle facts to determine others. Includes a couple of questions at the end of the notes. | t2 |
| [Trigonometric Graphs Spaghetti](https://vimeo.com/39923699) | Chris Smith | A very entertaining video that uses the circle and spaghetti to create a sine curve. | t2 |
| [Cosine Identity](https://www.geogebra.org/m/H9BbuSwX) | Geogebra | Cosine Identitylink the corresponding values of  on the graph  | t2 |
| [Sine Identity](https://www.geogebra.org/m/pb8Drtd5) | Geogebra | Sine Identitylink the corresponding values of  on the trig graph  | t2 |
| [Tangent Identity](https://www.geogebra.org/m/jpkTfgtk) | Geogebra | Tangent Identitylink the corresponding values of  on the graph . | t2 |
| [Trigonometry-Area Formula](https://www.geogebra.org/m/BZCJrmnz) | Geogebra | Trigonometry area formulaShows how to find the area of a triangle including step by step working. | t3 |
| [Trigonometric Laws](http://www.malinc.se/math/trigonometry/lawsen.php) | Malin Christersson’s Math Site | Demonstration of area of triangle and the sine/cosine rules. | t3 and t4 |
| [Sine Rule - find a side](https://www.geogebra.org/m/wcGshBdK) | Geogebra | Interactive exercise that allows different triangles to be investigated with step by step working. | t4 |
| [Sine Rule find an angle](https://www.geogebra.org/m/KK9tTH43) | Geogebra | Interactive exercise that allows different triangles to be investigated with step by step working. | t4 |
| [Cosine Rule find a side](https://www.geogebra.org/m/NXHPNrN6) | Geogebra | Interactive exercise that allows different triangles to be investigated with step by step working. | t4 |
| [Cosine rule find an angle](https://www.geogebra.org/m/vBbeNVGY) | Geogebra | Interactive exercise that allows different triangles to be investigated with step by step working. | t4 |
| [Trig ID Movie (I)](https://www.geogebra.org/m/eySKrAbP) | Geogebra | Shows the step by step proof for the first of the Pythagorean Identities using geogebra | t6 |
| [Unit Circles Exact Values](https://www.geogebra.org/m/G7xgNRxm) | Geogebra | Gives the exact values of the trig functions around the circle. | t8 |
| [Slices of](https://undergroundmathematics.org/trigonometry-triangles-to-functions/slices-of-pi)   | Undergound Mathematics | This problem looks at how ,  and  are related to each other. By working on the task, students can combine graph sketching, the unit circle, identities and solving equations. | t8 |
| [General solutions](https://undergroundmathematics.org/trigonometry-triangles-to-functions/general-solutions/interactive-graphs) | Underground Mathematics | By asking students what they can say about  and  if , this resource introduces the general solutions of trigonometric equations such as . The interactive graphs could be used to explore these and similar equations, revealing how the symmetry and periodicity of the functions comes into play. Students may also connect the graphs and solutions with the unit circle. | t8 |
| [Trigonometry](file:///%5C%5Cfilestorage%5COCR%5CPD%5CProdSup%5CProjMan%5CProjects%5CGCE%5CMaths%5CMaths_T3%5CMathsB_MEIonlyresources%5CDG_Trigonometry%5CPub%5Cmei.org.uk%5Cfiles%5Csow%5C22-trigonometry.pdf) | MEI | Scheme of work planning notes from MEI, teaching ideas, sample resources and common misconceptions. | t8-t12 |
| [Cosine and Inverse Cosine](https://www.geogebra.org/m/AW5tKn8x) | Geogebra | Graphs shown side by side with animated demonstration of how the values map across. | t9 |
| [Sine and Inverse Sine](https://www.geogebra.org/m/MPTCvTHj) | Geogebra | Graphs shown side by side with animated demonstration of how the values map across. | t9 |
| [Tangent and Inverse tangent](https://www.geogebra.org/m/tqMHfgPj) | Geogebra | Graphs shown side by side with animated demonstration of how the values map across. | t9 |
| [Radians](http://www.malinc.se/math/trigonometry/radiansen.php) | Malin Christersson’s Math Site | Introduces the idea of radians in context.Includes a couple of questions at the end of the notes. | t10 |
| [Exploring Radians](https://www.geogebra.org/m/wYVbkGzq) | Geogebra | Interactive geogebra activity where learners must put the angles, given in terms of  , in the right place. | t10 |
| [Sector spirals](https://undergroundmathematics.org/circles/sector-spirals) | Underground Mathematics | This resource provides an opportunity for students to practise calculating arc lengths and sector areas. Students who look at the problem as a whole before jumping in with calculations will notice patterns and relationships between the sectors that provide more efficient routes through the task. | t11 |
| [Trig tables](https://undergroundmathematics.org/trigonometry-triangles-to-functions/trig-tables) | Underground Mathematics | Non-calculator task investigating exact trig values. | t13 |
| [6,7 - Geo - cosec, sec & cot Graphs](https://www.geogebra.org/m/JF77rnxC) | Geogebra | Cosec sec and cot graphs. | t13 |
| [Trigonometric Functions](file:///%5C%5Cfilestorage%5COCR%5CPD%5CProdSup%5CProjMan%5CProjects%5CGCE%5CMaths%5CMaths_T3%5CMathsB_MEIonlyresources%5CDG_Trigonometry%5CPub%5Cmei.org.uk%5Cfiles%5Csow%5C26-trigonometric-functions.pdf) | MEI | Scheme of work planning notes from MEI, teaching ideas, sample resources and common misconceptions. | t13-t15 |
| [Going round in circles](https://undergroundmathematics.org/trigonometry-triangles-to-functions/going-round-in-circles) | Underground Mathematics | Investigation of the 6 trig ratios. | t14 |
| [Trig ID Movie (II)](https://www.geogebra.org/m/b7NnV8fC) | Geogebra | Show the step by step proof  using geogebra. | t15 |
| [Trig ID Movie (III)](https://www.geogebra.org/m/X37yjcFb) | Geogebra | Show the step by step proof of  using geogebra. | t15 |
| [Trigonometric Identities](file:///%5C%5Cfilestorage%5COCR%5CPD%5CProdSup%5CProjMan%5CProjects%5CGCE%5CMaths%5CMaths_T3%5CMathsB_MEIonlyresources%5CDG_Trigonometry%5CPub%5Cmei.org.uk%5Cfiles%5Csow%5C28-trigonometric-identities.pdf) | MEI | Scheme of work planning notes from MEI, teaching ideas, sample resources and common misconceptions. | t16-t20 |
| [Equation or identity? (II)](https://undergroundmathematics.org/trigonometry-compound-angles/equation-or-identity-ii) | Underground Mathematics | Opportunity to practice manipulating trig expressions. | t16 |
| [Double angle formula via area](https://www.geogebra.org/m/X5P9BGEj) | Geogebra | Geogebra animated demonstration of double angle formula using area of rectangle and rhombus.  | t17 |
| [The double angle formulae](https://www.geogebra.org/m/joLKGc3a) | Geogebra | Numerical demonstration of double angle formulae using unit circle. | t17 |
| [Proving half-angle formulae](https://undergroundmathematics.org/trigonometry-compound-angles/proving-half-angle-formulae/possible-proof) | Underground Mathematics | This resource provides a collection of diagrams that students can use to help them give a geometric proof of trig formula | t17 |
| [Rcos(x-a) form](https://www.geogebra.org/m/f45vAUd7) | Geogebra | Rcos(x-a) formA graphical look at this formula and what it does visually. | t18 |
| [Equation or identity? (I)](https://undergroundmathematics.org/trigonometry-triangles-to-functions/equation-or-identity-i) | Underground Mathematics | Opportunity to practice manipulating trig expressions. | t19 |
| [Make a Spirograph](http://www.malinc.se/math/trigonometry/spirographen.php) | Malin Christersson’s Math Site | Investigates complex curves and the trig functions that can be used to define them. | t20 |
| [Music](http://www.malinc.se/math/trigonometry/musicen.php) | Malin Christersson’s Math Site | Application of trigonometry in music.  | t20  |
| [Waves](http://www.malinc.se/math/trigonometry/wavesen.php) | Malin Christersson’s Math Site | These provide some background applications.Application of trigonometry in waves. | t21 |
| [Mathematical Harmonies](file:///%5C%5Cfilestorage%5COCR%5CPD%5CProdSup%5CProjMan%5CProjects%5CGCE%5CMaths%5CMaths_T3%5CMathsB_MEIonlyresources%5CDG_Trigonometry%5CPub%5Camath.colorado.edu%5Cpub%5Cmatlab%5Cmusic%5CMathMusic.pdf) | University of Colorado | A brief study of music and the sine curve. | t21 |
| [Ocean Mathematics](http://passyworldofmathematics.com/ocean-mathematics/) | Passy World of Mathematics | A nice series of web pages that are not overly mathematical but offer an insight into the application and uses of waves in a variety of contexts. | t21 |
| [Teacher Package Trigonometry](https://plus.maths.org/content/os/issue55/package/index#hands) | +plus magazine | A series of articles that examine a variety of applications of Trigonometry. | t21 |

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