

**ADVANCED SUBSIDIARY GENERAL CERTIFICATE OF EDUCATION
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MATHEMATICS

**LIST OF FORMULAE
AND
STATISTICAL TABLES
(List MF1)**

Pure Mathematics

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Trigonometry

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Arithmetic Series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

Geometric Series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Binomial Series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1.2.3\dots r}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Complex Numbers

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n(\cos n\theta + i \sin n\theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \dots, n-1$

Maclaurin's Series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots + \frac{x^r}{r!}f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 < x < 1)$$

Hyperbolic Functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\{x + \sqrt{(x^2 - 1)}\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln\{x + \sqrt{(x^2 + 1)}\}$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Coordinate Geometry

The perpendicular distance from (h, k) to $ax + by + c = 0$ is $\frac{|ah + bk + c|}{\sqrt{(a^2 + b^2)}}$

The acute angle between lines with gradients m_1 and m_2 is $\tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\text{For } t = \tan \frac{1}{2}A: \sin A = \frac{2t}{1+t^2}, \cos A = \frac{1-t^2}{1+t^2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Vectors

The resolved part of \mathbf{a} in the direction of \mathbf{b} is $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\frac{\lambda \mathbf{a} + \mu \mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

If A is the point with position vector $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ and the direction vector \mathbf{b} is given by

$\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$, then the straight line through A with direction vector \mathbf{b} has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \quad (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1 \mathbf{i} + n_2 \mathbf{j} + n_3 \mathbf{k}$ has cartesian equation

$$n_1 x + n_2 y + n_3 z + d = 0, \quad \text{where } d = -\mathbf{a} \cdot \mathbf{n}$$

The plane through non-collinear points A, B and C has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector \mathbf{a} and parallel to \mathbf{b} and \mathbf{c} has equation $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1 x + n_2 y + n_3 z + d = 0$ is $\frac{|n_1 \alpha + n_2 \beta + n_3 \gamma + d|}{\sqrt{(n_1^2 + n_2^2 + n_3^2)}}$

Matrix transformations

Anticlockwise rotation through θ about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Differentiation

| $f(x)$ | $f'(x)$ |
|--------------------------|----------------------------------|
| $\tan kx$ | $k \sec^2 kx$ |
| $\sin^{-1} x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\cos^{-1} x$ | $-\frac{1}{\sqrt{1-x^2}}$ |
| $\tan^{-1} x$ | $\frac{1}{1+x^2}$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |
| $\tanh x$ | $\operatorname{sech}^2 x$ |
| $\sinh^{-1} x$ | $\frac{1}{\sqrt{1+x^2}}$ |
| $\cosh^{-1} x$ | $\frac{1}{\sqrt{x^2-1}}$ |
| $\tanh^{-1} x$ | $\frac{1}{1-x^2}$ |

If $y = \frac{f(x)}{g(x)}$ then $\frac{dy}{dx} = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}$

Integration (+ constant; $a > 0$ where relevant)

| | |
|--------------------------------|---|
| $f(x)$ | $\int f(x) dx$ |
| $\sec^2 kx$ | $\frac{1}{k} \tan kx$ |
| $\tan x$ | $\ln \sec x $ |
| $\cot x$ | $\ln \sin x $ |
| $\operatorname{cosec} x$ | $-\ln \operatorname{cosec} x + \cot x = \ln \left \tan \frac{1}{2}x \right $ |
| $\sec x$ | $\ln \sec x + \tan x = \ln \left \tan \left(\frac{1}{2}x + \frac{1}{4}\pi \right) \right $ |
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |
| $\tanh x$ | $\ln \cosh x$ |
| $\frac{1}{\sqrt{(a^2 - x^2)}}$ | $\sin^{-1} \left(\frac{x}{a} \right) \quad (x < a)$ |
| $\frac{1}{a^2 + x^2}$ | $\frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$ |
| $\frac{1}{\sqrt{(x^2 - a^2)}}$ | $\cosh^{-1} \left(\frac{x}{a} \right) \text{ or } \ln \{x + \sqrt{(x^2 - a^2)}\} \quad (x > a)$ |
| $\frac{1}{\sqrt{(a^2 + x^2)}}$ | $\sinh^{-1} \left(\frac{x}{a} \right) \text{ or } \ln \{x + \sqrt{(x^2 + a^2)}\}$ |
| $\frac{1}{a^2 - x^2}$ | $\frac{1}{2a} \ln \left \frac{a+x}{a-x} \right = \frac{1}{a} \tanh^{-1} \left(\frac{x}{a} \right) \quad (x < a)$ |
| $\frac{1}{x^2 - a^2}$ | $\frac{1}{2a} \ln \left \frac{x-a}{x+a} \right $ |

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Area of a sector

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

$$A = \frac{1}{2} \int \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right) dt \quad (\text{parametric form})$$

Numerical Mathematics

Numerical integration

The trapezium rule: $\int_a^b y dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Simpson's Rule: $\int_a^b y dx \approx \frac{1}{3}h\{(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\}$,
where $h = \frac{b-a}{n}$ and n is even

Numerical Solution of Equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Mechanics

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

Centres of Mass (for uniform bodies)

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius r : $\frac{3}{8}r$ from centre

Hemispherical shell, radius r : $\frac{1}{2}r$ from centre

Circular arc, radius r , angle at centre 2α : $\frac{r \sin \alpha}{\alpha}$ from centre

Sector of circle, radius r , angle at centre 2α : $\frac{2r \sin \alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h : $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h : $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Moments of Inertia (for uniform bodies of mass m)

Thin rod, length $2l$, about perpendicular axis through centre: $\frac{1}{3}ml^2$

Rectangular lamina about axis in plane bisecting edges of length $2l$: $\frac{1}{3}ml^2$

Thin rod, length $2l$, about perpendicular axis through end: $\frac{4}{3}ml^2$

Rectangular lamina about edge perpendicular to edges of length $2l$: $\frac{4}{3}ml^2$

Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre: $\frac{1}{3}m(a^2 + b^2)$

Hoop or cylindrical shell of radius r about axis: mr^2

Hoop of radius r about a diameter: $\frac{1}{2}mr^2$

Disc or solid cylinder of radius r about axis: $\frac{1}{2}mr^2$

Disc of radius r about a diameter: $\frac{1}{4}mr^2$

Solid sphere, radius r , about diameter: $\frac{2}{5}mr^2$

Spherical shell of radius r about a diameter: $\frac{2}{3}mr^2$

Parallel axes theorem: $I_A = I_G + m(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the x - y plane)

Probability & Statistics

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A)$$

$$P(A | B) = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | A')P(A')}$$

$$\text{Bayes' Theorem: } P(A_j | B) = \frac{P(A_j)P(B | A_j)}{\sum P(A_i)P(B | A_i)}$$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities p_i

$$\text{Expectation (mean): } E(X) = \mu = \sum x_i p_i$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$$

$$\text{For a function } g(X): E(g(X)) = \sum g(x_i) p_i$$

The probability generating function of X is $G_X(t) = E(t^X)$, and

$$E(X) = G'_X(1)$$

$$\text{Var}(X) = G''_X(1) + G'_X(1) - \{G'_X(1)\}^2$$

For $Z = X + Y$, where X and Y are independent: $G_Z(t) = G_X(t)G_Y(t)$

Standard discrete distributions

| Distribution of X | $P(X = x)$ | Mean | Variance | P.G.F. |
|--|-------------------------------------|---------------|-------------------|-----------------------|
| Binomial $B(n, p)$ | $\binom{n}{x} p^x (1-p)^{n-x}$ | np | $np(1-p)$ | $(1-p + pt)^n$ |
| Poisson $Po(\lambda)$ | $e^{-\lambda} \frac{\lambda^x}{x!}$ | λ | λ | $e^{\lambda(t-1)}$ |
| Geometric $\text{Geo}(p)$ on $1, 2, \dots$ | $p(1-p)^{x-1}$ | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ | $\frac{pt}{1-(1-p)t}$ |

Continuous distributions

For a continuous random variable X having probability density function f

$$\text{Expectation (mean): } E(X) = \mu = \int xf(x) dx$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$$

$$\text{For a function } g(X): E(g(X)) = \int g(x)f(x) dx$$

$$\text{Cumulative distribution function: } F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$$

The moment generating function of X is $M_X(t) = E(e^{tX})$ and

$$E(X) = M'_X(0)$$

$$E(X^n) = M_X^{(n)}(0)$$

$$\text{Var}(X) = M''_X(0) - \{M'_X(0)\}^2$$

For $Z = X + Y$, where X and Y are independent: $M_Z(t) = M_X(t)M_Y(t)$

Standard continuous distributions

| Distribution of X | P.D.F. | Mean | Variance | M.G.F. |
|-----------------------------------|---|---------------------|-----------------------|---------------------------------------|
| Uniform (Rectangular) on $[a, b]$ | $\frac{1}{b-a}$ | $\frac{1}{2}(a+b)$ | $\frac{1}{12}(b-a)^2$ | $\frac{e^{bt} - e^{at}}{(b-a)t}$ |
| Exponential | $\lambda e^{-\lambda x}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | $\frac{\lambda}{\lambda - t}$ |
| Normal $N(\mu, \sigma^2)$ | $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ | μ | σ^2 | $e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ |

Expectation algebra

Covariance: $\text{Cov}(X, Y) = E((X - \mu_X)(Y - \mu_Y)) = E(XY) - \mu_X\mu_Y$

$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \pm 2ab \text{Cov}(X, Y)$

Product moment correlation coefficient: $\rho = \frac{\text{Cov}(X, Y)}{\sigma_X\sigma_Y}$

If $X = aX' + b$ and $Y = cY' + d$, then $\text{Cov}(X, Y) = ac \text{Cov}(X', Y')$

For independent random variables X and Y

$$E(XY) = E(X)E(Y)$$

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

Sampling distributions

For a random sample X_1, X_2, \dots, X_n of n independent observations from a distribution having mean μ and variance σ^2

\bar{X} is an unbiased estimator of μ , with $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2 , where $S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \quad (\text{also valid in matched-pairs situations})$$

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p , and $Y = \frac{X}{n}$, then

$$E(Y) = p \quad \text{and} \quad \text{Var}(Y) = \frac{p(1-p)}{n}$$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\left(\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}\right)}} \sim N(0, 1)$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then $\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\left\{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y}\right)\right\}}} \sim t_{n_x+n_y-2}$,

$$\text{where } S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{(S_{xx} S_{yy})}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{(\Sigma(x_i - \bar{x})^2)(\Sigma(y_i - \bar{y})^2)\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left\{ \left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} \right) \left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n} \right) \right\}}}$$

Spearman's rank correlation coefficient is $r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

Distribution-free (non-parametric) tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_v$

Approximate distributions for large samples

Wilcoxon Signed Rank test: $T \sim N\left(\frac{1}{4}n(n+1), \frac{1}{24}n(n+1)(2n+1)\right)$

Wilcoxon Rank Sum test (samples of sizes m and n , with $m \leq n$):

$$W \sim N\left(\frac{1}{2}m(m+n+1), \frac{1}{12}mn(m+n+1)\right)$$

CUMULATIVE BINOMIAL PROBABILITIES

| $n = 25$ | p | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 | |
|----------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | | 0.2774 | 0.0718 | 0.0172 | 0.0105 | 0.0038 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | | 0.6424 | 0.2712 | 0.0931 | 0.0629 | 0.0274 | 0.0070 | 0.0016 | 0.0005 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | | 0.8729 | 0.5371 | 0.2537 | 0.1887 | 0.0982 | 0.0321 | 0.0090 | 0.0035 | 0.0021 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | | 0.9659 | 0.7636 | 0.4711 | 0.3816 | 0.2340 | 0.0962 | 0.0332 | 0.0149 | 0.0097 | 0.0024 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | | 0.9928 | 0.9020 | 0.6821 | 0.5937 | 0.4207 | 0.2137 | 0.0905 | 0.0462 | 0.0320 | 0.0095 | 0.0023 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | | 0.9988 | 0.9666 | 0.8385 | 0.7720 | 0.6167 | 0.3783 | 0.1935 | 0.1120 | 0.0826 | 0.0294 | 0.0086 | 0.0020 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | | 0.9998 | 0.9905 | 0.9305 | 0.8908 | 0.7800 | 0.5611 | 0.3407 | 0.2215 | 0.1734 | 0.0736 | 0.0258 | 0.0073 | 0.0016 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | | 1.0000 | 0.9977 | 0.9745 | 0.9553 | 0.8909 | 0.7265 | 0.5118 | 0.3703 | 0.3061 | 0.1536 | 0.0639 | 0.0216 | 0.0058 | 0.0012 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 8 | | 1.0000 | 0.9995 | 0.9920 | 0.9843 | 0.9532 | 0.8506 | 0.6769 | 0.5376 | 0.4668 | 0.2735 | 0.1340 | 0.0539 | 0.0174 | 0.0043 | 0.0008 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 9 | | 1.0000 | 0.9999 | 0.9979 | 0.9953 | 0.9827 | 0.9287 | 0.8106 | 0.6956 | 0.6303 | 0.4246 | 0.2424 | 0.1148 | 0.0440 | 0.0132 | 0.0029 | 0.0016 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | | 1.0000 | 1.0000 | 0.9995 | 0.9988 | 0.9944 | 0.9703 | 0.9022 | 0.8220 | 0.7712 | 0.5858 | 0.3843 | 0.2122 | 0.0960 | 0.0344 | 0.0093 | 0.0056 | 0.0018 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 11 | | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9985 | 0.9893 | 0.9558 | 0.9082 | 0.8746 | 0.7323 | 0.5426 | 0.3450 | 0.1827 | 0.0778 | 0.0255 | 0.0164 | 0.0060 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 12 | | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9966 | 0.9825 | 0.9585 | 0.9396 | 0.8462 | 0.6937 | 0.5000 | 0.3063 | 0.1538 | 0.0604 | 0.0415 | 0.0175 | 0.0034 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 13 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9940 | 0.9836 | 0.9745 | 0.9222 | 0.8173 | 0.6550 | 0.4574 | 0.2677 | 0.1254 | 0.0918 | 0.0442 | 0.0107 | 0.0015 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 14 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9982 | 0.9944 | 0.9907 | 0.9656 | 0.9040 | 0.7878 | 0.6157 | 0.4142 | 0.2288 | 0.1780 | 0.0978 | 0.0297 | 0.0056 | 0.0012 | 0.0005 | 0.0000 | 0.0000 | 0.0000 |
| 15 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9984 | 0.9971 | 0.9868 | 0.9560 | 0.8852 | 0.7576 | 0.5754 | 0.3697 | 0.3044 | 0.1894 | 0.0713 | 0.0173 | 0.0047 | 0.0021 | 0.0001 | 0.0000 | 0.0000 |
| 16 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9992 | 0.9957 | 0.9826 | 0.9461 | 0.8660 | 0.7265 | 0.5332 | 0.4624 | 0.3231 | 0.1494 | 0.0468 | 0.0157 | 0.0080 | 0.0005 | 0.0000 | 0.0000 |
| 17 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9988 | 0.9942 | 0.9784 | 0.9361 | 0.8464 | 0.6939 | 0.6297 | 0.4882 | 0.2735 | 0.1091 | 0.0447 | 0.0255 | 0.0023 | 0.0000 | 0.0000 |
| 18 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9927 | 0.9742 | 0.9264 | 0.8266 | 0.7785 | 0.6593 | 0.4389 | 0.2200 | 0.1092 | 0.0695 | 0.0095 | 0.0002 | 0.0000 |
| 19 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9980 | 0.9914 | 0.9706 | 0.9174 | 0.8880 | 0.8065 | 0.6217 | 0.3833 | 0.2280 | 0.1615 | 0.0334 | 0.0012 | 0.0000 |
| 20 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9977 | 0.9905 | 0.9680 | 0.9538 | 0.9095 | 0.7863 | 0.5793 | 0.4063 | 0.3179 | 0.0980 | 0.0072 | 0.0000 |
| 21 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9976 | 0.9903 | 0.9851 | 0.9668 | 0.9038 | 0.7660 | 0.6184 | 0.5289 | 0.2364 | 0.0341 | 0.0000 |
| 22 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9979 | 0.9965 | 0.9910 | 0.9679 | 0.9018 | 0.8113 | 0.7463 | 0.4629 | 0.1271 | 0.0000 |
| 23 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 | 0.9984 | 0.9930 | 0.9726 | 0.9371 | 0.9069 | 0.7288 | 0.3576 | 0.0000 |
| 24 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9992 | 0.9962 | 0.9895 | 0.9828 | 0.7226 | 0.0000 | 0.0000 |
| 25 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

CUMULATIVE POISSON PROBABILITIES

| λ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.9900 | 0.9802 | 0.9704 | 0.9608 | 0.9512 | 0.9418 | 0.9324 | 0.9231 | 0.9139 |
| 1 | 1.0000 | 0.9998 | 0.9996 | 0.9992 | 0.9988 | 0.9983 | 0.9977 | 0.9970 | 0.9962 |
| 2 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 |
| 3 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 |
| 1 | 0.9953 | 0.9825 | 0.9631 | 0.9384 | 0.9098 | 0.8781 | 0.8442 | 0.8088 | 0.7725 |
| 2 | 0.9998 | 0.9989 | 0.9964 | 0.9921 | 0.9856 | 0.9769 | 0.9659 | 0.9526 | 0.9371 |
| 3 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9982 | 0.9966 | 0.9942 | 0.9909 | 0.9865 |
| 4 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9986 | 0.9977 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9997 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.3679 | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 | 0.1496 |
| 1 | 0.7358 | 0.6990 | 0.6626 | 0.6268 | 0.5918 | 0.5578 | 0.5249 | 0.4932 | 0.4628 | 0.4337 |
| 2 | 0.9197 | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 | 0.7834 | 0.7572 | 0.7306 | 0.7037 |
| 3 | 0.9810 | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 | 0.9212 | 0.9068 | 0.8913 | 0.8747 |
| 4 | 0.9963 | 0.9946 | 0.9923 | 0.9893 | 0.9857 | 0.9814 | 0.9763 | 0.9704 | 0.9636 | 0.9559 |
| 5 | 0.9994 | 0.9990 | 0.9985 | 0.9978 | 0.9968 | 0.9955 | 0.9940 | 0.9920 | 0.9896 | 0.9868 |
| 6 | 0.9999 | 0.9999 | 0.9997 | 0.9996 | 0.9994 | 0.9991 | 0.9987 | 0.9981 | 0.9974 | 0.9966 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9994 | 0.9992 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 2.00 | 2.10 | 2.20 | 2.30 | 2.40 | 2.50 | 2.60 | 2.70 | 2.80 | 2.90 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.1353 | 0.1225 | 0.1108 | 0.1003 | 0.0907 | 0.0821 | 0.0743 | 0.0672 | 0.0608 | 0.0550 |
| 1 | 0.4060 | 0.3796 | 0.3546 | 0.3309 | 0.3084 | 0.2873 | 0.2674 | 0.2487 | 0.2311 | 0.2146 |
| 2 | 0.6767 | 0.6496 | 0.6227 | 0.5960 | 0.5697 | 0.5438 | 0.5184 | 0.4936 | 0.4695 | 0.4460 |
| 3 | 0.8571 | 0.8386 | 0.8194 | 0.7993 | 0.7787 | 0.7576 | 0.7360 | 0.7141 | 0.6919 | 0.6696 |
| 4 | 0.9473 | 0.9379 | 0.9275 | 0.9162 | 0.9041 | 0.8912 | 0.8774 | 0.8629 | 0.8477 | 0.8318 |
| 5 | 0.9834 | 0.9796 | 0.9751 | 0.9700 | 0.9643 | 0.9580 | 0.9510 | 0.9433 | 0.9349 | 0.9258 |
| 6 | 0.9955 | 0.9941 | 0.9925 | 0.9906 | 0.9884 | 0.9858 | 0.9828 | 0.9794 | 0.9756 | 0.9713 |
| 7 | 0.9989 | 0.9985 | 0.9980 | 0.9974 | 0.9967 | 0.9958 | 0.9947 | 0.9934 | 0.9919 | 0.9901 |
| 8 | 0.9998 | 0.9997 | 0.9995 | 0.9994 | 0.9991 | 0.9989 | 0.9985 | 0.9981 | 0.9976 | 0.9969 |
| 9 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9991 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.0498 | 0.0450 | 0.0408 | 0.0369 | 0.0334 | 0.0302 | 0.0273 | 0.0247 | 0.0224 | 0.0202 |
| 1 | 0.1991 | 0.1847 | 0.1712 | 0.1586 | 0.1468 | 0.1359 | 0.1257 | 0.1162 | 0.1074 | 0.0992 |
| 2 | 0.4232 | 0.4012 | 0.3799 | 0.3594 | 0.3397 | 0.3208 | 0.3027 | 0.2854 | 0.2689 | 0.2531 |
| 3 | 0.6472 | 0.6248 | 0.6025 | 0.5803 | 0.5584 | 0.5366 | 0.5152 | 0.4942 | 0.4735 | 0.4532 |
| 4 | 0.8153 | 0.7982 | 0.7806 | 0.7626 | 0.7442 | 0.7254 | 0.7064 | 0.6872 | 0.6678 | 0.6484 |
| 5 | 0.9161 | 0.9057 | 0.8946 | 0.8829 | 0.8705 | 0.8576 | 0.8441 | 0.8301 | 0.8156 | 0.8006 |
| 6 | 0.9665 | 0.9612 | 0.9554 | 0.9490 | 0.9421 | 0.9347 | 0.9267 | 0.9182 | 0.9091 | 0.8995 |
| 7 | 0.9881 | 0.9858 | 0.9832 | 0.9802 | 0.9769 | 0.9733 | 0.9692 | 0.9648 | 0.9599 | 0.9546 |
| 8 | 0.9962 | 0.9953 | 0.9943 | 0.9931 | 0.9917 | 0.9901 | 0.9883 | 0.9863 | 0.9840 | 0.9815 |
| 9 | 0.9989 | 0.9986 | 0.9982 | 0.9978 | 0.9973 | 0.9967 | 0.9960 | 0.9952 | 0.9942 | 0.9931 |
| 10 | 0.9997 | 0.9996 | 0.9995 | 0.9994 | 0.9992 | 0.9990 | 0.9987 | 0.9984 | 0.9981 | 0.9977 |
| 11 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9994 | 0.9993 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

CUMULATIVE POISSON PROBABILITIES

| λ | 4.00 | 4.10 | 4.20 | 4.30 | 4.40 | 4.50 | 4.60 | 4.70 | 4.80 | 4.90 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.0183 | 0.0166 | 0.0150 | 0.0136 | 0.0123 | 0.0111 | 0.0101 | 0.0091 | 0.0082 | 0.0074 |
| 1 | 0.0916 | 0.0845 | 0.0780 | 0.0719 | 0.0663 | 0.0611 | 0.0563 | 0.0518 | 0.0477 | 0.0439 |
| 2 | 0.2381 | 0.2238 | 0.2102 | 0.1974 | 0.1851 | 0.1736 | 0.1626 | 0.1523 | 0.1425 | 0.1333 |
| 3 | 0.4335 | 0.4142 | 0.3954 | 0.3772 | 0.3594 | 0.3423 | 0.3257 | 0.3097 | 0.2942 | 0.2793 |
| 4 | 0.6288 | 0.6093 | 0.5898 | 0.5704 | 0.5512 | 0.5321 | 0.5132 | 0.4946 | 0.4763 | 0.4582 |
| 5 | 0.7851 | 0.7693 | 0.7531 | 0.7367 | 0.7199 | 0.7029 | 0.6858 | 0.6684 | 0.6510 | 0.6335 |
| 6 | 0.8893 | 0.8786 | 0.8675 | 0.8558 | 0.8436 | 0.8311 | 0.8180 | 0.8046 | 0.7908 | 0.7767 |
| 7 | 0.9489 | 0.9427 | 0.9361 | 0.9290 | 0.9214 | 0.9134 | 0.9049 | 0.8960 | 0.8867 | 0.8769 |
| 8 | 0.9786 | 0.9755 | 0.9721 | 0.9683 | 0.9642 | 0.9597 | 0.9549 | 0.9497 | 0.9442 | 0.9382 |
| 9 | 0.9919 | 0.9905 | 0.9889 | 0.9871 | 0.9851 | 0.9829 | 0.9805 | 0.9778 | 0.9749 | 0.9717 |
| 10 | 0.9972 | 0.9966 | 0.9959 | 0.9952 | 0.9943 | 0.9933 | 0.9922 | 0.9910 | 0.9896 | 0.9880 |
| 11 | 0.9991 | 0.9989 | 0.9986 | 0.9983 | 0.9980 | 0.9976 | 0.9971 | 0.9966 | 0.9960 | 0.9953 |
| 12 | 0.9997 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9992 | 0.9990 | 0.9988 | 0.9986 | 0.9983 |
| 13 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9997 | 0.9996 | 0.9995 | 0.9994 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9998 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 5.00 | 5.50 | 6.00 | 6.50 | 7.00 | 7.50 | 8.00 | 8.50 | 9.00 | 9.50 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.0067 | 0.0041 | 0.0025 | 0.0015 | 0.0009 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0001 |
| 1 | 0.0404 | 0.0266 | 0.0174 | 0.0113 | 0.0073 | 0.0047 | 0.0030 | 0.0019 | 0.0012 | 0.0008 |
| 2 | 0.1247 | 0.0884 | 0.0620 | 0.0430 | 0.0296 | 0.0203 | 0.0138 | 0.0093 | 0.0062 | 0.0042 |
| 3 | 0.2650 | 0.2017 | 0.1512 | 0.1118 | 0.0818 | 0.0591 | 0.0424 | 0.0301 | 0.0212 | 0.0149 |
| 4 | 0.4405 | 0.3575 | 0.2851 | 0.2237 | 0.1730 | 0.1321 | 0.0996 | 0.0744 | 0.0550 | 0.0403 |
| 5 | 0.6160 | 0.5289 | 0.4457 | 0.3690 | 0.3007 | 0.2414 | 0.1912 | 0.1496 | 0.1157 | 0.0885 |
| 6 | 0.7622 | 0.6860 | 0.6063 | 0.5265 | 0.4497 | 0.3782 | 0.3134 | 0.2562 | 0.2068 | 0.1649 |
| 7 | 0.8666 | 0.8095 | 0.7440 | 0.6728 | 0.5987 | 0.5246 | 0.4530 | 0.3856 | 0.3239 | 0.2687 |
| 8 | 0.9319 | 0.8944 | 0.8472 | 0.7916 | 0.7291 | 0.6620 | 0.5925 | 0.5231 | 0.4557 | 0.3918 |
| 9 | 0.9682 | 0.9462 | 0.9161 | 0.8774 | 0.8305 | 0.7764 | 0.7166 | 0.6530 | 0.5874 | 0.5218 |
| 10 | 0.9863 | 0.9747 | 0.9574 | 0.9332 | 0.9015 | 0.8622 | 0.8159 | 0.7634 | 0.7060 | 0.6453 |
| 11 | 0.9945 | 0.9890 | 0.9799 | 0.9661 | 0.9467 | 0.9208 | 0.8881 | 0.8487 | 0.8030 | 0.7520 |
| 12 | 0.9980 | 0.9955 | 0.9912 | 0.9840 | 0.9730 | 0.9573 | 0.9362 | 0.9091 | 0.8758 | 0.8364 |
| 13 | 0.9993 | 0.9983 | 0.9964 | 0.9929 | 0.9872 | 0.9784 | 0.9658 | 0.9486 | 0.9261 | 0.8981 |
| 14 | 0.9998 | 0.9994 | 0.9986 | 0.9970 | 0.9943 | 0.9897 | 0.9827 | 0.9726 | 0.9585 | 0.9400 |
| 15 | 0.9999 | 0.9998 | 0.9995 | 0.9988 | 0.9976 | 0.9954 | 0.9918 | 0.9862 | 0.9780 | 0.9665 |
| 16 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9990 | 0.9980 | 0.9963 | 0.9934 | 0.9889 | 0.9823 |
| 17 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9984 | 0.9970 | 0.9947 | 0.9911 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9993 | 0.9987 | 0.9976 | 0.9957 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 | 0.9989 | 0.9980 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9991 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

CUMULATIVE POISSON PROBABILITIES

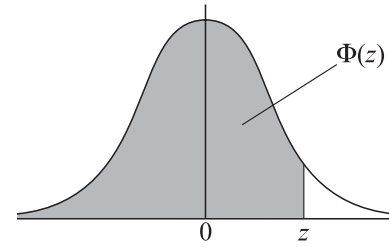
| λ | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0028 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0103 | 0.0049 | 0.0023 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0293 | 0.0151 | 0.0076 | 0.0037 | 0.0018 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 5 | 0.0671 | 0.0375 | 0.0203 | 0.0107 | 0.0055 | 0.0028 | 0.0014 | 0.0007 | 0.0003 | 0.0002 |
| 6 | 0.1301 | 0.0786 | 0.0458 | 0.0259 | 0.0142 | 0.0076 | 0.0040 | 0.0021 | 0.0010 | 0.0005 |
| 7 | 0.2202 | 0.1432 | 0.0895 | 0.0540 | 0.0316 | 0.0180 | 0.0100 | 0.0054 | 0.0029 | 0.0015 |
| 8 | 0.3328 | 0.2320 | 0.1550 | 0.0998 | 0.0621 | 0.0374 | 0.0220 | 0.0126 | 0.0071 | 0.0039 |
| 9 | 0.4579 | 0.3405 | 0.2424 | 0.1658 | 0.1094 | 0.0699 | 0.0433 | 0.0261 | 0.0154 | 0.0089 |
| 10 | 0.5830 | 0.4599 | 0.3472 | 0.2517 | 0.1757 | 0.1185 | 0.0774 | 0.0491 | 0.0304 | 0.0183 |
| 11 | 0.6968 | 0.5793 | 0.4616 | 0.3532 | 0.2600 | 0.1848 | 0.1270 | 0.0847 | 0.0549 | 0.0347 |
| 12 | 0.7916 | 0.6887 | 0.5760 | 0.4631 | 0.3585 | 0.2676 | 0.1931 | 0.1350 | 0.0917 | 0.0606 |
| 13 | 0.8645 | 0.7813 | 0.6815 | 0.5730 | 0.4644 | 0.3632 | 0.2745 | 0.2009 | 0.1426 | 0.0984 |
| 14 | 0.9165 | 0.8540 | 0.7720 | 0.6751 | 0.5704 | 0.4657 | 0.3675 | 0.2808 | 0.2081 | 0.1497 |
| 15 | 0.9513 | 0.9074 | 0.8444 | 0.7636 | 0.6694 | 0.5681 | 0.4667 | 0.3715 | 0.2867 | 0.2148 |
| 16 | 0.9730 | 0.9441 | 0.8987 | 0.8355 | 0.7559 | 0.6641 | 0.5660 | 0.4677 | 0.3751 | 0.2920 |
| 17 | 0.9857 | 0.9678 | 0.9370 | 0.8905 | 0.8272 | 0.7489 | 0.6593 | 0.5640 | 0.4686 | 0.3784 |
| 18 | 0.9928 | 0.9823 | 0.9626 | 0.9302 | 0.8826 | 0.8195 | 0.7423 | 0.6550 | 0.5622 | 0.4695 |
| 19 | 0.9965 | 0.9907 | 0.9787 | 0.9573 | 0.9235 | 0.8752 | 0.8122 | 0.7363 | 0.6509 | 0.5606 |
| 20 | 0.9984 | 0.9953 | 0.9884 | 0.9750 | 0.9521 | 0.9170 | 0.8682 | 0.8055 | 0.7307 | 0.6472 |
| 21 | 0.9993 | 0.9977 | 0.9939 | 0.9859 | 0.9712 | 0.9469 | 0.9108 | 0.8615 | 0.7991 | 0.7255 |
| 22 | 0.9997 | 0.9990 | 0.9970 | 0.9924 | 0.9833 | 0.9673 | 0.9418 | 0.9047 | 0.8551 | 0.7931 |
| 23 | 0.9999 | 0.9995 | 0.9985 | 0.9960 | 0.9907 | 0.9805 | 0.9633 | 0.9367 | 0.8989 | 0.8490 |
| 24 | 1.0000 | 0.9998 | 0.9993 | 0.9980 | 0.9950 | 0.9888 | 0.9777 | 0.9594 | 0.9317 | 0.8933 |
| 25 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9974 | 0.9938 | 0.9869 | 0.9748 | 0.9554 | 0.9269 |
| 26 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9987 | 0.9967 | 0.9925 | 0.9848 | 0.9718 | 0.9514 |
| 27 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9983 | 0.9959 | 0.9912 | 0.9827 | 0.9687 |
| 28 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 | 0.9978 | 0.9950 | 0.9897 | 0.9805 |
| 29 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9989 | 0.9973 | 0.9941 | 0.9882 |
| 30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9986 | 0.9967 | 0.9930 |
| 31 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9993 | 0.9982 | 0.9960 |
| 32 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9990 | 0.9978 |
| 33 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9995 | 0.9988 |
| 34 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 |
| 35 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 |
| 36 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 37 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 38 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z , the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \leq z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



| z | | | | | | | | | | | ADD | | | | | | | | |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----|---|----|----|----|----|----|----|----|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that

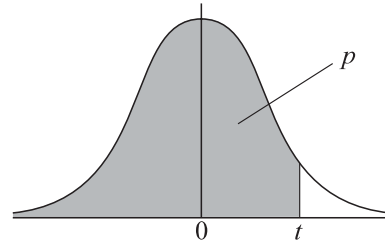
$$P(Z \leq z) = p.$$

| | | | | | | | | | |
|-----|-------|-------|-------|-------|-------|-------|--------|-------|--------|
| p | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
| z | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

CRITICAL VALUES FOR THE t DISTRIBUTION

If T has a t distribution with ν degrees of freedom then, for each pair of values of p and ν , the table gives the value of t such that

$$P(T \leq t) = p.$$

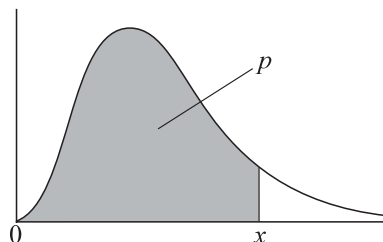


| p | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|-----------|-------|-------|-------|-------|-------|-------|--------|-------|--------|
| $\nu = 1$ | 1.000 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 4.773 | 5.894 | 6.869 |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.197 | 3.610 | 3.922 |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.057 | 3.421 | 3.689 |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.038 | 3.396 | 3.660 |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 2.860 | 3.160 | 3.373 |
| ∞ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

CRITICAL VALUES FOR THE χ^2 DISTRIBUTION

If X has a χ^2 distribution with ν degrees of freedom then, for each pair of values of p and ν , the table gives the value of x such that

$$P(X \leq x) = p.$$



| p | 0.01 | 0.025 | 0.05 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 |
|-----------|-----------------------|-----------------------|-----------------------|-------|-------|-------|-------|-------|-------|
| $\nu = 1$ | 0.0 ³ 1571 | 0.0 ³ 9821 | 0.0 ² 3932 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 | 10.83 |
| 2 | 0.02010 | 0.05064 | 0.1026 | 4.605 | 5.991 | 7.378 | 9.210 | 10.60 | 13.82 |
| 3 | 0.1148 | 0.2158 | 0.3518 | 6.251 | 7.815 | 9.348 | 11.34 | 12.84 | 16.27 |
| 4 | 0.2971 | 0.4844 | 0.7107 | 7.779 | 9.488 | 11.14 | 13.28 | 14.86 | 18.47 |
| 5 | 0.5543 | 0.8312 | 1.145 | 9.236 | 11.07 | 12.83 | 15.09 | 16.75 | 20.51 |
| 6 | 0.8721 | 1.237 | 1.635 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 | 22.46 |
| 7 | 1.239 | 1.690 | 2.167 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 | 24.32 |
| 8 | 1.647 | 2.180 | 2.733 | 13.36 | 15.51 | 17.53 | 20.09 | 21.95 | 26.12 |
| 9 | 2.088 | 2.700 | 3.325 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 | 27.88 |
| 10 | 2.558 | 3.247 | 3.940 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 | 29.59 |
| 11 | 3.053 | 3.816 | 4.575 | 17.28 | 19.68 | 21.92 | 24.73 | 26.76 | 31.26 |
| 12 | 3.571 | 4.404 | 5.226 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 | 32.91 |
| 13 | 4.107 | 5.009 | 5.892 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 | 34.53 |
| 14 | 4.660 | 5.629 | 6.571 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 | 36.12 |
| 15 | 5.229 | 6.262 | 7.261 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 | 37.70 |
| 16 | 5.812 | 6.908 | 7.962 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 | 39.25 |
| 17 | 6.408 | 7.564 | 8.672 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 | 40.79 |
| 18 | 7.015 | 8.231 | 9.390 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 | 42.31 |
| 19 | 7.633 | 8.907 | 10.12 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 | 43.82 |
| 20 | 8.260 | 9.591 | 10.85 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 | 45.31 |
| 21 | 8.897 | 10.28 | 11.59 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 | 46.80 |
| 22 | 9.542 | 10.98 | 12.34 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 | 48.27 |
| 23 | 10.20 | 11.69 | 13.09 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 | 49.73 |
| 24 | 10.86 | 12.40 | 13.85 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 | 51.18 |
| 25 | 11.52 | 13.12 | 14.61 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 | 52.62 |
| 30 | 14.95 | 16.79 | 18.49 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 | 59.70 |
| 40 | 22.16 | 24.43 | 26.51 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 | 73.40 |
| 50 | 29.71 | 32.36 | 34.76 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 | 86.66 |
| 60 | 37.48 | 40.48 | 43.19 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 | 99.61 |
| 70 | 45.44 | 48.76 | 51.74 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 | 112.3 |
| 80 | 53.54 | 57.15 | 60.39 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 | 124.8 |
| 90 | 61.75 | 65.65 | 69.13 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 | 137.2 |
| 100 | 70.06 | 74.22 | 77.93 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 | 149.4 |

WILCOXON SIGNED RANK TEST

P is the sum of the ranks corresponding to the positive differences,
 Q is the sum of the ranks corresponding to the negative differences,
 T is the smaller of P and Q .

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of T

| | Level of significance | | | | |
|---------|-----------------------|--------------|---------------|--------------|---------------|
| | One Tail Two Tail | 0.05 0.10 | 0.025 0.05 | 0.01 0.02 | 0.005 0.01 |
| $n = 6$ | 2 | 0 | | | |
| 7 | 3 | 2 | 0 | | |
| 8 | 5 | 3 | 1 | 0 | |
| 9 | 8 | 5 | 3 | 1 | |
| 10 | 10 | 8 | 5 | 3 | |
| 11 | 13 | 10 | 7 | 5 | |
| 12 | 17 | 13 | 9 | 7 | |
| 13 | 21 | 17 | 12 | 9 | |
| 14 | 25 | 21 | 15 | 12 | |
| 15 | 30 | 25 | 19 | 15 | |
| 16 | 35 | 29 | 23 | 19 | |
| 17 | 41 | 34 | 27 | 23 | |
| 18 | 47 | 40 | 32 | 27 | |
| 19 | 53 | 46 | 37 | 32 | |
| 20 | 60 | 52 | 43 | 37 | |

For larger values of n , each of P and Q can be approximated by the normal distribution with mean $\frac{1}{4}n(n + 1)$ and variance $\frac{1}{24}n(n + 1)(2n + 1)$.

WILCOXON RANK SUM TEST

The two samples have sizes m and n , where $m \leq n$.

R_m is the sum of the ranks of the items in the sample of size m .

W is the smaller of R_m and $m(m + n + 1) - R_m$.

For each pair of values of m and n , the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of W

| | Level of significance | | | | | | | | | | | |
|----------|-----------------------|-------|------|---------|-------|------|---------|-------|------|---------|-------|------|
| | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 |
| One Tail | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 |
| Two Tail | 0.1 | 0.05 | 0.02 | 0.1 | 0.05 | 0.02 | 0.1 | 0.05 | 0.02 | 0.1 | 0.05 | 0.02 |
| n | $m = 3$ | | | $m = 4$ | | | $m = 5$ | | | $m = 6$ | | |
| 3 | 6 | – | – | | | | | | | | | |
| 4 | 6 | – | – | 11 | 10 | – | | | | | | |
| 5 | 7 | 6 | – | 12 | 11 | 10 | 19 | 17 | 16 | | | |
| 6 | 8 | 7 | – | 13 | 12 | 11 | 20 | 18 | 17 | 28 | 26 | 24 |
| 7 | 8 | 7 | 6 | 14 | 13 | 11 | 21 | 20 | 18 | 29 | 27 | 25 |
| 8 | 9 | 8 | 6 | 15 | 14 | 12 | 23 | 21 | 19 | 31 | 29 | 27 |
| 9 | 10 | 8 | 7 | 16 | 14 | 13 | 24 | 22 | 20 | 33 | 31 | 28 |
| 10 | 10 | 9 | 7 | 17 | 15 | 13 | 26 | 23 | 21 | 35 | 32 | 29 |

| | Level of significance | | | | | | | | | | | |
|----------|-----------------------|-------|------|---------|-------|------|---------|-------|------|----------|-------|------|
| | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 |
| One Tail | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 | 0.05 | 0.025 | 0.01 |
| Two Tail | 0.1 | 0.05 | 0.02 | 0.1 | 0.05 | 0.02 | 0.1 | 0.05 | 0.02 | 0.1 | 0.05 | 0.02 |
| n | $m = 7$ | | | $m = 8$ | | | $m = 9$ | | | $m = 10$ | | |
| 7 | 39 | 36 | 34 | | | | | | | | | |
| 8 | 41 | 38 | 35 | 51 | 49 | 45 | | | | | | |
| 9 | 43 | 40 | 37 | 54 | 51 | 47 | 66 | 62 | 59 | | | |
| 10 | 45 | 42 | 39 | 56 | 53 | 49 | 69 | 65 | 61 | 82 | 78 | 74 |

For larger values of m and n , the normal distribution with mean $\frac{1}{2}m(m + n + 1)$ and variance $\frac{1}{12}mn(m + n + 1)$ should be used as an approximation to the distribution of R_m .

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