GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Reports on the Units

January 2010

3890-2/7890-2/R/10J

Oxford Cambridge and RSA Examinations

This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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REPORTS FOR THE UNITS

GCE Mathematics and Further Mathematics Certification

OPTIMISING GRADES FOR GCE MATHEMATICS QUALIFICATIONS

Centres are reminded that when candidates certificate for a GCE qualification in Mathematics they are strongly advised to recertificate for any GCE Mathematics qualification for which they have previously certificated.

For example

- a candidate certificating for A level Mathematics is advised to recertificate for AS Mathematics if this has been certificated in a previous session.
- a candidate certificating for A level Further Mathematics is advised to recertificate (or certificate) for AS Mathematics, A level Mathematics and AS Further Mathematics.

The reason for this is to ensure that all units are made available to optimise the grade for each qualification.

Certification entries are free of charge.

MANUAL CERTIFICATION FOR FURTHER MATHEMATICS

It is permissible for candidates to enter for GCE Further Mathematics with the OCR specification if they have previously entered (or are simultaneously entering) for GCE Mathematics with another specification or Awarding Body. In this case OCR has to check that there is no overlap between the content of the units being used for the GCE Mathematics qualification and the GCE Further Mathematics qualification.
A Manual Certification Form must be completed for each candidate. A copy of the form is available on the GCE Mathematics pages on the OCR website. If you wish to have an electronic copy of the form email your request to fmathsmancert@ocr.org.uk

The table below summarises this.

Qualification	
7890	Candidates are **strongly advised** to apply for recertification for 3890 in the same series as certificating for 7890 if this has been certificated in a previous session.
3892	Candidates are **strongly advised** to apply for recertification (or certification) for 3890 (and 7890 if enough units have been sat) in the same session as certificating for 3892. If a candidate has certificated or is certificating for AS Mathematics or A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* **must** be completed and returned to OCR.
7892	Candidates are **strongly advised** to apply for recertification (or certification) for 3890, 7890 and 3892 in the same series as certificating for 7892. If a candidate has certificated or is certificating for A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* **must** be completed and returned to OCR.

*A copy of the Manual Certification form is available on the GCE Mathematics pages on the OCR website. It may be photocopied as required, and should be returned to:
The Qualification Manager for Mathematics, OCR, 1 Hills Road, Cambridge, CB1 2EU; Fax: 01223 553242.
An electronic copy of the form may be requested by emailing fmathsmancert@ocr.org.uk When completed the form can be returned by email to the same address.

Chief Examiner's Report – Pure Mathematics

At this session, candidates for Core Mathematics 1 wrote their solutions in Printed Answer Books and the marking was carried out online. This will be extended to other units at future sessions with Core Mathematics 2 being assessed in this way at the next session. In general, the assessment for Core Mathematics 1 proceeded well. Centres and candidates are advised to note the following points with respect to answering in a printed answer book.

(a) Avoid placing negative signs, decimal points, straight lines of a diagram, etc. so that they coincide with the rulings of the page. They may not be visible on the scanned version of the page.
(b) Keep strictly to the space allotted for each solution. Do not allow a solution to extend into the space allotted for the next question. If there is insufficient space for the solution, continue on an extra single sheet.
(c) To make a second attempt at a question, use the allotted space if there is still room for the second attempt. Otherwise use an extra single sheet.
(d) Do not use graph paper.
(e) Do not use a pen containing ink which tends to seep through paper. This can affect what appears on the reverse side of the page.

The following piece of advice applies to candidates for other units providing solutions in the conventional way using Answer Booklets. The front page of such a booklet has nine lines where candidates can write solutions. Candidates are advised not to use this space for the first question or part question if the solution is likely to extend for more than nine lines. It is common for candidates to misread their own work when turning over the page to continue the solution.

4721 Core Mathematics 1

General Comments

In general, candidates were well prepared for this paper. Most made an attempt at every question, although questions 4(ii), 6(ii) and 11(iii) proved to be demanding for many candidates. The majority of scripts showed an appropriate amount of working and the response to the proof in question 11(ii) was encouraging.

As in previous sessions, many candidates of all abilities showed a lack of understanding of the rules for manipulating indices (question 4). There were also many occurrences of errors resulting from careless use (or omission) of brackets. Some candidates tried to solve every quadratic equation on the paper by completing the square, when factorisation would have been considerably quicker and more likely to yield the correct results.

Most candidates appeared to have enough time to complete the paper and many scripts showed evidence of checking and correcting answers. The full range of marks from 0 to 72 was awarded.

Comments on Individual Questions

1) This opening question was answered correctly by almost every candidate and incorrect expressions, usually $(x-6)^2-37$ or $(x-6)^2+37$, were rarely seen.

2) (i) The sketch was generally done well although some candidates put no values at all on their axes so it was impossible to establish whether they had transformed the original graph correctly. The most common error was in the choice of scale factor: there were many horizontal stretches seen, usually between −4 and 8 on the *x*-axis, but also vertical stretches with scale factor ½. In some otherwise correct diagrams, the point that should have been at (2, 2) was labelled as (2, 1) or (2, 3). A small number of candidates transformed the original graph by translating it, some vertically and others horizontally.

(ii) It was pleasing to note that the correct word ('translation') to define the transformation was much more in evidence than in previous sessions. However, there were still many cases of 'move' or 'shift' and 'squeeze' was also seen. Although most candidates realised that the translation was parallel to the *x*-axis and to the right, too many lost marks with imprecise definitions such as '1 unit towards the *x*-axis' or 'transformed in the *x* direction by scale factor 1'.

3) The majority of solutions were concise and accurate. A few candidates forgot to use the negative reciprocal of −4 in their straight line equation and some failed to rearrange the final equation into the required form. There was the occasional careless slip made in differentiating but most candidates scored well on this question. Some candidates recognised that differentiation was required but thought that they needed to equate either $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ to zero to find the gradient of the curve.

4) (i) This question was almost always correctly answered, with only a very few instances of the wrong power of 3 being given.

(ii) As in previous sessions, manipulation of indices proved to be a poorly understood topic. It was very common to see $18p^2 = 24$, resulting in $p = \sqrt{\frac{2}{3}}$. Some candidates chose to square 24 and they often calculated 24^2 and then $576 \div 36$ correctly. However, whichever method was chosen, only a small minority of candidates remembered to give both 2 and −2 as their final values.

(iii) This question on indices was also poorly done. $25^{2n+4} = 25$ was very commonly seen and there were also instances of $5^{n(n+4)} = 25$. A large number of candidates could only tackle the question by trial and improvement and they were fortunate that the required value was an integer.

5) The majority of candidates recognised that the given equation was a disguised quadratic and made the correct substitution for $\sqrt{x}$. They were generally successful at solving the resulting quadratic but then forgot to square their roots to find the solutions of the original equation. Those few who did try to square often expanded $(4 \pm \sqrt{3})^2$ as 16 ± 3 or $25 \pm 8\sqrt{3}$. Only the most able candidates scored all 7 marks for this question while over a quarter of the candidates scored no marks at all here.

6) (i) This question was well answered with the vast majority of responses gaining both marks.

(ii) Candidates found this question surprisingly difficult, possibly because the coordinate geometry skills being tested were set in an unfamiliar context. A failure to use brackets in the gradient formula often resulted in a quadratic equation that could not be factorised. Many candidates who got the correct quadratic did not have the confidence to proceed, presumably put off by the decimal coefficients. Some tried the quadratic formula but could rarely find the square root of 0.09 correctly.
An alternative approach was to find the equation of the line with gradient 2.3 through (1, 6). This was not sufficient to earn any marks unless the coordinates of point B were substituted into the resulting equation to form a quadratic equation in *a*. Of the incorrect approaches, by far the most common was to use the expression from part (i) and solve $2x = 2.3$. There were also scripts where candidates interpreted the coordinates of B incorrectly, substituting $x = a, a^2$ and $y = 5$ into their gradient expression!

(iii) Although a significant minority of candidates failed to make any attempt at this question, there were many correct values between 2 and 2.3, with some candidates simply stating $2 <$ gradient < 2.3, which showed the understanding required to earn the mark.

7) (i)(a) Although all but the very weakest candidates were able to match at least two of the equations with the graphs, a significant number thought that the answer to part (a) was Figure 2 (rather than Figure 3), presumably because they recognised the repeated root but failed to realise that the quadratic term was positive.

(b) Almost all candidates matched this equation correctly.

(c) Again, a successful response was common.

(ii) The final part of this question was not well answered. A method mark was available for those who had made errors in part (i) but then gave a correct equation for their unmatched curve. Fewer than one third of the candidates gave a fully correct equation for Figure 2, although many gave a partially correct expression with a negative x^2 term and either a repeated root of 3 or a *y* intercept of −9, which gained them 1 mark. Some very good candidates omitted the '$y =$' in their answer and thus lost a mark.

8) (i) The response to this circle question was a definite improvement on previous papers where the equation of a circle was given in expanded form. Many candidates scored full marks, with all but the weakest identifying the correct centre. However, the radius was not always found correctly, usually because of errors with signs of the three constants in the completed square equation. The most common wrong radius was 3 but 4 was also given when 9 + 4 + 4 was calculated as 16. Answers of 2, 17 and 17^2 were also seen.

(ii) Not all candidates recognised that this question required the solution of two simultaneous equations. Some drew a sketch or referred to the gradient of a radius and were unable to make any progress at all. Those who started correctly by substituting to obtain an equation in one variable generally did well. There were some errors in expanding $(3x+4)^2$ but this was largely done accurately. It was pleasing to see so many candidates able to factorise the resulting quadratic equation correctly, many halving to get a simpler quadratic starting with $5x^2$. Those who tried to use the formula often made computational errors and those who chose to complete the square were even less likely to be successful.

9) (i) Differentiation is usually well done by candidates of all abilities. Although more demanding than usual, with both negative and fractional powers of *x*, this question was done carefully and accurately by most. Only a small minority of candidates failed correctly to express $\frac{1}{x}$ and $\sqrt{x}$ in the form x^n before differentiating. A few left +3 in their differentiated expression or wrote +3*x*. A large number of candidates went on to score all 8 marks on this question.

(ii) Although most candidates tackled this second differentiation well, the notation was clearly unfamiliar to a significant number. Some multiplied one of their expressions by 4, others substituted 4 into f(*x*) or $f'(x)$ and then attempted to differentiate. A few thought that they had to solve $f''(x) = 4$. A good proportion of those who obtained $f''(x)$ correctly were able to evaluate their expression correctly but some could not simplify their fractions without error, e.g. $2x^{-3}$ became 8^{-3} in too many scripts.

10) Most candidates knew that they needed to consider the discriminant, although not all equated it to zero. The 2 occurrences of *k* caused difficulties to many candidates, with $-30^2 - 100k$ seen frequently. A disappointingly large number of candidates wrote $900 - 100k^2 = 0$ followed by $30 = 100k$ and many solutions omitted the negative square root (as in question 4(ii)). Even so, the majority of candidates scored at least half marks on this question. There were some rare but interesting alternative methods: as well as attempts to factorise the equation (or the equation divided through by *k*) into 2 identical brackets, a few candidates used the fact that, for a quadratic equation with a repeated root, the gradient of the graph must equal zero at the repeated root.

11) (i) It was disappointing to see such poor algebra among those who realised that Pythagoras' theorem was required to find the length of the diagonal side in the diagram. Careless omission of brackets led to $7x^2$ in very many scripts. Then $\sqrt{7x^2}$ was deemed to be 7*x*, which was by far the most common wrong length. It did not seem to occur to candidates that the diagonal of a rectangle with sides of 3*x* and 4*x* could not be 7*x*. Expressions for perimeter which contained x^2 terms were also frequently seen, which had repercussions in part (iii).

(ii) This proof was done well and the majority of candidates scored both marks. Some candidates used the formula for area of a trapezium, while others added the areas of the rectangle and the triangle or subtracted the triangle from a larger rectangle. Most solutions showed an appropriate amount of working. However, once again, too many candidates omitted essential brackets, writing $2 + 5x - 2 + x = 6x$ and then using $6x$ as the base of the triangle.

(iii) This final question proved quite challenging and only the best candidates scored all 7 marks although marks of 4 and 5 were reasonably common. Most could form the inequalities with the correct inequality signs and go on to solve the linear one. However, even those who had solved quadratics correctly in earlier questions did not always get the correct critical points here because of poor algebra. Good candidates solved the quadratic equation by factorisation. Those who used the formula or tried to complete the square were much less likely to be successful. It was pleasing to see many candidates sketch the graph to decide on the correct set of values for x.
Having solved both inequalities, the better candidates realised that they needed to combine their answers and it was encouraging to see number lines drawn to aid their thinking.

4722 Core Mathematics 2

General Comments

This paper was accessible to candidates, who seemed generally well-prepared and familiar with the topics being tested. The vast majority were able to make an attempt at every question and the paper gave them an opportunity to demonstrate their knowledge.

It is important that candidates can express their mathematical ideas clearly, both in words and using algebra, to convey meaning. The majority of scripts were well-presented and showed clear detail of the methods used, but sufficient working was not always provided when proving a given answer. It is also important that candidates show clear detail so that method marks can be awarded even when the final answer is incorrect. Examiners are unable to guess what may have been done when only an answer is provided. Candidates should also ensure that they delete unwanted work where several attempts have been made at a question. Unless otherwise indicated, the final attempt will be the only one that is marked by the examiner.

It was disappointing that a significant number of marks were lost through an inability to manipulate algebraic expressions, often through lack of care when expanding brackets. Some candidates also struggled to make efficient use of their calculators, often not appreciating how expressions are evaluated, in particular the need to close brackets on some of them. There was also an over-reliance on calculators to evaluate numerical expressions that could have been easily, and correctly, done mentally. This was particularly apparent when evaluating limits in question 5.

Some candidates attempted simplification to make expressions and equations easier to manage. In particular, the binomial expansion in question 3 was divided through by 16 to reduce the coefficients, the expressions in question 5 were multiplied by x^2 and in question 9(iii) there were incorrect attempts to divide though by 6. This is a good strategy if employed appropriately, such as when simplifying the simultaneous equations in question 6, but some candidates do not seem to realise when such a step is wrong.

Comments on Individual Questions

1) (i) This was a straightforward first question and most candidates gained full marks by substituting the required identity and rearranging to show the given equation. Some candidates lost a mark by failing to put their expression equal to 0, and others struggled to rearrange an initially correct identity.

(ii) Most candidates used factorisation to solve the given equation though some used the quadratic formula. Both methods were usually successful, though there were occasional sign errors. The first angle was nearly always correct, but some candidates struggled to find the second angle with 240° or 330° being common mistakes.

2) (i) Whilst some candidates attempted to use the equation of a straight line, with either a numerical or algebraic gradient, most appreciated the need to integrate and could do so successfully. The majority then continued to substitute the given coordinates, but a significant minority failed to gain the final mark by giving their final answer as an expression rather than as an equation involving y.

(ii) Most candidates could then equate their equation to 5 and attempt to solve, though a surprising number struggled with the relatively straightforward factorisation required. However the majority could successfully find both values for p, though some then rejected the correct answer in favour of $p = 2$.

3) (i) Candidates seemed familiar with the binomial expansion and the majority could make a

good attempt at the question. The most successful candidates made effective use of brackets whereas those who ignored this method often obtained a third term of the incorrect sign. Others simply ignored the negative sign from the start and made no attempt to include it in their expansion. A few candidates ignored the instruction to give ascending powers of x and attempted the wrong terms and others attempted to multiply out the brackets, to no avail. Occasionally candidates attempted to first take out a factor of 2^7, but this was rarely successful and candidates must appreciate the need to use the most effective method for the question posed.

(ii) This part of the question proved to be much more challenging for candidates. A number struggled to even identify which term to use, with powers of 4 or 6 being common errors. Of those who could pick out the correct term, a number neglected to use brackets. Of those who clearly indicated their intention to cube the entire term, the majority failed to do so, and sign errors were also common.

4) (i) Candidates seemed familiar with the trapezium rule and most could make a good attempt to apply it. It was very pleasing to see so many fully correct solutions. There were the usual errors of using incorrect x values, attempting integration first, and omitting the necessary brackets. The logarithm function also caused problems with some candidates attempting an initial rearrangement, often resulting in $\log 2 + \log x$, or using natural logarithms instead.

(ii) Only the most able candidates appreciated what was required and provided a convincing reason involving logarithm laws. A few more did divide their answer to part (i) by 2, but without providing any explanation. However, the majority of candidates either calculated the square root of their previous answer or simply redid the entire trapezium rule on the new function.

5) This question was generally very well done and a number of candidates gained full marks. The more successful candidates carried out two separate integrations before subtracting, as using the reverse method often resulted in sign errors from lack of brackets or a negative value for the area. Integrating $9x^{-2}$ caused problems for a few, but the majority could do this correctly. However, it was also disappointing to see a number of candidates who obviously knew what they were doing fail to get full marks through basic errors. Common errors also included equating the two expressions first, and sometimes even rearranging to a quartic before integrating, and incorrect use of limits, either subtracting in the wrong order or using y-coordinates instead.

6) (i) Most candidates seemed familiar with the factor and remainder theorems and could apply them to the given problem, though a few struggled with where to place the 35 when using the remainder theorem. Most then attempted to solve the two equations simultaneously but algebraic errors were surprisingly common when manipulating the equations, as indeed were more fundamental errors in applying an incorrect method. Some candidates attempted a coefficient matching method but this often resulted in confusion when a and b were also used as coefficients in their quadratic quotient. Attempts involving long division were rarely successful, and a surprising number of candidates used the factor theorem to find the first equation but then long division for the second.

(ii) Many candidates used long division successfully in this part of the question, with others using coefficient matching. Some candidates tried to divide by inspection, but with no method shown it was impossible to award method marks when the final answer was wrong. Most candidates left the answers as factors without identifying the quotient, but this was not penalised on this occasion. A significant minority merely evaluated f(-3) to demonstrate that $(x + 3)$ was a factor rather than actually attempting the division.

7) (i) Most candidates were able to quote and apply the correct cosine rule to show the given result, though there were some algebraic errors when rearranging. A surprising number of candidates found the angle in degrees and then converted it, rather than using their calculator in radian mode. A few chose a method based on verification of 1.1 radians, but did not gain full credit as they hadn't fully justified their answer.

(ii) This was generally very well done, with many candidates gaining full marks. Finding the arc length was nearly always done correctly though some introduced rounding errors by working in degrees with fractions of the circle. Most candidates then made a good attempt at the perimeter though a few failed to subtract 4 from the two relevant sides, and others subtracted the perimeter of the sector from the perimeter of the triangle.

(iii) Again, this question was very well done. Finding the area of the sector was generally correct, though a few omitted the $^1/_2$ from the formula. The area of the triangle posed a few more problems with some using incorrect sides. A surprising minority felt the need first to find another angle in the triangle before using this in the formula and others first attempted the perpendicular height of the triangle.

8) (i) This was a very straightforward question and most candidates obtained full marks, either by listing terms or finding the 5th term of the A.P.

(ii) This part of the question caused more problems, though many still obtained the correct answer. The more able candidates could easily write down the *n*th term expression, though $3n + 8$ was a common error. Other candidates used a series of equations that they solved simultaneously to find the required values.

(iii) The vast majority of candidates were able to identify the given sequence correctly, though some included extra descriptions. This was condoned as long as they did not contradict the previous answer.

(iv) This proved to be very challenging for candidates, and fully correct solutions were rare. Most correctly identified that sums were required, could quote the relevant formula and find a correct expression for the sum of N terms. However the sum of $2N$ terms caused more problems and the majority failed to use $2N$ consistently throughout the formula. Others struggled to identify the values for a and d. Although most candidates recognised the need to set their subtraction equal to 1256, mistakes were frequently made when the resulting equation was rearranged, usually as a result of omission of brackets. They often then struggled to solve the resulting quadratic, whatever method was employed. A number of candidates assumed that S_{2N} was equal to $2S_N$ and simply equated their S_N to 1256 which gained only one mark. This was a question where most candidates seemed to appreciate what was required but were let down by errors in basic algebraic skills.

9) (i) Most candidates seemed to have a basic understanding of the shape of the required graph, but did not show due care when sketching it. Examiners expected to see a reasonable amount of the graph in both quadrants, clearly asymptotic in the second quadrant and a steadily increasing gradient throughout. A number of candidates had attempted to calculate coordinates and plot these; given the numbers involved this often led to graphs that were vertical in the first quadrant and horizontal in the second, especially when graph paper was used. Most candidates could correctly identify the point of intersection on the y-axis, though a number gave the coordinates as (6, 0).

(ii) This was a straightforward, routine question and the majority of candidates scored full marks. Most candidates still take logarithms of both sides and then rearrange but the approach involving log to the base 9 is becoming increasingly common as calculators are able to evaluate this.

(iii) Most candidates were able to gain some marks on this question but only the most able were able to gain full credit, and indeed some elegant and creative solutions were seen. Most gained the first mark for introducing logarithms, though some failed to include the 6, and then the second mark for dropping the power. However, most candidates then failed to correctly separate the two terms on the left-hand side and thus struggled to make any further progress. Subsequent steps were not fully justified, for example log9 becoming 2 with no base 3 shown. Candidates must take extra care when the answer is given, but many failed to show every step convincingly.

4723 Core Mathematics 3

General Comments

This paper proved to be quite challenging for many candidates and the final three questions did, to an extent, assess candidates' understanding and their ability to apply their knowledge to the solution of more searching questions. At this level, candidates should expect an examination to do more than just ask them to reproduce well-rehearsed responses to requests of a routine nature. Candidates need to have a thorough understanding of all topics and techniques so that they can readily choose the appropriate response when faced with a situation which may be slightly unfamiliar. So it is disappointing to record that many candidates seemed to have no idea of connected rates of change (as required in question 7), to have no appreciation of the way in which iterative sequences operate (question 8) and to have very uncertain ideas about trigonometry (question 9). By contrast, the first six questions were, in general, answered well.

The standard of the algebraic skills displayed on this paper was varied. The solution of the equation in question 6 needed considerable care and many candidates did proceed to a correct value for *k*. But there were also serious errors such as a step in question 4 from $y = 2 - \sqrt[3]{x+1}$ to $y^3 = 8 - x + 1$. Equations such as $\frac{8\cos\theta}{1-\cos^2\theta} = 3$ and $p = \frac{2\tan 5°}{1-\tan^2 5°}$ occurred in questions 2 and 9 and it was common for candidates not to notice that rearrangement would lead to a quadratic equation. It was also evident that many candidates failed to transfer basic algebraic principles to expressions and equations involving trigonometric ratios. For example, in question 2, $6(2\sin\theta\cos\theta)$ was not infrequently rewritten as $12\sin\theta\, 6\cos\theta$ and, in question 9, division of $3\sin\theta\cos 10°$ by $\cos\theta$ sometimes resulted in $3\tan\theta\frac{\cos 10°}{\cos\theta}$.

Comments on Individual Questions

1) This was a straightforward opening question for the majority of candidates although many did lose one mark by not including the constant of integration. Incorrect attempts included $-20(2x-7)^{-1}$ and many answers involved either $(2x-7)^{-3}$ or a natural logarithm.

2) Part (i) was answered well by most candidates. The identity for $\sin 2\theta$ was either written down or derived from expansion of $\sin(\theta+\theta)$ and $\cos\theta$ was cancelled from the equation, sometimes with a pertinent comment to the effect that $\cos\theta$ could not be zero. A few candidates, having reached $12\sin\theta\cos\theta = 5\cos\theta$, decided that squaring both sides was a necessary next step.

Part (ii) was answered well by many candidates but it did pose more problems for candidates. Incorrect identities for $\operatorname{cosec}^2\theta$ were tried and those using the correct $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$ often had difficulty manipulating the equation to a solvable form. Candidates using $\operatorname{cosec}^2\theta \equiv \frac{1}{\sin^2\theta}$ were more successful and had no difficulty reaching a quadratic equation in $\cos\theta$. Candidates concluding with $\frac{1}{3}$ and -3 as possible values for $\cos\theta$ did not earn the final mark; no justification for omitting -3 was required but many candidates creditably offered a sound reason for doing so. One error which did occur occasionally raised questions about candidates' understanding of the nature of equations; having reached $\frac{8\cos\theta}{\sin^2\theta} = 3$, some candidates promptly substituted the value of $\sin\theta$ found in part (i).

3) This question was answered very well and many candidates earned all seven marks without difficulty. There were a few errors in part (i) and the second mark was not earned if there were mistakes in the working such as $60\ln 20 - 60\ln 10 = 60\frac{\ln 20}{\ln 10} = 60\ln 2$. The calculation in part (ii) was carried out efficiently and accurately by the majority of candidates and the only errors to occur with any frequency involved either the use of values of $60\ln x$ or the use of a formula of the form $\frac{1}{3}h(y_0 + 2y_1 + y_2)$. Candidates were not always able to answer part (iii) convincingly and some did not appreciate the need to use the value $\frac{125}{3}$ from part (ii).

4) This question was also a good source of marks for many candidates and examiners were often able to award full marks. Most candidates had no difficulty with part (i) although some were not sure about finding the cube root of a negative number. Candidates opting to find an expression for $\mathrm{ff}(x)$ before substitution of -126 were not quite as successful as those carrying out successive numerical calculations. Part (ii) was the most troublesome part of this question and some candidates embarked on usually fruitless attempts to solve involved equations. Many candidates did recognise that the point where the graph of $y = \mathrm{f}(x)$ meets the *x*-axis was relevant; some had the awareness to realise that their work in part (i) had already given them the required value and were able to write down the answer $x \le 7$ immediately.

The vast majority of candidates knew, in general terms, how to find the inverse function. Those using an algebraic approach were usually more successful than those using the method using arrow diagrams to represent the functions. Sign slips were common in both approaches but a more serious error was committed by those candidates who started by replacing $y = 2 - \sqrt[3]{x+1}$ with $y^3 = 8 - (x+1)$. Most candidates earned the mark in part (iv); there were a few references to reflection in the *x*-axis or *y*-axis but most referred, in some acceptable form, to reflection in the line $y = x$.

5) This question tested some basic differentiation techniques and many candidates were able to record full marks without undue difficulty. However, many other candidates showed a lack of familiarity with some or all of the aspects involved. In part (i), most realised that the chain rule was necessary and produced the correct first derivative. Confirming that a stationary point occurs when $x = 0$ was then a straightforward task; some candidates did this by substituting the value 0. Not all earned the mark for showing that this is the only stationary point; it was only necessary to state, in some form, that $x^2 + 1 = 0$ does not give a real root but this was omitted by many candidates.

In part (ii), it was surprising how many candidates did not recognise that further differentiation required the use of the product rule; these candidates could gain no credit in this part. A few candidates tried to use the quotient rule here, perhaps because the quotient rule is given in the *List of Formulae* booklet whereas the product rule is not. Many candidates did produce a correct expression for the second derivative although there were some algebraic errors to be seen. Substitution of $x = 0$ into the expression proved surprisingly awkward for some and the incorrect answer 17 appeared quite often. A significant number of candidates either did not read the question carefully or revealed considerable doubt about stationary points by equating their second derivative to zero and attempting to solve the resulting equation.

6) This question needed accurate integration and also great care in dealing with brackets and signs when trying to solve the resulting equation. Many candidates did provide meticulous and clear solutions but many others did make errors, particularly when attempting to subtract the terms associated with the lower limit of 0. Some candidates did not attempt integration at all but just substituted the limits into the given integrand. Others produced $3k\mathrm{e}^{3x} - \frac{1}{2}(k-2)\mathrm{e}^{-\frac{1}{2}x}$ as their attempt at integration. A further common error was a failure to use the limit $x = 0$ at all, presumably due to a mistaken belief that substitution of 0 would lead to a value of 0.

7) The response to these two questions on rates of change was disappointing. In particular, many candidates seemed to have at best a very hazy notion of connected rates of change and struggled to record any marks in part (a). A mark for stating $\frac{\mathrm{d}A}{\mathrm{d}t} = 250$ was often the only mark earned and was typically followed by various haphazard calculations. Some candidates tried to set up an exponential function to describe the situation and, surprisingly, a number of candidates did not know the formula for the area of a circle. Clear solutions linking the three derivatives $\frac{\mathrm{d}A}{\mathrm{d}r}$, $\frac{\mathrm{d}A}{\mathrm{d}t}$ and $\frac{\mathrm{d}r}{\mathrm{d}t}$ in the appropriate way were not common.

There was more success with part (b) although correct solutions were in the minority. Most candidates realised that an expression for $\frac{\mathrm{d}m}{\mathrm{d}t}$ was needed but often this was equated to 3 rather than to –3. Attempts to find an expression for *t* were often not very convincing; some candidates with a correct and acceptable expression for *t* of $-\frac{1}{k}\ln(\frac{1}{50k})$ were suspicious of the minus sign and adjusted their solutions.

8) Part (i) of this question proved to be the most demanding part of the paper and few candidates realised that the two transformations involved corresponded to the curve $y = \sqrt{x}$ first becoming $y = \sqrt{2}\sqrt{x}$ and then $y = \sqrt{2}\sqrt{x + \frac{3}{2}}$. Most wrong answers referred to a stretch with factor $\frac{1}{2}$ and a translation in the negative x-direction by 3 units. The sketches in part (ii) were disappointing in many cases. The graph of $y = \sqrt{2x+3}$ was sometimes correct but often it was shown only for $x \geq 0$ or it was the whole parabola, the equation of which is $y^2 = 2x + 3$, which was drawn. Attempts at the graph of $y = \frac{N}{x^3}$ were no better in general. Sometimes the graph was shown only for $x > 0$ and, in other cases, the attempts appeared to be based on $y = x^3$, $y = -x^3$ or $y = \sqrt[3]{x}$.

Part (iii) tested understanding of the iterative process. Some candidates demonstrated that they had a sound grasp and produced two concise and accurate solutions. Other candidates had little idea and resorted to trial and improvement methods. In part (a), some candidates did understand what was required and proceeded to solve the equation $1.9037 = N^{\frac{1}{3}}(2 \times 1.9037 + 3)$ although not all took note of the fact that *N* was given to be an integer. Often, though, the same technique was again used in part (b), using each given value separately to find two values of *N*. No marks were available for this incorrect approach.

9) This was a more demanding question requiring some thought by candidates on the choice of appropriate methods in the three parts. Many candidates did make sensible choices and there was some ingenuity shown, particularly in part (ii). Most candidates were able to record at least a few marks.

There was much confusion about p; many attempts at part (i) started with $55 = 45 + p$ or involved expansions using $\tan p$. There were also many attempts at the first two parts which merely claimed $\tan 55° = 5.5p$ and $\tan 5° = \frac{1}{2}p$ or which used a calculator to give $\tan 55° = 8.1p$ and $\tan 5° = 0.496p$. However, the correct answer of $\frac{1+p}{1-p}$ in part (i) was obtained by a pleasing number of candidates.

There were several viable methods available in part (ii) and candidates proved to be quite resourceful in devising ways to find $\tan 5°$. One effective approach used the expansion of $\tan(15° - 10°)$ with an exact value of $\tan 15°$ used. Another approach depended on the result from part (i) with the expansion of $\tan(60° - 55°)$. Other methods were based on either $\tan 5° = \tan(10° - 5°)$ or $\tan 10° = \frac{2\tan 5°}{1 - \tan^2 5°}$; these approaches needed the solution of a quadratic equation and, in many cases, this was either not attempted at all or was not concluded correctly. Whatever method had been adopted, candidates were expected to present their final answer in a suitably simplified form.

Many candidates took the appropriate first step in part (iii) and, with the exception of some sign errors, the expansions were usually correct. But not so many candidates then realised that dividing each of the four terms in the equation by $\cos\theta\cos 10°$ would lead to an equation involving p and $\tan\theta$.

4724 Core Mathematics 4

General Comments

There was a wide range of responses; many were excellent, obtaining the top marks but 10% of the candidature understood little of the examination. Misreading and carelessness are becoming more prevalent and simple algebraic and arithmetic errors continue to be present.

Candidates should be aware that the way in which questions are set is meant to be helpful and, certainly, not intended to deceive. In question 3, the instruction was given "By expressing $\cos 2x$ in terms of $\cos x, \ldots$" and so they were expected to write down an expression for $\cos 2x$ and then use it in the definite integral. The question did not suggest expressing $\cos^2 x$ in terms of $\cos 2x$, a device often seen. In question 8(i), candidates were asked for the derivative of $e^{\cos x}$; in part (ii), it is then unlikely that, in the split for integration by parts, u will be equal to $e^{\cos x}$ because the first operation to be done will be to differentiate it – and it is not likely that the same operation will be needed twice so quickly.

Comments on Individual Questions

1) This gave an excellent start for the vast majority of candidates, enabling them to proceed confidently to the more difficult questions. As has been mentioned in previous reports, the long division method seems more reliable than the identity method, particularly since the latter demands knowledge of the form of the remainder.

2) (i) Most candidates realised that they needed to calculate direction vectors and the majority used the correct pair. However, subtraction of the corresponding elements was often poor and the multiplication needed in the scalar products was likewise prone to error.

(ii) Two methods were used in this part – considering ratios of the direction vectors and the standard use of the angle between two lines. The majority, creditably, used ratios and this produced the answer rapidly. The minority, using the angle between two lines, generally became involved in a complicated quadratic equation which they were unable to solve. Others decided that the scalar product should be equal to 1 or –1.

3) This relatively straightforward question did not prove to be easy for the majority of candidates. At the start, most did use $\cos 2x \equiv 2\cos^2 x - 1$. Many other candidates expressed $\cos^2 x$ in terms of $\cos 2x$ which, of course, then made the denominator more complicated. Using the standard method, it was surprising how few candidates were able to simplify $\dfrac{2\cos^2 x - 1}{\cos^2 x}$ to $2 - \sec^2 x$. The final stage involving the limits often produced sign errors.

4) This question was answered correctly by most candidates. One common error involved an initial misreading of the denominator as $t(2 + \ln t)$; examiners applied the misread rule which led to a penalty of one mark. At a later stage, the careless transcription of $t(2 + \ln t)^2$ as $t(2 + \ln t)$ also occurred a number of times.

5) (i) This was answered well with any mistakes usually occurring in the third term.

(ii) As might have been anticipated, the first error was the extraction of the 8 as 8. The idea of x in part (i) being replaced by $2x$ was common although some candidates used the binomial theorem again. Those candidates replacing x by $2x$ frequently failed with the third term where $-\frac{1}{9}x^2$ became $-\frac{1}{9}\times 2x^2$.

Part (b) was generally correct but equality signs did appear sometimes and the use of $-1<16x<1$ was not uncommon.

6) There were not many good attempts at this question. Most candidates knew what to do but were careless with the differentiation or with the process of forming a reciprocal. Few correct differentiations of $\ln(9t)$ were seen, the most common response being $\frac{1}{9t}$. Rather more correct differentiations of $\ln(t^3)$ were seen because candidates often changed it to $3\ln t$ before differentiating. Candidates producing $9-\frac{1}{9t}$ for $\frac{\mathrm{d}x}{\mathrm{d}t}$ almost unanimously then produced $\frac{1}{9}-9t$ for $\frac{\mathrm{d}t}{\mathrm{d}x}$. Those candidates working correctly throughout usually managed to show clearly why $t=-3$ was an invalid solution.

7) This type of question is becoming very popular although many candidates did not make it as easy as they might have done. Having differentiated, candidates had to substitute $x=2$ and $y=1$ and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or *vice-versa*. The easier way is to substitute first and then solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$, a process which avoids various algebraic pitfalls. The equation of the tangent at (2, 1) was produced by a few candidates. Some solutions began with $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ but no candidate actually made use of this extraneous $\frac{\mathrm{d}y}{\mathrm{d}x}$ and this was definitely an improvement on similar attempts in previous sessions.

8) This question was answered poorly in nearly every case, even by candidates otherwise recording very high marks. Candidates appeared confused by the hint given in part (i); surely they should have realised that, if the derivative is requested in part (i), then it is unlikely to be used directly again in part (ii); rather it is the converse of part (i) which is more likely. Many candidates were unable to state the derivative in part (i) correctly and $\mathrm{e}^{\cos x}$, $\sin x\,\mathrm{e}^{\cos x}$, $\cos x\,\mathrm{e}^{\cos x}$ and $\sin \mathrm{e}^{\cos x}$ were all seen. Those candidates using their mathematical knowledge well were able to move steadily through the integration by parts and, usually, produced the correct answer.

9) Many candidates showed that they were not really conversant with equations of the form **r** = **a** + *t* **b**; they were unaware of what the **a** and **b** represented. It seemed obvious that, with the equation given and the value 1 for *t* specified in part (i), the value would be substituted; but that proved not to be obvious to the majority of candidates. Nor was it obvious to candidates which direction vectors should be used to the find the angle. Parts (ii) and (iii) were more successful for candidates although the equation $4t+6=0$ did lead to a surprising number of wrong solutions.

10) The first part was usually answered well although some candidates used A and B without stating to what they referred; and some, having obtained the values of A and B correctly, did not produce the correct fractions. Separation of variables was carried out well; other candidates rearranged the differential equation to the form $\frac{dt}{dx} = ...$ although sufficient care was often not taken with the position of k in the rearrangement. The change to partial fractions was almost universal although the separation of variables and resolution into partial fractions were sometimes combined, a risky procedure.

The integration was done well, with relatively few candidates omitting the negative sign when integrating; however, omission of the constant of integration was much too frequent. In the final part, candidates were expected to use the given value of k together with their own value of c. The manipulations here were generally done well although cases of $\ln p - \ln q = \ln r$ implying $p - q = r$ were not uncommon.

4725 Further Pure Mathematics 1

General Comments

Most candidates were able to provide good attempts at the majority of the questions, showing a good understanding of most parts of the syllabus. Correct solutions to all questions were seen and the presentation was generally better than in previous January papers. There was little evidence that candidates were short of time and most answered the questions sequentially.

Comments on Individual Questions

1) (i) Most candidates answered this correctly, with only odd slips being made. Some candidates thought that **I** was $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(ii) Many correct solutions were seen, the common errors being to use the determinant of **A** – 4**I**, while some did not know how to find a determinant correctly.

2) (i) This was generally done well, but some simple algebraic errors were quite common.

(ii) Quite a number of candidates did not appreciate how the required value was linked to the coefficients of the new cubic equation and many tried to use the symmetric functions of the original cubic.

3) Most knew the conjugate, z^*, and made good attempts to equate real and imaginary parts in order to solve the equation.

4) Most candidates showed the correct procedures for obtaining a correct unsimplified expression for the required sum, but then did not see a common factor of $n(n + 1)$ and so had difficulty in obtaining a completely factorised answer. Misreads, e.g. $(r + 2)$ for $(r – 2)$, were seen but usually candidates were able to score most of the marks.

5) (i) This part was not answered correctly by many candidates. A clearly labelled diagram would have shown that the transformation was a rotation, rather than a reflection.

(ii) As in (i) few clearly labelled diagrams were seen. Those who attempted to find the matrix first often had the order of matrix multiplication the wrong way round.

6) (i) Most candidates used 5 + i as the other complex root and then used $\sum \alpha\beta$ to find the real root. Others found the quadratic factor and solved both parts quite easily.

(ii) Those who had used $\sum \alpha\beta$ generally used the other symmetric functions, with sign errors being the most common cause of loss of marks. The approach of substituting 5 – i or 5 + i into the equation and equating real and imaginary parts often broke down when arithmetic errors occurred.

7) (i) Most candidates answered this correctly.

(ii) The method of differences was generally well used and explained, although a small number of candidates attempted to use the standard result for $\sum r^2$, and so scored no marks.

(iii) Most candidates knew how to find a sum to infinity, but many failed to spot where the series started.

8) (i) The majority of candidates knew how to find square roots of a complex number and solved correctly. Sign errors in setting up the simultaneous equations and sign errors in the final answers were quite common.

(ii) Most candidates realised that the locus was a circle in both cases, but failed to see that both circles pass through the origin. The perpendicular bisector of the line joining the two centres was the most common wrong sketch.

9) (i) The majority of candidates found det**A** and then the adjoint matrix, demonstrating that the techniques were well understood. Sign errors and errors in one of the minors were the common mistakes and few candidates tried to check their answer by matrix multiplication of **A** with $\mathbf{A}^{-1}$ or the adjoint.

(ii) Most candidates used their inverse matrix correctly; the most common error was to attempt to post-multiply a column matrix instead of pre-multiplying. Those who attempted to solve the equations directly either gave up rather quickly or made an algebraic error at an early stage.

10) (i) Most candidates found both matrices correctly, $\mathbf{M}^3 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$ being the most common error.

(ii) A sensible suggestion for $\mathbf{M}^n$ from their (i) was usually made.

(iii) A minority of candidates attempted to add two matrices without realising that the leading diagonal is now incorrect. Those who thought that 2^n rather than $2n$ was the crucial element of $\mathbf{M}^n$ failed to see that this would lead to an element of $(2^n + 2)$, which is not the required result and so should have reviewed their answers to (i) and (ii). Many candidates did not give a clear statement of the induction conclusion with statements like
" It is true for $n = 1$, $n = k$ and $n = k + 1$, so true for all n"
being very common and it is hoped that centres will emphasise this point to future candidates.

(iv) Some candidates gave two transformations, rather than a single transformation as asked for in the question. Many candidates thought that the transformation was a stretch, while those who recognised a shear sometimes were too vague in their description, the best being " x-axis invariant" and the image of one point being given.

4726 Further Pure Mathematics 2

General Comments

Candidates answered the questions in the order set, with no question proving particularly difficult. Candidates appeared to be better prepared, with a sound knowledge of most of the specification items. Most candidates could gain some marks in each question. As a result, there were few very poor scripts. However, there were also relatively few outstanding scripts and it was disappointing how many marks were lost due to carelessness and a lack of precision. Indeed, candidates often lost marks by basic errors in, for example, integrating $e^{-4\theta}$ or squaring brackets. There was some evidence that poor or overlong methods meant that some candidates struggled to finish the final question. Candidates did waste time by not reading questions well enough, by not being able to simplify methodically and by choosing inappropriate methods.

Comments on Individual Questions

1) (i) The majority of candidates were able to number-crunch their way to both marks, with only a minority using degree mode rather than radian mode. Perhaps because it was the first question, many candidates took a long time to write down the question as given, to show and use the given equation and to repeat the process twice. All that was required were the answers taken directly from a calculator.

(ii) Most candidates were able to gain one mark by subtracting correctly. At this point many candidates stopped, obviously unaware of what was being tested. A few other candidates divided the errors the wrong way round. Nonetheless, the question as a whole usually produced at least 3 marks for most candidates.

2) (i) This part was done well, with most candidates able to differentiate twice. Again, some time was lost in applying the chain or quotient rules, the former often requiring a substitution rather than just the written answer. However, most candidates scored well. As usual, it was important to derive fully any answers which were given.

(ii) Again, this part was done well. Candidates unable to find $f''(0)$ sensibly used the given answer in this part and such candidates could gain both marks.

3) (i) It was surprising how few candidates knew the derivation of the Newton-Raphson method and many candidates left this part out. Other candidates recognised the negative gradient but were unsure as to how to deal with it. This resulted in a slightly unconvincing answer which could gain 2 marks. The best solutions came from candidates using gradient equals y-step over corresponding x-step or finding the equation of the tangent at $(x_1, f(x_1))$ and substituting $(x_2, 0)$ into their equation. The negative gradient then took care of itself.

(ii) An accurate sketch was often provided, with only a few candidates wrongly producing a cobweb or staircase approach. The ability to "describe" was limited and often produced some confusion. Two brief sentences describing the tangent at $(x_1,f(x_1))$ meeting the x-axis at $x=x_2$ followed by a tangent at $(x_2,f(x_2))$ meeting the x-axis at $x=x_3$ were all that was required.

(iii) This part was generally well answered with only minor arithmetical errors seen, possibly from difficulties in evaluating sinh1 and cosh1.

4) (i) Curves were often too small. Candidates are encouraged to exaggerate the scale if necessary to ensure that all the relevant points are seen. Too many candidates showed a pole and had the curve passing through it. Other candidates who did not show a pole or who gave a brief table of values including $e^{-2\pi}$ were more successful. The greatest value of r and the corresponding θ had to be clearly seen or implied as a separate item.

(ii) A surprising number of candidates did not know the area formula or could not square $e^{-2\theta}$ or could not integrate $e^{-4\theta}$ correctly. Such basic errors were disappointing. Other candidates used the limits in the wrong order or confused limits of θ as limits of r. However, most candidates were able to gain at least 3 marks on this part.

5) (i) This part was done well. Only a minority of candidates were unable to make the deduction, often resorting to a long method involving the exponential definitions for which no marks were awarded.

(ii) The majority of candidates were able to produce a quadratic equation in sech$x$ or cosh$x$. Some candidates then went on to "solve" sech$x = -1$, although most candidates were able to reject this equation for a variety of reasons (or by just crossing it out), with only a few knowing $0 < \text{sech}x \leq 1$. Credit was given for its omission for whatever reason. Candidates getting cosh$x = 2$ often produced one answer only. Again, the basic properties of the hyperbolic functions were not known well enough. Other candidates who resorted to the exponential definition to solve cosh$x = 2$ usually produced both answers but wasted some time in getting there.

6) (i) Most candidates knew the correct form for the partial fractions, with only a few candidates using $C/(x^2 + 1)$. The most successful and quickest method in this case involved substituting $x = 1, -1, 0$ and 2 (say) to find the required constants. Other candidates multiplied out and then equated coefficients, often accurately, but again such candidates wasted time. A mixture of the two methods often proved successful. There were some interesting approaches using (x^2-1) instead of $(x-1)(x+1)$, but the final answer was then left with the (x^2-1) as a denominator. This method could be used in the second part however.

(ii) There were four main approaches. The first involved a quoted answer for the integral given directly from the formulae sheet, evidently from candidates who did not recognise a connection between the two parts. The second, from a surprisingly large number of candidates, involved $\int (1/(1-x))\,dx = \ln(1-x)$. The third involved writing the top limit as $\sqrt{3}/3$ and then attempting to reach the answer by dubious means. The fourth involved a fully correct and relatively short method, using previous results and the limits as given.

7) (i) The explanations given were generally better than in previous years. The best answers came from candidates who showed that the LHS represented the total area of the rectangles and who explained that the RHS represented the total area under the curve between the given limits. Candidates who wrote, for example, "rectangle = $\sqrt[3]{1} + \sqrt[3]{2} + \ldots$" or stated that the RHS was the "actual area" gained few marks.

(ii) Although there was some confusion as to what happened between $x = 0$ and $x = 1$ (despite the limits given in the integral), most diagrams showed the correct rectangles, although not all diagrams showed them going up to $x = n+1$. There were some good explanations using a right shift of the rectangles in the given diagram. Again some candidates lost marks due to a lack of precision.

(iii) It was surprising how many candidates failed to integrate $\sqrt[3]{x}$ correctly, whilst other candidates missed the fact that the limits in the two integrals were different. Candidates were asked to give the final answer correct to 2 significant figures and were not expected to approximate before that.

8) (i) Some candidates spent time in finding or explaining the asymptotes. The two asymptotes

could be "written down" at once. The asymptote at $y = 0$ was missed by a significant minority.

(ii) The most common and most efficient method used was to set up a quadratic in x and use $b^2 \geq 4ac$ for all real x, although it was apparent that some candidates did not appreciate why this worked. Nevertheless, many candidates gained at least 3 marks. The final stage from $4ky + k^2 \geq 0$ to the required answer was often unconvincing, with only a few candidates justifying division by k from the information given in the question. Candidates using differentiation at this stage were less successful, although marks could be gained in part (iii) even if seen here.

(iii) Most candidates started again and used differentiation, not realising that $y \geq -\frac{1}{4}k$ meant that a minimum occurred at $y = -\frac{1}{4}k$ and that feeding this back into their quadratic in x gave the required result. Candidates who used differentiation often divided by k and stopped, not showing that a stationary point existed. Even those candidates who went on to show that $x = -1$ (and -1 only) failed to produce the corresponding y-value. The sketch was generally poor, with candidates often omitting asymptotes they had found and/or the coordinates of the minimum point. It was evident that in some cases a graphical calculator had been used, a useful tool to give an idea of the shape of the curve.

9) (i) Although the majority of candidates scored well on this part, a significant minority were unable to produce what is a basic piece of bookwork.

(ii)a) There were various methods available in this part, with the quickest answers coming from candidates who used the clue in the question and discussed $\tanh x = b/a$. The explanation was sometimes lacking, with candidates using $x = \tanh^{-1}(b/a)$ and the inequalities given in part (i) being most successful. Other candidates used $\tanh x = b/a$ but then resorted to exponential or logarithmic approaches, often successfully. Candidates using arguments based on $\cosh x > \sinh x$ for all x were equally successful, although the argument did need some precision for all marks.

(ii)b) Three marks were immediately available from using the earlier parts of the question to find the x-coordinate of the minimum point. The corresponding y-value often took some time and effort, and many candidates struggled with this part. Some candidates expressed $y = a\cosh x - \sinh x$ as $\cosh x(a - \tanh x)$, which reduced the work somewhat. Other candidates replaced $\cosh x$ by $1/\text{sech}\, x$ and used the connection between sech and tanh, although any problems with $\pm$ were largely ignored. Few candidates successfully showed it to be a minimum, with statements such as "$f''(x) = a\cosh x - \sinh x > 0$" all too common. Again the substitution of the x-value proved difficult. The few candidates who noted that $f''(x) = f(x)$ are to be commended, as are the candidates who used $R\cosh(x - \alpha)$ throughout this part of the question.

4727 Further Pure Mathematics 3

General Comments

As usual in the January session this paper attracted a small entry, although the total was higher than in previous years. On this occasion there were a good number of very well prepared candidates, but there were also several candidates who could do very little of what was asked. The mathematics covered in this paper is demanding, and a full year's work is usually required for preparation. All the questions were accessible to those who had studied the topics, and there did not appear to be any problems with the time allocated for the paper. Presentation was generally fair to good, but a small number of scripts contained work which was scattered over the page or written so badly as to be illegible.

Comments on Individual Questions

1) This question was done very well by the majority of candidates. The technique of using parametric equations and attempting to find whether the resulting equations were consistent was well known, and the solution was usually carried out correctly. Most found the values of the parameters from the x and y equations and then substituted into the z equation to obtain a contradiction. Very few noticed that the x and z equations were contradictory, which would have given the answer straight away. Method 1 of the mark scheme was not used by any candidates. One or two attempts were seen which used only the cartesian equations; this method will work, but it needs care.

2) (i) Most candidates correctly multiplied two different general elements together to show that an element of the same type was produced. Only a very small minority used the same element twice or used numerical values, neither of which was acceptable.

(ii) The identity was usually written down correctly, either as 1 or as $1+0\sqrt{5}$.

(iii) A large number of candidates thought that it was sufficient to state that the identity was $\frac{1}{a+b\sqrt{5}}$. As elements of the group are of the form $p+q\sqrt{5}$, it was essential to attempt to obtain the inverse in that form. A minority used the definition of an identity and solved the resulting equations (the second alternative in the mark scheme), but this took longer and was not always successful.

(iv) Many gave a correct answer, that $\sqrt{5}$ was not rational, or 5 was not a square, or 5 was a prime number, but some did not attempt this part.

3) This differential equation was solved easily by most candidates, and full marks were common. A few integrated wrongly or made algebraic slips, but it was encouraging that the majority worked through the various stages confidently and accurately.

4) (i) Having earned a good number of marks in the first three questions, a majority of candidates struggled to gain much credit for this question. The answers to the first part were intended to be written down, with little or no working. But it was disappointing to find candidates at this level appearing to think that there were only two roots of this quartic equation, both real. The phrase "in cartesian form" should have been a clue that complex numbers were involved and those who gave only real roots earned no marks. Some credit was available, here and in part (ii), to those who found the real positive fourth root of 16 incorrectly.

(ii) Even those who did part (i) correctly often did not spot the connection with the equation in w, and blanks in the answer booklets were much in evidence. For those who saw what to do, it was quite easy to replace z by $\frac{w}{1-w}$ in the four answers to part (i), and to solve the resulting equations for w. Few chose to do this in general, but took more time and used more paper by doing more or less the same thing four times.

5) (i) Vector questions are often done well, perhaps because the topic has been well practised, and examiners were very pleased with many of the responses to this one. There was no doubt that most candidates knew what to do in order to find the equation of the plane ABC, but it had been expected that the proliferation of surds would cause difficulties. However, most answers managed all aspects of the calculation confidently. It helped that the answer was given, but in many cases simplification to the required form was carried out accurately.

(ii) There was much less certainty about the geometrical reason for being able to write down the equation of the other plane at sight. Few sketches were drawn, which would have helped the symmetry to be seen. There were very few who stated either symmetry or reflection and also wrote down the correct plane of symmetry. Many earned just a single mark for saying something about the signs and magnitudes of the y coordinates of C and D, but this omitted consideration of the tetrahedron as a whole.

(iii) Many correct answers were obtained to this straightforward calculation of the angle between the faces of the tetrahedron. The negative answer, implying an obtuse angle, was much more common than the positive answer. The angle between the faces should really have been acute, but as the calculation led directly to the obtuse angle, either was accepted. In the course of the calculation the product of the moduli of the vectors became 2 × the modulus of one of them in several cases. A few candidates either chose to use different planes or spent time deriving the normal to the plane ABD, thereby giving themselves much extra work.

6) (i) Few candidates had problems with this second order differential equation. The complementary function was usually correct, with the main errors being to write the auxiliary equation as $m^2 + 16m = 0$ and to retain complex exponential terms in the answer.

(ii) The stages in the calculation of p were usually carried out correctly. There were some errors in differentiation and a small number of candidates included an unnecessary term $qx\cos 4x$ which was not penalised. Some follow-through marks were available for those who made mistakes early in the working.

(iii) The final part, involving substitution of the initial conditions, was quite straightforward, especially with several zero terms being present, and most answers scored well.

7) (i) Although a good number of candidates found all the required roots of $\cos 6\theta = 0$, there were too many who found only the smallest one, $\frac{1}{12}\pi$. In view of the rest of the question, it should have been clear that there were several more to be found.

(ii) A fair amount of detailed work was needed to obtain the identity, but the method was well known and there were few attempts at other methods which either started from the wrong aspect of de Moivre or did not use the theorem at all. When the substitution $s^2 = 1 - c^2$ had to be cubed it was unusual to see the use of binomial coefficients. The final stage, of obtaining the identity from the polynomial for $\cos 6\theta$, was usually written down at sight, which was accepted. Although no answer used Method 2 of the mark scheme, there were one or two quite ingenious answers which used de Moivre's theorem correctly in a non-direct way.

(iii) Questions of this type have been set before and have usually been found to be quite demanding. The starting point was to give a value to $\cos 6\theta$ which, in view of the earlier parts, was obviously 0. The technique, as in Method 1, was then to associate the six roots from part (i) with the two factors found in part (ii). The value of the product of cosines required came from the quartic equation, either by using the product of roots directly or, more often, by solving the equation. To obtain full credit, however, it was also necessary to justify the cosines used by showing that the ones not included in the required product came from the quadratic factor. This part was solved fully only by a few of the best candidates, one or two of whom used Method 2 which is perhaps neater than Method 1. The majority of other answers earned only a mark or two at the most. The word "Hence" meant that other methods, such as using a calculator or finding the values of the cosines without reference to the previous parts, were not accepted.

8) (i) Examiners were impressed by the ability of many candidates to answer the first part of this function group question. The majority of candidates knew exactly what to do. It was not difficult and both answers were given, but care was needed in sorting out the algebra, and this was shown.

(ii) Most wrote down the correct orders, probably using what the results of part (i).

(iii) In contrast, correct answers for the inverse of f were much less common. Most candidates failed to realise that, by making $\mathrm{f\,h}(x) = x$, they should have been using composition of functions. The usual incorrect answer was $2 - 2x$.

(iv) The final part of this question proved to be easy for many. Although some did quite a lot of working, it was sufficient to write down the operation table. 11 of the 16 entries came from previous parts of the question, and the others followed by the latin square property. The question did not actually state that the four elements were a subgroup of the group *K*, but this was probably assumed, correctly, by candidates.

Chief Examiner's Report – Mechanics

Much work of high quality was seen in the three mechanics papers sat this session. There was little evidence of ignorance of the topics in the relevant papers. Where questions had been posed in an unfamiliar way, candidates found it hard to make progress even though evidence in other questions indicated the possession of appropriate knowledge.

One perennial concern is that candidates lose marks through carelessness with basic pure mathematics techniques, typically solving equations. This detracts from the performance their knowledge of mechanics would justify. There was also evidence of a weakness in deciding whether composite quantities, eg momentum or kinetic energy, were vector or scalar entities.

4728 Mechanics 1

General Comments

A majority of candidates were able to cope with the demands of most of the paper, but there were some who found questions very taxing if set in an unfamiliar way. When candidates did fully understand mechanical principles clearly, it was disappointing to see marks lost through mistakes in solving elementary equations. Happily, there were few sine/cosine errors when resolving vectors.

Comments on Individual Questions

1) (i) This presented an undemanding start to the paper

(ii) Nearly all candidates gained both marks.

(iii) Though most scripts contained correct solutions, many candidates lost both marks by using 5 as the distance, rather than (17.325-5) as the distance travelled.

2) Candidates found this question difficult, with some commenting that two horizontal forces could not have an angle between them. Solutions based on vectors and the cosine rule usually began with the diagram for *subtracting* two vectors. When components were used, the detailed values put into Pythagoras' Theorem were frequently wrong, at times only one of a pair of components being used.

(i) The cosine rule solutions usually included cos60. Pythagoras' Theorem was often applied to forces not perpendicular to each other.

(ii) Though the angle to be found was specified in the question, candidates often attempted to find a different one, particularly if the approach in (i) had been the use of a right angled triangle with sides incorporating the components of the 12N force.

3) (i)(a) This was well done, the only significant error being an incorrect sign before $3.5m$ in the "after" momentum. Obtaining a wrong answer from a correct equation was a common reason for losing a mark.

(i)(b) This was attempted successfully by most candidates, though subtracting a negative quantity was not invariably done.

(ii) The only common mistake was to work with Q travelling at 2 ms^{-1} when striking R (travelling at 2.75 ms^{-1}). Most candidates however obtained the correct answer.

4) (i) Very often solutions ignored the $0.3g$ N tension acting on P. There was a noted reluctance to use, in part (b), a value of R less than the value of F, and many candidates lost marks by trying to contrive a situation in which the opposite was true.

(ii) In this part of the question a situation quite distinct from (i) had been created. Though many candidates appreciated this, a significant minority did not set up the two standard equations applying Newton's Second Law to particles attached
to a string passing over a pulley. In this situation, calculating acceleration from "difference in weights divided by sum of masses" is not regarded as a valid method. Candidates who did try this approach very rarely were able to find a correct value for the tension.

5) (i) This was answered very well.

(ii) The distinction between travelling quicker and travelling further was appreciated by nearly all candidates. Many correct solutions were seen, and the only notable error was the use of $s = 3t + 0.4t^2/2$ for the distance travelled by B for the entire time until A is overtaken.

(iii) Most candidates scored all 3 marks for their graphs.

6) (i) Nearly all candidates scored both marks.

(ii) Many correct solutions were seen from candidates who realised that the minimum speed would occur at the time when the acceleration is zero. A minority of scripts showed solutions based on the minimum being when $t = 0$ or $t = 28.4$.

(iii) Most candidates obtained the correct value for the length, though some had made no reference to the value of a constant of integration.

7) (i) Newton's Second Law was often correctly applied, though omitting the frictional force, the component of weight down the plane, or the mass times acceleration term were all seen.

(ii)(a) This was done well.

(ii)(b) It was notable how many candidates structured their solution to this long final part of the question clearly and correctly. An accumulation of rounding errors caused the final accuracy mark to be lost in some cases.

4729 Mechanics 2

General Comments

A large number of able candidates were entered for this paper, most of whom were well prepared. Few candidates were incorrectly entered and showed minimal understanding. As is often the case, poor or non existent diagrams frequently led to misunderstanding, particularly with question 3(ii) and question 7.

Comments on Individual Questions

1) This question was usually well answered. Only a minority failed to convert the time to 120 seconds.

2) (i) Most candidates were able to find the two relevant speeds using either constant acceleration or energy. However a significant number did not take into account the vector nature of momentum when calculating the impulse.

(ii) Well answered by the majority.

3) (i) This was well answered, with very few attempts at "fiddling" the given answer.

(ii) The use of moments about the vertex was expected in this question. Those who found the slant height of the cone to be 0.5 m and resolved the tension perpendicular to this distance were usually more successful. Those who chose to resolve the tension in another direction invariably used only the moment of one component. Some did not realise that a force does act at the point of suspension and resolved forces vertically only.

4) (i) This question was usually well answered, and candidates who used Newton's Second Law were very successful. More informal methods were spoilt by the omission of the 400N force or subtracting it from 700 × 0.5.

(ii) This was well answered by the majority. Those using a wrong answer from part (i) were credited with "follow-through" marks.

(iii) Although candidates demonstrated in part (ii) how to calculate the power, a significant number didn't realise that the change in situation would result in a different driving force and continued with a value of 750N rather than the 400N that was needed.

(iv) All but a minority of candidates successfully used their power value to calculate the angle required.

5) Candidates needed to set up two equations, a linear equation from conservation of momentum and a quadratic equation from the loss in kinetic energy. A common error was to neglect the vector nature of momentum (again), as was to use an increase of kinetic energy of 81 J. Weaker candidates used $m(v-u)^2/2$ for the change in kinetic energy. However the majority of problems arose from poor algebraic manipulation when solving the two equations simultaneously. Candidates who did solve them correctly to get two pairs of solutions usually chose the correct pair.

6) (i) This question was very well done, although some very long solutions were seen.

(ii) The time of flight of P was usually well answered, but some who found the time to the maximum height forgot to double this time for a complete method. A few candidates omitted to find the time of flight of Q.

(iii) The method for solving this question was well understood by the majority.

7) Examiners saw few correct solutions to this question. Candidates would have been more successful if full force diagrams, with an acceleration indicated, had been used.

(i) The common error was to resolve in the direction of the normal reaction. This would have also required the use of a component of the acceleration. Resolving perpendicular to the acceleration was the most successful strategy.

(ii) The radius of the circular motion should have been 4m, but it was more common to see 5m or even 2m used. Also the horizontal component of the normal reaction needed to be used in the solution of this question, but invariably a component was not used.

(iii) This part was a success for only the most able candidate. Most who attempted this did not appreciate that the change in situation meant that a new normal reaction had to be calculated and persevered with the value in (i).

4730 Mechanics 3

General Comments

Almost all candidates made a very good start to the paper, with a majority scoring full marks in question 1 and similarly for question 2. Not surprisingly with a paper having an incline of difficulty by design, candidates found that questions 3 and 4 were more difficult than the first two questions and less difficult than the last three questions.

Almost all candidates completed the paper in time, although a few candidates may have had insufficient time. Question 7 seems to have, like all of the other questions, its maximum mark as the modal mark for the distribution of marks for the question.

Where symbols are defined in a question, such as θ in question 1, v in question 2 , θ in question 6 and x in question 7 (defined in the secondary stem following part (i)), candidates should avoid using the same symbol for a different quantity in that question. This feature occurred most frequently in questions 1 and 7.

Comments on Individual Questions

1) The two methods of solution indicated in the marking scheme proved to be equally popular, and the corresponding two sets of candidates equally successful.

The question was very well attempted, the only recurring error being the omission, sometimes consistently and sometimes occasionally, of the factor 0.4 representing the mass.

2) The features that

- the components of B's velocity after the collision, parallel to and perpendicular to the line of centres, are equal
- each of these components is equal to v

were well understood by candidates. Furthermore the understanding was usually conveyed in a clear and concise way.

3) This was the least well attempted of the early (first four) questions.

Almost all candidates answered part (i) clearly and correctly.

The two most common methods used by successful candidates in part (ii) are those shown in the marking scheme. Clear diagrams with AB and BC shown separately, and with the forces acting on the rod shown in each case, often prefaced a clear and correct solution.

4) The early stages of this question, including part (i) and solving the differential equation given in part (i), were very well attempted. However in attempting to convert $\ln(1 + v) = -x + \ln 3$ in part (ii), many candidates wrote $1 + v = -e^x + 3$, possibly encouraged by the appearance of $3 - e^x$ in the required answer. Most candidates realised the need to replace v with dx/dt but mistakes in transposition were common.

In part (iii) a significant number of candidates failed to benefit from the hint available in providing the given answer in (ii) in the form $[f'(x)/f(x)]dx/dt = -1$. Candidates who started simply by removing the minus signs from both sides, almost always correctly reintroduced it on the left hand side on integration. However integration of the left hand side often produced the erroneous $-e^x\ln(3 - e^x)$.

$t = 1.27$ from $\ln|3 - e| = -t$ was a common incorrect answer in part (iii).

5) Candidates who calculated the elastic energy loss and the potential energy gain in part (i),

based on the assumption that the distance AB is indeed 4m, were almost always successful in showing the two quantities to be equal and were thus able to draw the relevant conclusion.

Candidates who attempted to verify by finding the distance AB were partitioned into three sets, determined by first finding the extension of the string at B, or the height of B above O, or by finding the distance AB directly. The three strategies were equally popular and equally successful, there being a significant number of correct answers for each case and a significant number of messy attempts leading to incorrect quadratic equations.

In part (ii) candidates were equally divided in using the principle of conservation of energy to find the speed at the equilibrium point and at the point where the string becomes slack. The same answer, correct to three significant figures, is reached by either method, but in the latter case the answer is of course fortuitous.

6) Although this is the best attempted of the last three questions, a very large minority of candidates had the factor $\sin\theta$ instead of just θ in the expression for the potential energy lost by *Q*. In every other respect part (i) was well attempted, although a few candidates had kinetic energy terms for *P* and *Q* on opposite sides of the energy equation.

Part (ii) was well attempted, although candidates with an incorrect answer in part (i) could not perforce reach the given answer.

Part (iii) was answered correctly by almost all of the candidates who attempted it, but a large proportion of candidates made no attempt. The method used in the marking scheme is the most favoured, although the small number of candidates who used the iterative equation $\alpha_{n+1} = \frac{89}{58}\sin\alpha_n$ were also universally successful.

7) Not surprisingly this proved to be the most difficult question. In part (i) almost all candidates realised the need to resolve forces parallel to the plane, but in very many cases the sum of the extensions in the two strings differed from 1.6 m, so that the correct answer could not possibly be achieved.

This feature was also apparent in many cases of answers in part (ii), so that $\ddot{x} = -49x$ could not be reached, although almost all candidates applied Newton's second law.

A significant minority of candidates did not attempt part (iii); the method of the marking scheme was favoured by most of those who did. The errors made by such candidates arose from an incorrect value of ω^2 or of the amplitude, or from omission of the negative answer. Candidates who used $\dot{x} = -A\omega\sin\omega t$, after differentiating $x = A\cos\omega t$, usually found difficulty with handling the numerical work and might profitably have avoided finding values of *t* in favour of finding the value of $\sin^2\omega t$ and to then substituting into
$x^2 = A^2(1 - \sin^2\omega t)$.

Chief Examiner's Report – Statistics

From year to year the same areas of the Statistics specification continue to cause problems for candidates - permutations and combinations, use of formulae from MF1, continuity corrections and so on. However, standards in S2 and S3 in particular were very high this January and it is pleasing to see so much work of high quality. Many centres have taken note of the comments made in previous Reports. In particular, a majority of candidates now state the hypotheses and conclusions in appropriate forms. For those who are new to these reports, it is repeated that an answer such as "the mean height is 1.8" is too assertive and does not gain full credit, compared with a statement such as "there is insufficient evidence that the mean height is not 1.8".

Verbal answers continue to cause problems. Many candidates seem to be able to do no more than repeat phrases learnt parrot-fashion (such as "events must occur randomly, singly, independently and uniformly"); the object of these questions is to assess understanding, and mere repetition of formulaic answers is unlikely to achieve much credit. Some candidates do not pay sufficient attention to the wording of the question to see what sort of answer is required. Words such as "exact" or "numerical" are specific pointers to the method needed. However, many candidates showed that they could produce good qualitative answers to questions when required.

4732 Probability & Statistics 1

General Comments

Many candidates showed a reasonable understanding of a good proportion of the mathematics in this paper. There were some very good scripts, although very few candidates gained full marks. The overall performance of the candidates was not as good as usual, due to the last parts of questions 7, 8 and 9 (particularly 9) being found very difficult.

It was pleasing to note that very few candidates ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. However, in a few cases, a mark was lost through an incorrectly rounded answer without any previous answer being shown.

Responses to question 2 suggested that many candidates had not revised GCSE work recently. Also, many candidates appeared not to be aware of how to handle a discrete variable when drawing a histogram. Perhaps some centres have not appreciated that this is possibly just beyond what candidates will have met at GCSE and so have not covered this topic.

Question 9(iii) involved the sum to infinity of a GP, but no other question made a significant call upon candidates' knowledge of Pure Mathematics. Few candidates appeared to run out of time. In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Candidates would benefit from direct teaching on the proper use of MF1. They need to understand which formulae are easiest to use, where they can be found in MF1 and how to use them. Candidates should be encouraged to use e.g. $\text{Var}(X) = \sum x^2p - \mu^2$ and $S_{xx} = \sum x^2 - (\sum x)^2/n$ rather than those based on the basic definition of variance.
A few candidates used their calculator functions for standard deviation. Some gave fewer than three significant figures in their answer and could not be awarded any marks as no method was seen. Candidates who wish to use these functions should be advised to work through each calculation twice.

Comments on Individual Questions

1) (i) As usual, many candidates did not give the conditions of constant probability and independence in context.

(ii)(a) This was usually answered correctly.

(ii)(b) This was usually answered correctly by better candidates. Weaker candidates often used the longer, alternative method and sometimes omitted a term or had an extra one. Some did 1-(ii) .

(iii) Generally, candidates who scored full marks in (i) also scored full marks here.

2) (i)(a) The mean was usually answered successfully, some candidates dropped a mark for an incorrect mid-point, usually the last one. The standard deviation was less well done. Those who attempted $\sum(l\text{-}17.65)^2f/40$ usually went wrong.

(i)(b) This was usually correct.

(ii) A class width of 4 was often used. 5x20, 4x20, 5/20 and 4/20 were often seen.

(iii) Most correctly identified the class containing the median, but did not give a full explanation. Few pointed out that there were 14 readings before the median class.

(iv) These answers were usually correct.

3) (i) Most scored full marks in this part. A few forgot the square root in the denominator.

(ii) Most candidates did not comment on the value of r, some others did not comment on the diagram.

(iii)
(iv) These answers were usually correct.

4) (i) As usual, this topic proved to be the easiest on the paper. Those who used $\sum(x-\mu)^2p$ usually went wrong. A few, using the easier method, did not subtract the mean squared or sometimes subtracted the mean.

(ii) Most realised that a binomial distribution was required.

(iii) This was usually correct, but a few used 2 instead of 4.

5) (i) Most scored the first mark, but some left it there, others multiplied by 3 not 6.

(ii) Most scored at least one mark, many gaining all three. In both parts, there were a few who answered 'with replacement'.

6) (a) The impression was that many knew what they wanted to say, but couldn't express it very well. There were however many concise answers. The best answers were a diagram for A, and a comment that both coefficients must be 1 for B.

(b) The most popular method was to find the mean of y, then use the general equation for x on y via a'=x-$b'y$ .Weaker candidates had no idea what to do. Moderate candidates were let down by poor algebra e.g. $x = 0.96x + 3.7$ (error) - c.

7) (i) Almost all candidates scored at least one mark, usually for '25'.

(ii) Better candidates answered this correctly, using the standard method. There were some extremely convoluted attempts, often scoring M1 for 15/23 and sometimes another method mark. Others gave 15/60 x 39/59 which scored two marks.

8) (i) 5!=120 was seen far too often.

(ii) 4!x2 was a common wrong answer, but was allowed a method mark.

(iii) Those who used a tree diagram, or other probability method usually did better than those who tried to use arrangements.

9) (i)(ii) Weak candidates had no idea, others usually scored these marks.

(iii) Most scored zero. Those who knew what to do generally scored at least 4 marks. This was the hardest question on the paper. $[p/(1-q)]^2=p^2/(1-q^2)$ etc., a dubious attempt to produce the given answer, was seen several times.

4733 Probability & Statistics 2

General comments

Examiners were very impressed by the standard of performance on this paper. Many candidates scored more than 60 out of 72. It was particularly pleasing to see that conclusions to hypothesis tests were often very well stated, in accordance with previous instructions. The comments below must be read in the context of this high standard.

Some Centres appear to teach candidates to answer all hypothesis test questions by finding the critical region. This is good advice in the case of questions involving Type I and Type II errors but otherwise it is often more long-winded and error-prone than finding probabilities or z-values. Good candidates should be capable of both approaches,

Some Centres have responded to the verbal questions on this Specification by teaching candidates to use standard slogans that can be regurgitated. This is to be discouraged as it encourages candidates to apply such slogans in the wrong context, and the issues often demand a more sophisticated approach. While a mental "checklist" is clearly helpful, candidates need to think about which issues are relevant in each particular situation and decide what is relevant and what is not. On this paper verbal answers that included spurious as well as correct statements were penalised, and are likely to continue to be penalised in the future. A "scattergun" approach to such questions should be avoided (particularly in considering modelling assumptions for the Poisson distribution, though this was not tested on the present paper).

Comments on particular questions

1) Almost everyone scored full marks. Only a handful forgot the 5/4 factor.

2) (i) Most realised that not all candidates were equally likely to be chosen, though it was necessary to say more than that to score both marks. Some failed to understand that subtracting multiples of 400 merely allowed fewer random numbers to be discarded and thought that it made the process non-random. Some misunderstood the question or failed to read it carefully.

(ii) Most said that numbers bigger than 399, or 799, should be ignored. An equally valid unbiased method was to choose numbers in the range 000 to 1199, though this is plainly less convenient. Some wrote out a learnt answer about random sampling, which was not applicable here.

3) Fully correct answers were again common here. The usual errors were using nq instead of npq for the variance, and, of course, using the wrong, or no, continuity correction.

4) Those candidates who started off by stating hypotheses in terms of the sample mean instead of the population mean ("$H_0 : \mu = 58.9$", etc, when "$H_0 : \mu = 60$" is correct) usually lost a large number of marks. It is again emphasised that this confusion is a major error. The other common mistake was omission of the 80 factor; candidates who obtain a strange z-value would be well advised to double-check the value they used for the variance.

5) (i) In this question it is certainly easier to score full marks by calculating $P(\geq 19)$ than by finding the critical region. Relatively few committed the major error of finding $P(\leq 19)$ or $P(= 19)$. Hypotheses and conclusions were generally well stated.

(ii) The issue here is not whether the results are statistically unreliable ("one Saturday morning is not enough evidence" was a common answer) but that no cause-and-effect can be deduced from any hypothesis test conclusion whatsoever.

6) (i) In the past, candidates have been unwilling to answer questions such as this qualitatively. Here they were forced to do so, and produced many very good answers, apart from those who tried to use taught slogans such as symmetry arguments. Most appreciated that in (a) the probabilities needed to add up to 1. In (b) it was necessary to explain that P(> 70) had to be smaller than P(> 50) as it included a smaller range. Good answers to (c) included "P(< 70) + P(> 50) must be grater than 1 as it includes all values with an overlap" or "P(> 50) = 0.3 $\Rightarrow \mu < 50$, but P(< 70) = 0.3 $\Rightarrow \mu > 70$", or used the same method as in (b) on P(< 50) = 0.7. Some wrongly thought that P(> 50) could not equal P(< 70), but as part (ii) shows, they can. Those who used diagrams were often able to make their arguments more convincing.

(ii) This was an easy example of this entirely standard type of question, though it was not especially well answered. Despite its familiarity a number of candidates failed to obtain z values (and used 0.7 or $\Phi(0.7)$ instead), while as noted in previous Reports too many make heavy weather of the calculation by using substitution instead of elimination. Here $\mu = 60$ can be written down by inspection.

7) (i) Most drew a correct diagram, though there are still candidates who ignore the restriction on the non-zero part and draw a line continuing indefinitely. Weaker candidates thought the line had a slope or was a curve.

(ii) Most found $\mu = 8$ easily but a surprising number of generally competent candidates were unable to find the variance correctly, often missing or miscalculating the height of the PDF (which should be 1/6). The question requires integration, so those who wrote down the variance from the formula in MF1 scored no marks.

(iii) This was a less familiar question and the fact that many answered it well reflects credit on the candidates. Some omitted the 48 factor, and some attempted to use a continuity correction, which is plainly wrong here as the parent distribution is continuous. Some thought that the reason the answer was only an approximation was that the variance had been estimated, but this is false; the variance has been calculated exactly. The approximation is in the shape of the distribution, which is only approximately normal.

8) (i) Full marks were common here too, but many stated the critical region as "$R = 4$" and some failed to state the probability of a Type I error explicitly.

(ii) This was perhaps the least well-answered question. Some, as usual, attempted to find a new "critical region" corresponding to $p = 0.4$, and some used a region of ≤ 4 instead of its complement, > 4.

(iii) Correct answers were again not very common here. A variety of complicated calculations was seen, including $0.5 \times 0.3669 + 0.5 \times 0.9527$.

9) (i) Often correct, the usual mistake being, of course, P(> 7) = 1 – P(≤ 6).

(ii) Again, very often correct. Those who used tables were perhaps more likely to get it wrong by using the wrong rows.

(iii) N(60, 60) was used by almost everyone, and the only mistakes were the usual ones of the wrong, or no, continuity correction and using 60 instead of $\sqrt{60}$ as the variance.

(iv) (a) Correct versions of the formula were often seen, though those who merely wrote "1 – P(≤ 1)" scored no marks here.

(b) Strong candidates gave confident and accurate answers, comparing an appropriate evaluated expression with 0 (“sign change”) or 0.1 or 0.9. Weaker ones either failed to understand that a numerical solution was required and attempted an algebraic one, which was clearly doomed to fail, or were over-reliant on calculators, using built-in software for probabilities but failing to achieve the necessary degree of accuracy. To obtain full marks it was necessary to end with an appropriate comparison and conclusion.

4734 Probability & Statistics 3

General comments

The general performance on the paper was higher than in January 2009 with candidates showing proficiency in all areas of the specification. There were more questions involving statistical tests, which tend to follow a standard pattern, and these are generally found easier to answer than questions involving probability distributions.

The Chief Examiner's Report last June stated that the conclusion of a test should not be assertive and most centres seem to have taken note of this. Some candidates produce statements such as "there is sufficient (or insufficient) evidence to accept Ho". Candidates should be aware that the test is looking for evidence to reject Ho and the statement should reflect this. As is expected, most candidates gave their final statement in context.

In deciding whether or not to reject Ho candidates should give an explicit statement involving the test statistic and critical value. It was good to see this being observed.

In stating the conditions for validity of a test, the context was not always used.

Comments on particular questions

1) Most candidates were aware of the requirements in both parts but only the most able could negotiate the integration by parts required for finding E(X).

2) The majority of candidates were familiar with dealing with the normal distribution of $X_1 + X_2 + X_3 + X_4$ and $Y - 4X$ but many gave the variance of the former as 16X9 and the variance of the latter as Var(Y) - Var(4X). Most were able to earn the mark for stating the value of $P(Y > 4X)$.

3) (i) Several gave an incorrect variance although some then used the correct one in the test in part(ii).

(ii) Except for those who tried to use a t-test there were many good scores.

(iii) Only a small minority were unaware that the small sample sizes ruled out the application of the Central Limit Theorem.

4) (i) Using the CDF of V to find the CDF and PDF of Y, where $Y = 1/(1+V)$, is a standard procedure, but care is required and in this case there was some tricky algebra. Only the best candidates could manage this.

(ii) Since the CDF was given, most could obtain the 1 mark for attempting $E(1/Y^2)$.

5) This was the highest scoring question.

(i) The procedure for finding a confidence interval for p was mostly familiar but some gave their answer as limits, rather than an interval.

(ii) The main difficulty in the test for the equality of proportions is the estimate for the standard deviation. This requires a pooled estimate of p, and only a minority of candidates fell down on this.

6) (i) Some candidates confuse the conditions for the paired sample t-test and the two-sample test.

For the former test it is unnecessary for both populations to have normal distributions but only necessary that the population of differences has a normal distribution.

(ii) The test was often carried out accurately. Many knew how to find the required confidence interval, but the correct value of t was not always used.

7) (i) The chi-squared test of independence was carried out confidently and accurately.

(ii) This required the reduction of the 2X3 table to a 1X3 table and the test became goodness of fit. Very many candidates could see this and so there some very high scores for the question. Several, however, did not combine the M and F and then used E values of 20. This scored very little.

4736 Decision Mathematics 1

General Comments

A good proportion of the candidates were able to achieve many of the marks on the paper, although only a few achieved very high scores.

Most candidates showed a good understanding of the algorithms needed, although they did not always show sufficient details of their working.

Only a few candidates seemed to have run out of time, although some candidates left out large parts of some questions.

Comments on Individual Questions

1) (i) This was a simple application of Dijkstra's algorithm, and many candidates scored full marks on this section.

(ii) Most candidates identified *B* and *E* as the only odd nodes, but several of them thought that the shortest route from *B* to *E* had weight 4, instead of 3, resulting in a total of 29 instead of 28.

(iii) The question asked for the new least weight path, this was *A*-*B*-*E*-*F*, but many candidates instead gave the weight of this path. Most candidates realised that there was no need to repeat any arcs because the graph was now Eulerian.

2) (i) Several candidates were able to say that a graph cannot have an odd number of odd vertices, and some of these also explained that in this particular case there would have needed to be $7\frac{1}{2}$ arcs.

There was evidence from some candidates of confusion between the terms 'vertices' and 'arcs'.

(ii) Most candidates were able to say that they knew that the graph was semi-Eulerian because it had exactly two odd vertices. Many were able to deduce one of the two possible structures and write down a suitable trail, some candidates just wrote down a path that passed through all the vertices.

(iii) Most candidates chose to apply Prim's algorithm to the table but they did not always show any working. To achieve full marks it was necessary to see the arcs chosen and the order in which they were added to the tree, as well as the final tree and its weight.

3) (i) The variables x and y represented the number of people (clients) on programmes X and Y, respectively. Some candidates only said that they represented the programmes X and Y and some thought that the variables represented the weight lost on the programmes.

(ii) This part was beyond many candidates, others identified the important information but did not form inequalities or did so but did not simplify them, or they made numerical slips.

(ii) Some candidates realised that the variables also needed to be both non-negative and integer-valued.

(iv) Despite the errors in setting up the constraints, several candidates were able to produce graphs to represent the situation and many went on either to check the vertices or to use lines of constant profit to find the optimum vertex.

Those candidates who found that the optimal vertex was not an integer-valued point and then went on to check integer-valued feasible points were often able to get (3, 8) as the optimal vertex, only a few candidates interpreted this in context as meaning that Maggie should put 3 clients on X, 8 on Y and 1 on programme Z.

4) (i) Many candidates were able to list the items in order of decreasing weight ($15\times A$, $4\times D$, $3\times C$, $8\times B$) and then apply first-fit to this list to give three boxes each containing $5\times A$ and a fourth box containing the rest. This packing was not possible because the final box was too heavy.

(ii) Most candidates were able to list the items in order of decreasing volume ($8\times B$, $3\times C$, $4\times D$, $15\times A$) and then attempt to apply first-fit to this list. Some candidates forgot to go back to fit smaller items in earlier boxes that still had not reached their weight limits. Some candidates tried to accommodate the volume restrictions at the same time, this was not asked for. This packing required 9 boxes, it was not possible because the fifth box was too big.

(iii) Although the volumes have been taken account of, the dimensions have not. Items may fit by weight and volume but still be the wrong shape.

5) (i) Some candidates replaced a, b and c by the expressions given in each line of the LP problem to show that the problem was equivalent to the one given. Many gave up after dealing with the objective function and only a few also used the upper bounds for a, b and c to achieve the non-negativity constraints for x, y and z. The candidates who successfully converted the given minimisation problem into a maximisation often did so very efficiently, rarely using more than about half a page of working at most.

(ii) The majority of the candidates represent the given problem as a Simplex tableau. Some omitted the P-column, and often this resulted in sign errors in the objective row. The candidates with a correct tableau could then identify that the only column suitable for finding a pivot was x, because the y-column had a positive value in the objective row and the z-column had no positive entries. Most candidates who to pivot on the x-column also showed how they had used the ratios to choose the pivot row.

Most candidates showed how they had obtained the rows in their new tableau, albeit that some used abbreviated notations. Apart from the pivot row, each new row must be of the form *old row* $\pm$ *multiple of new pivot row*, where the multiples are chosen to give 0 entries in the pivot column.

Many candidates achieved a correct augmented tableau and were able to use it to read off the current values of the variables x, y and z. Provided none of these were negative, the values of x, y and z could then be used to find values for a, b and c.

(iii) Some candidates just guessed that a, b and c being non-negative meant that x, y and z were also non-negative, others correctly showed that x, y and z had limiting values of 20, 10 and 8. Usually candidates were able to state that these were upper bounds for x, y and z, respectively.

6) (i) Many candidates realised that there would be a maximum of 1+2+3+4 = 10 arcs, but often they generalised to either $2n$ or $\frac{1}{2}n(n+1)$, instead of $1+2+\ldots+(n-1) = \frac{1}{2}(n-1)(n)$.

(ii)a) The maximum number of passes would be 1 less than the maximum number of arcs, with 5 vertices there would be a maximum of 9 passes.

Many candidates recognised that the maximum number of comparisons in the first three passes would be 1, 2 and 3, respectively. Some candidates claimed 9, 8 and 7 (or 10, 9 and 8) but this would have been a bubble sort not a shuttle sort.

The maximum number of comparisons was $1+2+\ldots+9 = 45$.

b) The general result for the maximum number of comparisons came from $1+2+\ldots+[\frac{1}{2}(n-1)(n) -1] = \frac{1}{2}[\frac{1}{2}(n-1)(n) -1]\ [[\frac{1}{2}(n-1)(n) -1]+1]$ which tidied up to give $\frac{1}{2}[\frac{1}{2}(n-1)(n) -1]\ [\frac{1}{2}(n-1)(n)]$ and hence the given expression. Only the very best candidates were able to successfully achieve this part.

(iii) Many candidates were able to follow the algorithm to achieve the required result. Some candidates put the arc *AE* into list *M2* in the first pass but then did not show it in the table for the second pass. Some candidates chose the arcs correctly but did not list the vertices in lists *M1* and *M3* correctly, sometimes listing arcs instead.

(iv) Many candidates understood that for a quartic order algorithm scaling the size of the problem by a factor of 5 would scale the processing time by a factor of 5^4 approximately, giving a time of 18750 seconds.

4737 Decision Mathematics 2

General Comments

Several candidates achieved good marks on this paper. The candidates were, in general, well prepared and were able to show what they knew. A few candidates ran short of time on the last question, but the majority were able to complete the paper. Candidates should be reminded to read the questions carefully as several dropped marks for not answering exactly what had been asked.

Comments on Individual Questions

1) (i) Nearly all the candidates were able to draw the bipartite graph correctly.

(ii) The majority of candidates drew a second bipartite graph correctly showing the incomplete matching. Only the new matching was required, candidates' answers could not always be determined when they had drawn the arcs in the matching using pen and the arcs not in the matching using pencil. Several candidates did not write down the alternating path and some did not write down the resulting matching, in both cases referring to a diagram was not enough. A few candidates just paired *D* with *P* and others just wrote down the matching given in the question.

(iii) Most candidates were able to find the matching and state that *S* did not arise. A few candidates went back to the matching from the previous part and said that *P* did not arise. This did not match *B* to *T* and so did not fit the conditions given in the question.

2) Almost all candidates added a dummy row to make a square table, although some candidates chose to use the largest values in the columns rather than giving a dummy row of equal values. Most candidates reduced the rows of their table correctly and some stated that the columns could not be reduced, some reduced columns first. Many candidates lost the accuracy mark here, sometimes for arithmetic errors or for reducing columns first, but usually for not stating how each table was formed or for not saying that, in this case, once the rows had been reduced the columns were already reduced.

Candidates usually crossed out the zero elements correctly and augmented by 1 without arithmetic errors, most also said that they had then augmented or described the process. Finally candidates read off the final matching and costed it. A few candidates misread the column labels or thought that *J* was the last row (when it was the dummy), some made arithmetic errors and some omitted the units. A small number of candidates did not scale back up to the original units or scaled by too big a factor of 10, resulting in final bills from £96 to £96000.

3) (i) Candidates often used extra dummy activities to ensure that the precedences were correct and some did not give directed arcs, either of these issues resulted in the loss of the accuracy mark. Some candidates used activity on node, or a peculiar mixture of activity on arc and activity on node, and some gave diagrams where the precedences were violated or where a pair of nodes were joined by dummy activities both ways, these candidates did not get any credit for their networks.

(ii) The forward and backwards passes were often correct, allowing follow through from incorrect networks where possible. Some candidates made errors with using dummy activities on the backwards pass.

The minimum completion times should have been 10 hours, missing units were penalised here. The critical activities were *A*, *B*, *D*, *E* and *H*. Candidates who gave one of the critical paths *AEH* or *BDH* were given the method mark, provided no non-critical activities were also listed.

(iii) The resource histogram needed to be drawn on graph paper with appropriately scaled and labelled axes. A resource histogram has no 'holes' in the middle of it and has no blocks hanging out over gaps.

(iv) Many candidates stated that *F* could be delayed by 1 hour, even when they had made earlier errors. Some said 1 day (or some other units) but this was condoned, as was the answer 1 without units.

(v) Several candidates discussed what would happen if there were only 6 workers for the first 4 hours, or sometimes throughout the entire project, rather than from the first 4 hours. The point that needed to be identified was that *G* and *H* could not happen simultaneously and so one or the other needed to be delayed. This gave a minimum delay of 2 hours, some candidates gave the new project duration but did not state the delay.

Since *H* is the only activity that depends on *F*, the maximum delay on *F* is achieved by starting *H* as late as possible. This gives a 3 hour delay on *F* compared with its earliest start time from the earlier part.

4) (i) Most candidates were able to use the information given to draw an appropriate network. Some put in extra arcs joining (2;1) to (2;0) and (2;2), but this would have meant that these three states were not at the same stage, and some omitted the arcs joining (1; 0) to (2; 2) or (1; 1) to (2; 0). Some of the diagrams were so small that it was difficult to tell which weight was attached to which arc.

(ii) This is a maximin problem.

(iii) Candidates who had correct networks and who were able to set up the dynamic programming tabulation appropriately usually went on to achieve high marks on this question. Some candidates used (stage; state) labels instead of just giving the state values in the state column, and several made errors in the action values. The action value is the state of the vertex (in the next stage) that the arc joins to; this enables the solution to be traced back through the table without needing to refer to a network.

Several candidates worked forwards through their network, instead of working backwards, this was only given partial credit.

The heaviest truck had weight 8 tonnes and was transported using the route (0; 0) – (1; 0) – (2; 2) – (3; 0).

5) (i) The majority of candidates were able to show the row minima and column maxima and hence find the play-safe strategies, some did not explicitly show their working (the row minima and column maxima, or equivalent) and some did not write down the play-safe strategies (just marking the row/column is not enough). Most candidates were able to write down the helper that Robbie should choose if he knows that Conan will play-safe, following through from incorrect play-safe choices is necessary.

(ii) Some candidates left this part out, but most either gave the correct values of -1, 0, $\frac{1}{3}$ or just made one slip (the answer -3, 0 1 was treated as a single slip). A few candidates used the columns instead of the rows, this scored nothing.

(iii) Nearly all the candidates were able to write down appropriate expressions for the three probabilities, some made numerical slips and some applied dominance too soon. Several candidates then used a sketch graph to determine which pair of equations needed to be solved, although some used dominance and some checked the expected profit values at the intersections (although these candidates should also have considered the extreme points corresponding to $p = 0$ and $p = 1$). Any valid method that gave the optimum point $p = \frac{2}{7}$ was accepted.

(iv) Candidates who understood where the given expression had come from usually identified the use of the *G* column and said that 4 had been added to all the values, although some said that this was to make the values positive rather than to make them non-negative or to remove the negative values.

(v) Many correct answers, some candidates used $z = \frac{5}{7}$ to find the *m* values $\frac{39}{7}$, $\frac{25}{7}$ and $\frac{25}{7}$, but then subtracted 4 from $\frac{39}{7}$ instead of $\frac{25}{7}$. Others used elaborate methods to get to $z = \frac{5}{7}$, or sometimes to fail to get to this, and some then thought that the value for *z* was *m* or that the value for *m* was *M*.

6) (i) Most candidates were able to calculate the capacity of cut α as 12 although fewer were able to calculate the capacity of β as 15.

(ii) Most candidates were able to explain that because at least 3 litres per second must enter *A* from *SA* there must be more than 2 litres per second leaving *A*.

(iii) Most candidates were able to explain that because the maximum that can enter *B* is 4, using *SB*, there cannot be more than 4 leaving *B*, so arcs *BC* and *BD* must both be at their minimum capacities. Since *BD* is the only arc that feeds into *B*, it follows that *D* has 2 litres per second entering it and hence 2 litres per second leaving it, so *DE* and *DT* must also be at their minimum capacities.

(iv) This part was only successfully answered by a minority of candidates. Some candidates gave descriptions of why specific attempts at a flow of 10 litres per second failed, but did not give a complete reason that covered all cases. The best explanations either used the fact that at least 11 must flow from $\{S, A, B, C\}$ to $\{D, E, F, T\}$, or equivalent; or they explained that since 2 is flowing in *BD* and 2 in *BC* then if there is a flow of 10 it means that *SA* and *SC* carry 6 between them, but at least 1 litre per second must flow from *SA* to *AF*, so this only leaves 5+2 = 7 available to flow into *C*, but the minimum that must flow out of *C* is 8.
Many candidates stated that 11 was the minimum flow, and some showed that this was possible by giving a diagram to show an example of such a flow.
Many candidates stated that 12 was the maximum flow, stating that cut $\alpha = 12$ only shows that the minimum cut ≤ 12 and hence that the maximum flow ≤ 12 (so 13, for example, is not the maximum) but it was also necessary to show that 12 is possible, perhaps by augmenting the flow of 11 already given.
Some candidates tried to claim very large maximum flows or very small minimum flows. A few claimed maximum flows that were smaller than their minimum flows.

(v) The question had asked candidates to show the vertex restriction on a diagram, and a few candidates did this, but most just showed the flow with the additional restriction. Some candidates contravened the upper or lower capacities of the remaining arcs, for example by starting with 3 through each of *SA*, *SB* and *SC* which then caused problems with *BC* and *BD*; or they left out arcs completely, in particular the arc *CF*.

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
January 2010 Examination Series

Unit Threshold Marks

7890-7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	51	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	62	54	46	38	31	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	53	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	52	44	36	28	21	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	51	44	37	30	24	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	54	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	62	53	44	35	26	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	58	50	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	47	40	34	28	22	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	51	45	39	33	28	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	28.2	53.1	73.0	87.2	96.4	100	1385
3892	39.2	61.7	79.2	92.5	97.5	100	126
7890	30.8	60.1	83.8	95.0	99.3	100	459
7892	21.1	60.5	84.2	100	100	100	43

For a description of how UMS marks are calculated see: http://www.ocr.org.uk/learners/ums/index.html

Statistics are correct at the time of publication.

List of abbreviations

Below is a list of commonly used mark scheme abbreviations. The list is not exhaustive.

AEF	Any equivalent form of answer or result is equally acceptable
AG	Answer given (working leading to the result must be valid)
CAO	Correct answer only
ISW	Ignore subsequent working
MR	Misread
SR	Special ruling
SC	Special case
ART	Allow rounding or truncating
CWO	Correct working only
SOI	Seen or implied
WWW	Without wrong working
Ft or √	Follow through (allow the A or B mark for work correctly following on from previous incorrect result.)

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