

**ADVANCED GCE**

**MATHEMATICS (MEI)**

Methods for Advanced Mathematics (C3)

**4753/01**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Wednesday 20 January 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (36 marks)

1 Solve the equation  $e^{2x} - 5e^x = 0$ . [4]

2 The temperature  $T$  in degrees Celsius of water in a glass  $t$  minutes after boiling is modelled by the equation  $T = 20 + be^{-kt}$ , where  $b$  and  $k$  are constants. Initially the temperature is  $100^\circ\text{C}$ , and after 5 minutes the temperature is  $60^\circ\text{C}$ .

(i) Find  $b$  and  $k$ . [4]

(ii) Find at what time the temperature reaches  $50^\circ\text{C}$ . [2]

3 (i) Given that  $y = \sqrt[3]{1 + 3x^2}$ , use the chain rule to find  $\frac{dy}{dx}$  in terms of  $x$ . [3]

(ii) Given that  $y^3 = 1 + 3x^2$ , use implicit differentiation to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . Show that this result is equivalent to the result in part (i). [4]

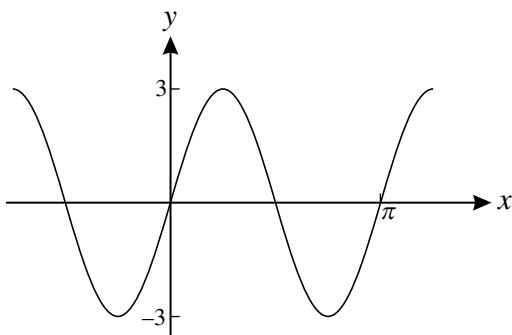
4 Evaluate the following integrals, giving your answers in exact form.

(i)  $\int_0^1 \frac{2x}{x^2 + 1} dx$ . [3]

(ii)  $\int_0^1 \frac{2x}{x + 1} dx$ . [5]

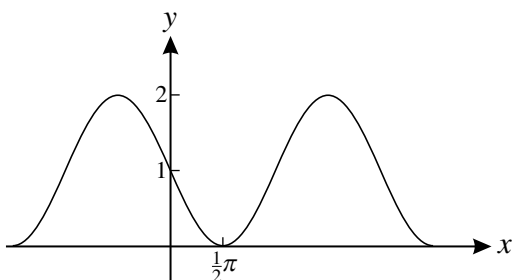
5 The curves in parts (i) and (ii) have equations of the form  $y = a + b \sin cx$ , where  $a$ ,  $b$  and  $c$  are constants. For each curve, find the values of  $a$ ,  $b$  and  $c$ .

(i)



[2]

(ii)

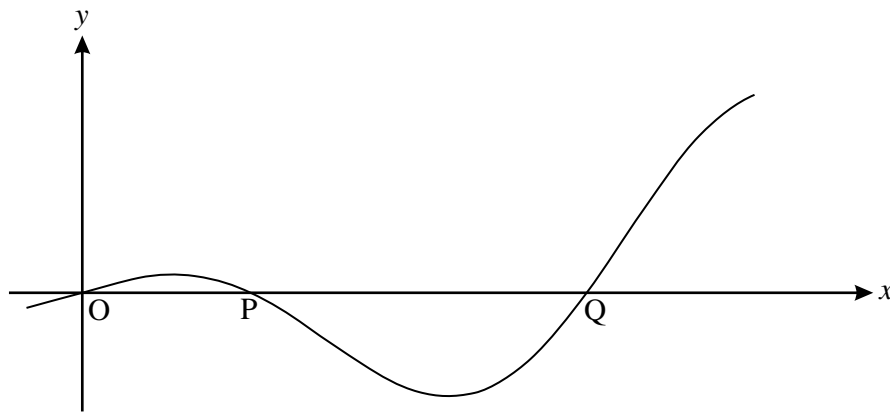


[2]

- 6 Write down the conditions for  $f(x)$  to be an odd function and for  $g(x)$  to be an even function.  
Hence prove that, if  $f(x)$  is odd and  $g(x)$  is even, then the composite function  $gf(x)$  is even. [4]
- 7 Given that  $\arcsin x = \arccos y$ , prove that  $x^2 + y^2 = 1$ . [Hint: let  $\arcsin x = \theta$ .] [3]

**Section B** (36 marks)

- 8 Fig. 8 shows part of the curve  $y = x \cos 3x$ .  
The curve crosses the  $x$ -axis at O, P and Q.

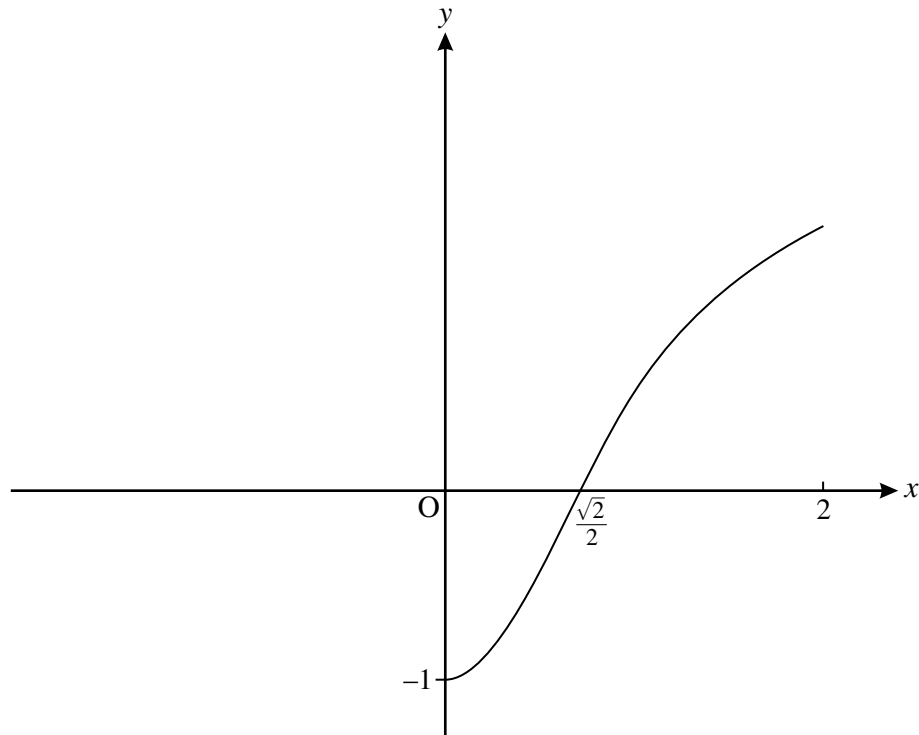


**Fig. 8**

- (i) Find the exact coordinates of P and Q. [4]
- (ii) Find the exact gradient of the curve at the point P.  
Show also that the turning points of the curve occur when  $x \tan 3x = \frac{1}{3}$ . [7]
- (iii) Find the area of the region enclosed by the curve and the  $x$ -axis between O and P, giving your answer in exact form. [6]

**[Question 9 is printed overleaf.]**

- 9 Fig. 9 shows the curve  $y = f(x)$ , where  $f(x) = \frac{2x^2 - 1}{x^2 + 1}$  for the domain  $0 \leq x \leq 2$ .



**Fig. 9**

- (i) Show that  $f'(x) = \frac{6x}{(x^2 + 1)^2}$ , and hence that  $f(x)$  is an increasing function for  $x > 0$ . [5]
- (ii) Find the range of  $f(x)$ . [2]
- (iii) Given that  $f''(x) = \frac{6 - 18x^2}{(x^2 + 1)^3}$ , find the maximum value of  $f'(x)$ . [4]

The function  $g(x)$  is the inverse function of  $f(x)$ .

- (iv) Write down the domain and range of  $g(x)$ . Add a sketch of the curve  $y = g(x)$  to a copy of Fig. 9. [4]
- (v) Show that  $g(x) = \sqrt{\frac{x+1}{2-x}}$ . [4]

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