# OCR - Oxford Cambridge and RSATeacher Delivery Guide Statistics: Statistical Hypothesis Testing

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **STATISTICS: STATISTICAL HYPOTHESIS TESTING (1)** | | | | | |
| Hypothesis testing | MH1 | Understand the process of hypothesis testing and the associated language. | Null hypothesis, alternative hypothesis. Significance level, test statistic, 1-tail test, 2-tail test. Critical value, critical region (rejection region), acceptance region, *p*-value. |  |  |
| H2 | Understand when to apply 1- tail and 2- tail tests. |  |  |  |
| H3 | Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis. | For a binomial hypothesis test, the probability of the test statistic being in the rejection region will always be less than or equal to the intended significance level of the test, and will usually be less than the significance level of the test. Learners will not be tested on this distinction. If asked to give the probability of incorrectly rejecting the null hypothesis for a particular binomial test, either the intended significance level or the probability of the test statistic being in the rejection region will be acceptable. |  |  |
| **Null and alternative hypotheses** | | | | | |
| The null hypothesis for a hypothesis test is the default position which will only be rejected in favour of the alternative hypothesis if the evidence is strong enough. Assuming the null hypothesis is true, as a default position, allows the calculation of values of the test statistic which would be unlikely (have low probability) if the null hypothesis were true; this is the critical region (rejection region). | | | | | |

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| **STATISTICS: STATISTICAL HYPOTHESIS TESTING (1)** | | | | | |
| Hypothesis testing for a binomial probability *p* | H4 | Be able to identify null and alternative hypotheses (H0 and H1) when setting up a hypothesis test based on a binomial probability model. | H0 of form *p* = a particular value, with *p* a probability for the whole population. | H0, H1 |  |
| H5 | Be able to conduct a hypothesis test at a given level of significance. Be able to draw a correct conclusion from the results of a hypothesis test based on a binomial probability model and interpret the results in context. |  |  | Normal approximation. |
| H6 | Be able to identify the critical and acceptance regions. |  |  |  |
| **STATISTICS: STATISTICAL HYPOTHESIS TESTING (2)** | | | | | |
| Hypothesis testing for a mean using Normal distribution | MH7 | Know that random samples of size *n* from  have the sample mean Normally distributed with mean and variance . |  | Sample mean,  Particular value of sample mean,  Population mean, | Central Limit Theorem |
| H8 | Be able to carry out a hypothesis test for a single mean using the Normal distribution and be able to interpret the results in context. | In situations where either (a) the population variance is known or (b) the population variance is unknown but the sample size is large Learners may be asked to use a *p*‑value or a critical region.  H0 of form *µ* = a particular value, where *µ* is the population mean.  Significance level will be given. |  |  |
| H9 | Be able to identify the critical and acceptance regions. |  |  |  |

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| **Specification** | **Ref.** | **Learning outcomes** | **Notes** | **Notation** | **Exclusions** |
| **STATISTICS: STATISTICAL HYPOTHESIS TESTING (2)** | | | | | |
| Informal hypothesis testing for correlation/ association | MH10 | Understand correlation as a measure of how close data points lie to a straight line.  Understand that a rank correlation coefficient measures the correlation between the data ranks rather than actual data values. | Learners are not required to know the names of particular correlation coefficients. | *r* | Calculation of correlation coefficient |
| H11 | Be able to use a given correlation coefficient for a sample to make an inference about correlation or association in the population for given *p*-value or critical value. | Association refers to a more general relationship between the variables.  The (often implicit) null hypothesis is of the form either that there is no correlation or no association in the population. Questions will use an appropriate correlation coefficient and indicate whether correlation or association is being tested for. | Questions may require understanding of notation from software; sufficient guidance will be given in the question. | Knowledge of bivariate Normal distribution |
| **Calculating correlation** | | | | | |
| Learners are expected to use technology to work with real data, including the pre-release data. Calculators, spreadsheets and other software will calculate correlation coefficients. Learners may be asked to interpret such correlation coefficients in the examination. The following points should be noted:   * A correlation coefficient measures the strength of a linear relationship. A correlation between the ranks of the data values may be used for a more general relationship. * Correlation coefficients will only be used for data where both variables are random (not, for example, for time series data where one variable occurs at set intervals). * Outliers or distinct sections of data in the scatter diagram can affect the value of the correlation coefficient. | | | | | |
| **Conclusion from a hypothesis test** | | | | | |
| Learners are expected to make non-assertive conclusions in context.  E.g. “There is not enough evidence to conclude that the proportion of... has increased.”  E.g. “There is enough evidence to indicate that the probability of ..... has changed.”  E.g. “There is insufficient evidence to indicate that the true mean of ..... is lower than......”  E.g. "There is sufficient evidence to suggest that there is positive correlation between..... and ....."  E.g. "There is not sufficient evidence to suggest that there is association between ... and ...." | | | | | |

# Thinking Conceptually

**General approaches**

Prior to working with statistical hypothesis testing, it would be beneficial if learners had a firm understanding of the different probability distributions. This should be a core component of the initial approach.

It is useful to make links with other subject areas; learners may have experience of using different tests to make inferences in these other subject areas.

It is useful to use graphs of the distributions as this may help learners understand some of the key concepts. Reference to interactive resources will help visualise the area of interest on the graph of the distribution.

It is important to emphasise that learners are using sample evidence to make a choice between a null and alternative hypothesis and it must be stressed that the null hypothesis stands unless the sample evidence clearly supports the alternative hypothesis. Leaners must have experience of lots of different examples to ensure that they understand the language needed to make the conclusions based on their calculations.

Learners must be encouraged to make a final and most important step to interpret, in the context of the problem, what the hypothesis test results suggest.

**Common misconceptions or difficulties learners may have**

There are a number of misconceptions that learners may hold, or develop regarding statistical hypothesis testing and care should be taken to avoid these becoming ingrained in learners.

One source of confusion for learners may be in the language used to interpret and create the hypotheses. It is imperative that learners are clear in how these are constructed. Being confused between a one-tail test and a two-tail test is a common error.

For Normal hypothesis testing and hypothesis testing using correlation coefficients, students sometimes get confused between the test statistic and the population parameter.

Students sometimes think that either the significance level or the *p*-value gives the probability that the null hypothesis is true. That is not the case.

Learners may find it difficult to understand why the probability of the whole “tail” is found – finding critical regions before working with observed values and probabilities can help conceptual understanding.

It would be useful to spend some time with examples of hypotheses and the final result for learners to then make their own interpretations. This would highlight misconceptions held and allow for discussion of the lack of absolute certainty with hypothesis testing.

It is important to highlight to learners the difference between significant and non-significant outcomes, it is important that they understand that a non-significant outcome means that the data does not suggest that the null hypothesis is false rather than being able to state definitely that the null hypothesis is true.

Learners should be encouraged to pay attention to the context of the data, referring back to the initial context given to give the result meaning. Teachers should encourage learners to check the reasonableness of their results within the context, and the context should remain at the centre of any learning.

It is often useful to present incorrect conclusions to the hypothesis test to learners to encourage them to highlight and discuss the errors.

**Conceptual links to other areas of the specification**

There is a lot of problem solving involved in statistical hypothesis testing; learners need to be able to extract the information they need from the questions and data given in order to arrive at a solution.

Learners must have a clear understanding of the basics of probability distributions and how they can be applied to perform statistical hypothesis testing.

Teachers should ensure that time is spent on longer questions so that learners have the opportunity to extract the data they need and ignore extraneous information.

It is strongly suggested that teachers provide as many real life uses of statistical hypothesis testing from other topic areas and different real world examples to emphasise the relevance of this area of mathematics to learners.

# Thinking Contextually

Learners need to see the relevance of their learning to real life events; they often struggle to understand the concepts in mathematics unless they can see the relevance.

The very nature of statistical hypothesis testing is contextual and many different areas can be used to enhance learners understanding, these can be as basic as a simple test to the need to collect data to test their own hypotheses. This can be a very useful way of checking their full understanding of the concepts covered in statistical hypothesis testing.

Learners will be more successful if they can see how the concepts can be used outside of the classroom. If scenarios are chosen that are meaningful to the learners this will help to maintain their interest and motivation. This will also help learners to focus on the mathematics and lead to independent thinking and greater retention of the skills.

# Resources

| **Title** | **Organisation** | **Description** | **Ref** |
| --- | --- | --- | --- |
| [Statistical hypothesis testing using the binomial distribution](http://www.mei.org.uk/files/sow/17-statistical-hypothesis-testing-using-the-binomial-distribution.pdf) | MEI | A commentary of the underlying mathematics, a sample resource, a use of technology, links with other topics, common errors and questions to promote mathematical thinking. | H1, H2, H3, H4, H5 and H6 |
| [A ducks story- introducing the idea of testing (statistical) hypotheses](http://serc.carleton.edu/sp/cause/conjecture/examples/18163.html) | CAUSE | A simple story and a worksheet with questions to guide the students. The teaching material is intended to be flexible depending on the time available. Instructors can choose to do just the interactive lecture type, interactive lecture + activity, or even add the optional material. | H1, H2, H3, H4, H5 and H6 |
| [Introducing Hypothesis testing](http://www.wiley.com/college/sc/lock/resources/Lock-Instructors_Manual_sample.pdf) | Wiley | A full set of class teacher’s notes and lesson plans on hypothesis testing that can be adapted.  All solutions to the examples are given. This goes on to confidence levels. | H1, H2, H3, H4, H5 and H6 |
| [Test for a Binomial Proportion](https://www.examsolutions.net/tutorials/test-binomial-proportion/?board=OCR&module=s2&topic=1876) | Exam Solutions | In this video, hypothesis testing for the binomial distribution is introduced along with notation used, one-tail tests and significance levels. The conditions that are needed to consider in order to accept or reject the null hypothesis are also covered. | H4, H5, H6 |
| [S2 - Hypothesis Testing - The Population Proportion p, using a Binomial Distribution - Example 3](https://www.youtube.com/watch?v=cTHO4bxOPAQ) | MathsAcademyUK3 | Video demonstration of Hypothesis Testing - The Population Proportion p, using a Binomial Distribution - 2 tailed test. Goes into splitting the Significance Level. And using the Critical Values Method. | H4, H5, H6 |
| [Hypothesis testing using the binomial distribution](https://www.geogebra.org/m/rTGEWmn7) | Geogebra | Interactive resource representing Hypothesis test using the binomial distribution. | H4, H5 and H6 |
| [Hypothesis Testing](http://www.mei.org.uk/files/sow/37-hypothesis-testing.pdf) | MEI | A commentary of the underlying mathematics, a sample resource, a use of technology, links with other topics, common errors and questions to promote mathematical thinking. | H7, H8, H9, H10 and H11 |
| [Testing Hypotheses (Means, Proportions, and Standard Deviations)](https://education.ti.com/en/activity/detail?id=0ADE3A000C4447C9A4E6DD38FA01E3A3) | Texas Instruments | A full lesson dedicated to testing hypothesis using GDC, this includes all the resources needed and teacher notes. Although this resource focuses upon TI graphical calculator, activity could be adapted to alternate GDC or using a combination of graphing software and scientific calculators. | H7, H8 and H9 |
| [Intro to Hypothesis Testing in Statistics - Hypothesis Testing Statistics Problems & Examples](https://www.youtube.com/watch?v=VK-rnA3-41c) | mathtutordvd | Video presentation of using normal distribution model. Part 1. Time spent define Null and Alternate Hypothesis (note this is an American source so Ho and Ha notation used). | H1, H2, H3, H7, H8 and H9 |
| [Null and Alternate Hypothesis - Statistical Hypothesis Testing - Statistics Course](https://www.youtube.com/watch?v=_Qlxt0HmuOo) | mathtutordvd | Video presentation of using normal distribution model. Part 2. Focuses on definition of Null and Alternate Hypothesis in greater detail(note this is an American source so Ho and Ha notation used). | H1, H2, H3, H7, H8 and H9 |
| [S2 - Hypothesis Testing - The Mean, µ using a Normal Distribution - Example 1](https://www.youtube.com/watch?v=9PkYISeHHI4&index=1&list=PL6KJ8qhTyY-GBB1BMc0ZmLkwqGduiCbTp) | MathsAcademyUK3 | Video demonstration of testing the population mean, µ of a continuous variable using the Normal Distribution. | H7, H8 and H9 |
| [S2 - Hypothesis Testing - The Mean, µ using a Normal Distribution - Example 2](https://www.youtube.com/watch?v=f8J5nIZgrYY&index=2&list=PL6KJ8qhTyY-GBB1BMc0ZmLkwqGduiCbTp) | MathsAcademyUK3 | Video demonstration of testing the population mean, µ of a continuous variable using the Normal Distribution. | H7, H8 and H9 |
| [Hypothesis Testing - Statistics](https://www.youtube.com/watch?v=0XXT3bIY_pw) | Math Meeting | Video example of a 2 tail hypothesis test undertaken on a data set modelled as a normal distribution. | H7, H8 and H9 |
| [Hypothesis Testing Example 1 with CASIO fx 991 ES](https://www.youtube.com/watch?v=_xhJl6vgWj8) | Wei Ching Quek | Examples of statistical hypothesis tests for data modelled as a normal distribution using Casio fx 991 ES. Can be adapted slightly for alternate calculator models. | H7, H8 and H9 |
| [Hypothesis Tests of Mean with Changing Sample Mean](https://www.geogebra.org/m/ZCxjHMj6#material/H7kgsXR7) | Geogebra | Interactive demonstration. | H7, H8 and H9 |
| [Why Correlations?](http://www.cpalms.org/Public/PreviewResourceLesson/Preview/53889) | CPALMS | A full lesson plan on using the correlation coefficient. Worked examples and assessments are included. | H10 and H11 |
| [Hypothesis testing with Pearson’s](https://www.youtube.com/watch?v=dkrtZ4pbygg) | ProfNoria | A video using an example to show how to use Pearson’s product moment correlation coefficient for hypothesis testing. | H10 and H11 |
| [Hypothesis testing with Pearson’s](https://www.youtube.com/watch?v=rR-jptLvhFw) r | statslectures | A video with a situation and full worked example including calculations for Pearson’s product moment correlation coefficient. | H10 and H11 |

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