OCR RECOGNISING ACHIEVEMENT

**ADVANCED GCE**

# MATHEMATICS

Further Pure Mathematics 3

**4727**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**
None

**Thursday 29 January 2009**
**Morning**

**Duration:** 1 hour 30 minutes

*4727*

**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**1** In this question $G$ is a group of order $n$, where $3 \leqslant n < 8$.

**(i)** In each case, write down the smallest possible value of $n$:

**(a)** if $G$ is cyclic, **[1]**

**(b)** if $G$ has a proper subgroup of order 3, **[1]**

**(c)** if $G$ has at least two elements of order 2. **[1]**

**(ii)** Another group has the same order as $G$, but is not isomorphic to $G$. Write down the possible value(s) of $n$. **[2]**

**2** **(i)** Express $\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[3]**

**(ii)** Hence find the smallest positive value of $n$ for which $\left(\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}}\right)^n$ is real and positive. **[2]**

**3** Two skew lines have equations

$$\frac{x}{2} = \frac{y+3}{1} = \frac{z-6}{3} \qquad \text{and} \qquad \frac{x-5}{3} = \frac{y+1}{1} = \frac{z-7}{5}.$$

**(i)** Find the direction of the common perpendicular to the lines. **[2]**

**(ii)** Find the shortest distance between the lines. **[4]**

**4** Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = 65\sin 2x. \qquad \textbf{[9]}$$

**5** The variables $x$ and $y$ are related by the differential equation

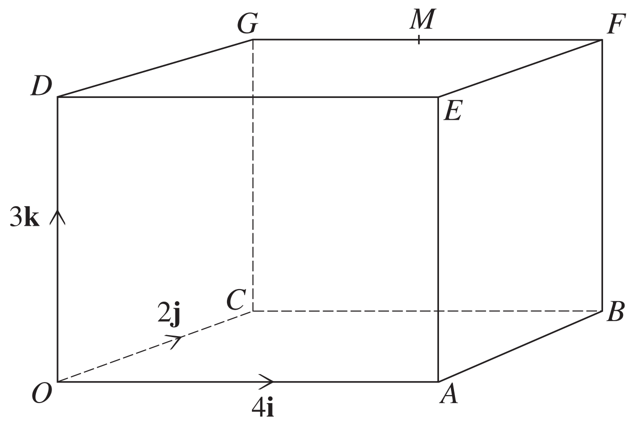
$$x^3\frac{\mathrm{d}y}{\mathrm{d}x} = xy + x + 1. \qquad \text{(A)}$$

**(i)** Use the substitution $y = u - \dfrac{1}{x}$, where $u$ is a function of $x$, to show that the differential equation may be written as

$$x^2\frac{\mathrm{d}u}{\mathrm{d}x} = u. \qquad \textbf{[4]}$$

**(ii)** Hence find the general solution of the differential equation (A), giving your answer in the form $y = \mathrm{f}(x)$. **[5]**

**6**




The cuboid $OABCDEFG$ shown in the diagram has $\overrightarrow{OA} = 4\mathbf{i}$, $\overrightarrow{OC} = 2\mathbf{j}$, $\overrightarrow{OD} = 3\mathbf{k}$, and $M$ is the mid-point of $GF$.

**(i)** Find the equation of the plane $ACGE$, giving your answer in the form $\mathbf{r.n} = p$. **[4]**

**(ii)** The plane $OEFC$ has equation $\mathbf{r.}(3\mathbf{i} - 4\mathbf{k}) = 0$. Find the acute angle between the planes $OEFC$ and $ACGE$. **[4]**

**(iii)** The line $AM$ meets the plane $OEFC$ at the point $W$. Find the ratio $AW : WM$. **[5]**

**7** **(i)** The operation $*$ is defined by $x * y = x + y - a$, where $x$ and $y$ are real numbers and $a$ is a real constant.

**(a)** Prove that the set of real numbers, together with the operation $*$, forms a group. **[6]**

**(b)** State, with a reason, whether the group is commutative. **[1]**

**(c)** Prove that there are no elements of order 2. **[2]**

**(ii)** The operation $\circ$ is defined by $x \circ y = x + y - 5$, where $x$ and $y$ are **positive** real numbers. By giving a numerical example in each case, show that two of the basic group properties are not necessarily satisfied. **[4]**

**8** **(i)** By expressing $\sin\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\sin^6\theta \equiv -\tfrac{1}{32}(\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10).$$ **[5]**

**(ii)** Replace $\theta$ by $\left(\tfrac{1}{2}\pi - \theta\right)$ in the identity in part **(i)** to obtain a similar identity for $\cos^6\theta$. **[3]**

**(iii)** Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \left(\sin^6\theta - \cos^6\theta\right) d\theta$. **[4]**

There are no questions printed on this page.