OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4758/01**

Differential Equations

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 18 May 2011
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72.**
- This document consists of **4** pages. Any blank pages are indicated.

1 The differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 13\cos 2t \qquad (*)$$

is to be solved.

(i) Find the general solution. **[9]**

(ii) Find the particular solution, given that when $t = 0$, y and $\frac{dy}{dt}$ are both zero. **[6]**

Now consider the differential equation

$$\frac{d^3z}{dt^3} + 4\frac{d^2z}{dt^2} + 3\frac{dz}{dt} = -26\sin 2t.$$

(iii) Show that the general solution may be expressed as $z = y + c$ where y is the general solution of $(*)$ and c is a constant. **[2]**

(iv) When $t = 0$, $z = 2$, $\frac{dz}{dt} = 0$ and $\frac{d^2z}{dt^2} = 13$. Use these conditions to find the particular solution. **[7]**

2 **(a)** A curve in the x-y plane satisfies the differential equation

$$\frac{dy}{dx} - \frac{2y}{x} = \sqrt{x}$$

for $x > 0$.

(i) Find the general solution for y in terms of x. **[8]**

The curve passes through $(1, 0)$.

(ii) Find the equation of this curve. **[2]**

(iii) Find the coordinates of the stationary point of this curve and find the values to which y and $\frac{dy}{dx}$ tend as $x \to 0$. Sketch the curve. **[6]**

(b) The differential equation

$$\frac{dy}{dx} = \sqrt{x^2 + y^2}$$

is to be solved approximately by using a tangent field.

(i) Describe the shape of the isocline for which $\frac{dy}{dx} = 1$. **[2]**

(ii) Sketch, on the same axes, the isoclines for the cases $\frac{dy}{dx} = 1$, $\frac{dy}{dx} = 2$, $\frac{dy}{dx} = 3$. Use these isoclines to draw a tangent field. **[3]**

(iii) Sketch the solution curve through $(0, 1)$. **[1]**

(iv) Sketch the solution curve through the origin. **[2]**

3 **(a)** A particle of mass 2 kg moves on a horizontal straight line containing the origin O. When its displacement is x m from O, it is subject to a force of magnitude $2k^2x$ N directed towards O, where k is a positive constant.

(i) Show that the velocity, $v \text{ m s}^{-1}$, of the particle satisfies the differential equation

$$v\frac{dv}{dx} = -k^2x.$$ **[3]**

The particle is at rest when $x = a$, where a is a positive constant.

(ii) Solve the differential equation, subject to this condition. Hence show that, while the particle moves in the negative direction,

$$\frac{dx}{dt} = -k\sqrt{a^2 - x^2}.$$ **[6]**

Initially the particle is at $x = a$.

(iii) Use the standard integral

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + c$$

to find x in terms of t, k and a. **[5]**

(b) At time t s, the angle, θ rad, that a pendulum makes with the vertical satisfies the differential equation

$$\omega\frac{d\omega}{d\theta} = -9\sin\theta$$

where $\omega = \dfrac{d\theta}{dt}$.

(i) Solve the differential equation for ω in terms of θ subject to the condition $\omega = 0$ when $\theta = \frac{1}{3}\pi$. Hence show that, while θ is decreasing,

$$\frac{d\theta}{dt} = -3\sqrt{2\cos\theta - 1}.$$ **[6]**

(ii) Starting from $\theta = \frac{1}{3}\pi$ when $t = 0$, use Euler's method with a step length of 0.1 to estimate θ when $t = 0.1$. The algorithm is given by $t_{r+1} = t_r + h$, $\theta_{r+1} = \theta_r + h\dot{\theta}_r$. State whether this algorithm can usefully be continued, justifying your answer. **[4]**

[Question 4 is printed overleaf.]

4 The quantities x and y at time t are modelled by the simultaneous differential equations

$$\frac{dx}{dt} = -3x - 2y + 3t,$$
$$\frac{dy}{dt} = 2x + y + t + 2.$$

(i) Show that $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = -5t - 1$. **[5]**

(ii) Find the general solution for x. **[8]**

(iii) Find the corresponding general solution for y. **[4]**

When $t = 0$, $x = 9$ and $y = 0$.

(iv) Find the particular solutions. **[4]**

(v) Find approximate expressions for x and y in terms of t, valid for large positive values of t. **[3]**