OCR
Oxford Cambridge and RSA

Wednesday 28 June 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4773/01 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the barcodes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

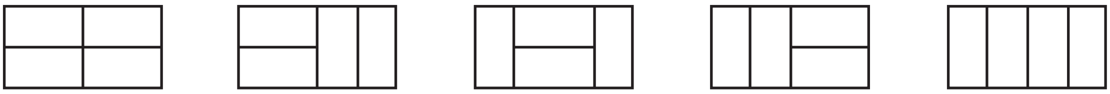
COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 A path is to be constructed using slabs which are 30 cm wide and 60 cm long. The path is to be 60 cm wide. Let u_n be the number of possible ways of constructing a path of length $30n$ cm.

So $u_1 = 1,\ u_2 = 2,\ u_3 = 3,\ u_4 = 5, \ldots$ etc.

For instance, here are the ($u_4 =$) 5 ways of constructing a path of length 120 cm …



(i) Explain why $u_{n+2} = u_{n+1} + u_n$. **[4]**

(ii) Construct a spreadsheet to find the values of u_n for $n = 1, 2, 3, \ldots 20$. **[2]**

(iii) Show that the auxiliary equation for the recurrence relation in part **(i)** is $\lambda^2 - \lambda - 1 = 0$. **[3]**

(iv) Find the solutions to the auxiliary equation and explain how to use them to construct a solution to the recurrence relation. Do not attempt to find a solution, but give two equations that need to be solved simultaneously. **[3]**

(v) Extend your spreadsheet to verify that $u_n = \left(\frac{1+\sqrt{5}}{2\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$ is a correct solution to the recurrence relation. **[1]**

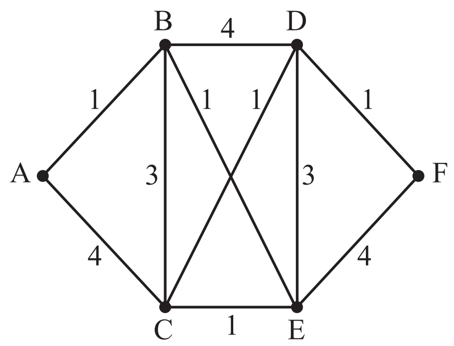
(vi) Find how many paths there are of length 15 m. **[1]**

Another path is to be constructed from slabs which are 30 cm wide and 90 cm long. This path is to be 90 cm wide.

(vii) How many such paths are there of lengths 30 cm, 60 cm and 90 cm? **[1]**

(viii) Use a spreadsheet to find how many of these 90 cm wide paths there are of length 15 m. **[3]**

2 **(a)** The network below is labelled with distances between vertices.



The following LP is formulated to solve a problem in the network …

Min AB+4AC+3BC+4BD+BE+3CB+CD+CE+4DB+DC+3DE+EB+EC+3ED+DF+4EF
st AB+AC=1
AB+CB+DB+EB−BC−BD−BE=0
AC+BC+DC+EC−CB−CD−CE=0
BD+CD+ED−DB−DC−DE−DF=0
BE+CE+DE−EB−EC−ED−EF=0
DF+EF=1
end

(i) What problem does the LP solve? **[2]**

(ii) What does the line AB+AC=1 achieve? **[1]**

(iii) What does the line AB+CB+DB+EB−BC−BD−BE=0 achieve? **[1]**

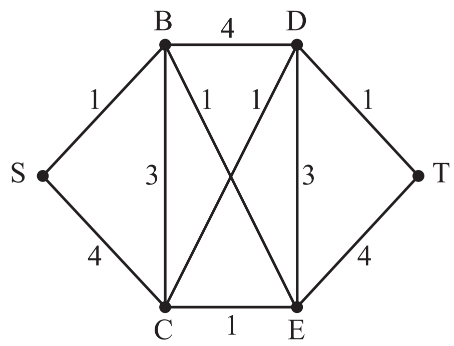
The solution includes the following output …

Objective value 5

Variable	AB	AC	BC	BD	BE	CB	CD	CE	DB	DC	DE	EB	EC	ED	DF	EF
Value	1	0	0	0	1	0	1	0	0	0	0	0	1	0	1	0

(iv) Interpret this output. **[2]**

(b) The network below is labelled with the capacities of pipes.



(i) Formulate an LP to find the maximum flow through the network from the source S to the sink T. **[6]**

(ii) Run your LP and interpret the solution. Provide a diagram showing your flows. **[5]**

(iii) Confirm that your flow from part **(ii)** is maximal by finding a cut with the same capacity as that flow. **[1]**

3 (a) Freda is packing her rucksack for a day's walking. She does not want to carry more than 3.5 kg in the rucksack. The table shows the items which she might pack and their masses. It also shows the value of each item, which is a measure which Freda has assigned to its importance. Freda wants to maximise the total value of the items which she will carry.

Item	Weight (g)	Value
map	90	150
compass	100	50
water	1530	250
food	600	150
chocolate	150	60
fruit	300	60
suntan cream	110	70
camera	320	30
change of clothing	640	15
waterproof clothing	800	75
sunglasses	60	15
socks	40	50

(i) Formulate this as a 0/1 integer programming problem. **[4]**

(You will need 12 variables, and you can specify all of them as 0/1 variables by adding "INT 12" to the end of your formulation.)

(ii) Run your program and interpret the output. **[3]**

Freda decides that she really must carry the waterproof clothing in addition to the items indicated by the above solution.

(iii) How much must Freda carry to achieve this? **[1]**

Freda decides that she must carry the waterproof clothing, but that she cannot carry more than 3.5 kg.

(iv) Modify your model to find the best solution for Freda, including the waterproof clothing, but including no more than 3.5 kg in total. Give that solution. **[4]**

(b) Freda and four of her friends go on a trekking holiday. They each have the same list of 12 items to pack as in part **(a)** of the question. They hire a porter to carry some of their items. The porter can carry up to 15 kg. The total value of the items carried by the porter is to be maximised.

(i) Formulate this as a (general) integer programming problem.

(You will need to add "GIN 12" at the end of your formulation.) **[2]**

(ii) Run your program and interpret the output. **[4]**

4 The number of customers arriving at the till of a gift shop during each 15-minute period during a day is a random variable shown in the table.

Number of customers	0	1	2	3	4	5
Probability	0.06	0.18	0.34	0.25	0.12	0.05

The number of gifts purchased by a customer is also a random variable, shown in the next table.

Number of gifts	1	2	3	4
Probability	0.32	0.41	0.16	0.11

The two random variables are independent.

(i) Give the maximum number of gifts which can be purchased in a 15-minute period. **[1]**

(ii) Construct a spreadsheet to simulate the number of gifts purchased in a 15-minute period. **[10]**

(iii) Run your simulation 20 times. **[2]**

(iv) Estimate the mean and standard deviation of the number of gifts purchased in a 15-minute period. **[2]**

(v) Compute an estimate of how many simulations will be needed to estimate the mean number of gifts purchased in a 15-minute period, to within an accuracy of ± 0.1.

(You will need n such that $\frac{2s}{\sqrt{n}} < 0.1$, where s is an estimate of the standard deviation of the number of gifts purchased.) **[3]**

END OF QUESTION PAPER

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