

**ADVANCED GCE
MATHEMATICS (MEI)**

Differential Equations

4758/01

Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

**Wednesday 18 May 2011
Morning**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 13 \cos 2t \quad (*)$$

is to be solved.

(i) Find the general solution. [9]

(ii) Find the particular solution, given that when $t = 0$, y and $\frac{dy}{dt}$ are both zero. [6]

Now consider the differential equation

$$\frac{d^3z}{dt^3} + 4\frac{d^2z}{dt^2} + 3\frac{dz}{dt} = -26 \sin 2t.$$

(iii) Show that the general solution may be expressed as $z = y + c$ where y is the general solution of (*) and c is a constant. [2]

(iv) When $t = 0$, $z = 2$, $\frac{dz}{dt} = 0$ and $\frac{d^2z}{dt^2} = 13$. Use these conditions to find the particular solution. [7]

2 (a) A curve in the x - y plane satisfies the differential equation

$$\frac{dy}{dx} - \frac{2y}{x} = \sqrt{x}$$

for $x > 0$.

(i) Find the general solution for y in terms of x . [8]

The curve passes through $(1, 0)$.

(ii) Find the equation of this curve. [2]

(iii) Find the coordinates of the stationary point of this curve and find the values to which y and $\frac{dy}{dx}$ tend as $x \rightarrow 0$. Sketch the curve. [6]

(b) The differential equation

$$\frac{dy}{dx} = \sqrt{x^2 + y^2}$$

is to be solved approximately by using a tangent field.

(i) Describe the shape of the isocline for which $\frac{dy}{dx} = 1$. [2]

(ii) Sketch, on the same axes, the isoclines for the cases $\frac{dy}{dx} = 1$, $\frac{dy}{dx} = 2$, $\frac{dy}{dx} = 3$. Use these isoclines to draw a tangent field. [3]

(iii) Sketch the solution curve through $(0, 1)$. [1]

(iv) Sketch the solution curve through the origin. [2]

- 3 (a) A particle of mass 2 kg moves on a horizontal straight line containing the origin O. When its displacement is x m from O, it is subject to a force of magnitude $2k^2x$ N directed towards O, where k is a positive constant.

(i) Show that the velocity, v m s⁻¹, of the particle satisfies the differential equation

$$v \frac{dv}{dx} = -k^2x. \quad [3]$$

The particle is at rest when $x = a$, where a is a positive constant.

(ii) Solve the differential equation, subject to this condition. Hence show that, while the particle moves in the negative direction,

$$\frac{dx}{dt} = -k\sqrt{a^2 - x^2}. \quad [6]$$

Initially the particle is at $x = a$.

(iii) Use the standard integral

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + c$$

to find x in terms of t , k and a .

[5]

- (b) At time t s, the angle, θ rad, that a pendulum makes with the vertical satisfies the differential equation

$$\omega \frac{d\omega}{d\theta} = -9 \sin \theta$$

where $\omega = \frac{d\theta}{dt}$.

(i) Solve the differential equation for ω in terms of θ subject to the condition $\omega = 0$ when $\theta = \frac{1}{3}\pi$. Hence show that, while θ is decreasing,

$$\frac{d\theta}{dt} = -3\sqrt{2 \cos \theta - 1}. \quad [6]$$

(ii) Starting from $\theta = \frac{1}{3}\pi$ when $t = 0$, use Euler's method with a step length of 0.1 to estimate θ when $t = 0.1$. The algorithm is given by $t_{r+1} = t_r + h$, $\theta_{r+1} = \theta_r + h\dot{\theta}_r$. State whether this algorithm can usefully be continued, justifying your answer. [4]

[Question 4 is printed overleaf.]

4 The quantities x and y at time t are modelled by the simultaneous differential equations

$$\frac{dx}{dt} = -3x - 2y + 3t,$$

$$\frac{dy}{dt} = 2x + y + t + 2.$$

(i) Show that $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = -5t - 1$. [5]

(ii) Find the general solution for x . [8]

(iii) Find the corresponding general solution for y . [4]

When $t = 0$, $x = 9$ and $y = 0$.

(iv) Find the particular solutions. [4]

(v) Find approximate expressions for x and y in terms of t , valid for large positive values of t . [3]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.