OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4753/01

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

475301

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Solve the equation $e^{2x} - 5e^{x} = 0$. **[4]**

2 The temperature T in degrees Celsius of water in a glass t minutes after boiling is modelled by the equation $T = 20 + be^{-kt}$, where b and k are constants. Initially the temperature is 100 °C, and after 5 minutes the temperature is 60 °C.

(i) Find b and k. **[4]**

(ii) Find at what time the temperature reaches 50 °C. **[2]**

3 **(i)** Given that $y = \sqrt[3]{1 + 3x^2}$, use the chain rule to find $\frac{dy}{dx}$ in terms of x. **[3]**

(ii) Given that $y^3 = 1 + 3x^2$, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y. Show that this result is equivalent to the result in part **(i)**. **[4]**

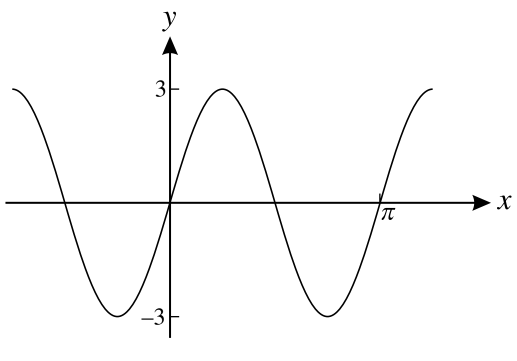
4 Evaluate the following integrals, giving your answers in exact form.

(i) $\displaystyle\int_0^1 \frac{2x}{x^2 + 1}\, dx$. **[3]**

(ii) $\displaystyle\int_0^1 \frac{2x}{x + 1}\, dx$. **[5]**

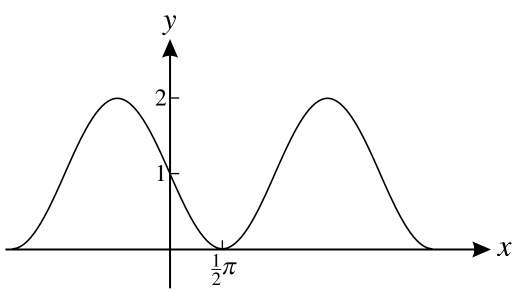
5 The curves in parts **(i)** and **(ii)** have equations of the form $y = a + b\sin cx$, where a, b and c are constants. For each curve, find the values of a, b and c.

(i)



[2]

(ii)



[2]

6 Write down the conditions for $f(x)$ to be an odd function and for $g(x)$ to be an even function.

Hence prove that, if $f(x)$ is odd and $g(x)$ is even, then the composite function $gf(x)$ is even. **[4]**

7 Given that $\arcsin x = \arccos y$, prove that $x^2 + y^2 = 1$. [Hint: let $\arcsin x = \theta$.] **[3]**

Section B (36 marks)

8 Fig. 8 shows part of the curve $y = x\cos 3x$.

The curve crosses the x-axis at O, P and Q.

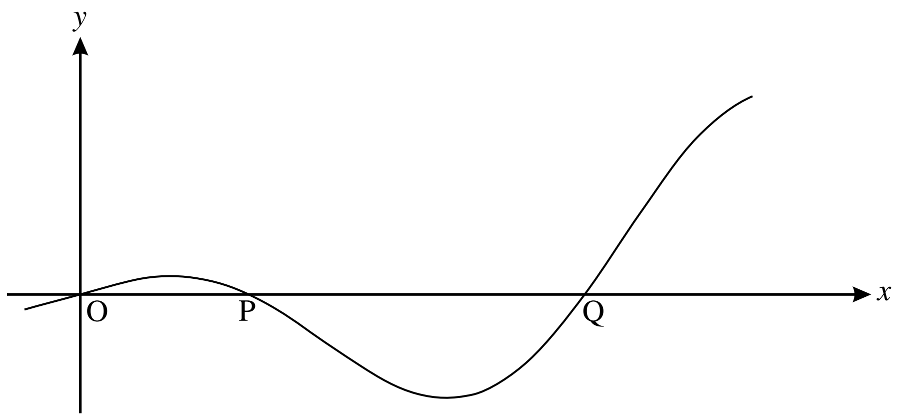


Fig. 8

(i) Find the exact coordinates of P and Q. **[4]**

(ii) Find the exact gradient of the curve at the point P.

Show also that the turning points of the curve occur when $x\tan 3x = \frac{1}{3}$. **[7]**

(iii) Find the area of the region enclosed by the curve and the x-axis between O and P, giving your answer in exact form. **[6]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2x^2 - 1}{x^2 + 1}$ for the domain $0 \leqslant x \leqslant 2$.

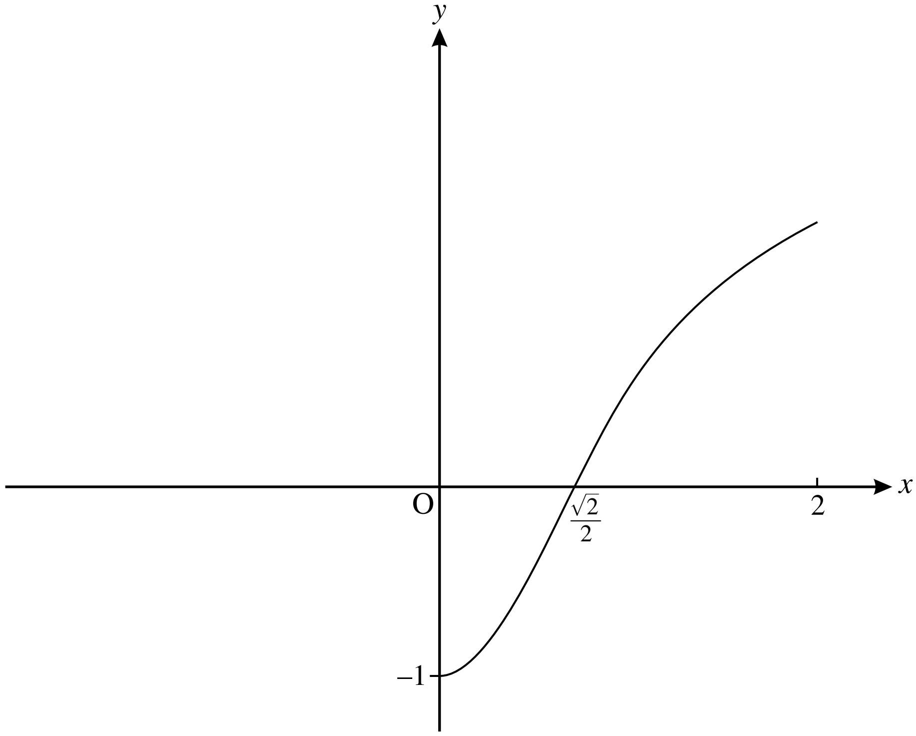


Fig. 9

(i) Show that $\mathrm{f}'(x) = \dfrac{6x}{(x^2 + 1)^2}$, and hence that $\mathrm{f}(x)$ is an increasing function for $x > 0$. **[5]**

(ii) Find the range of $\mathrm{f}(x)$. **[2]**

(iii) **Given** that $\mathrm{f}''(x) = \dfrac{6 - 18x^2}{(x^2 + 1)^3}$, find the maximum value of $\mathrm{f}'(x)$. **[4]**

The function $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

(iv) Write down the domain and range of $\mathrm{g}(x)$. Add a sketch of the curve $y = \mathrm{g}(x)$ to a copy of Fig. 9. **[4]**

(v) Show that $\mathrm{g}(x) = \sqrt{\dfrac{x + 1}{2 - x}}$. **[4]**