



Mathematics

Advanced GCE

Unit 4737: Decision Mathematics 2

Mark Scheme for January 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2012

Any enquiries about publications should be addressed to:

OCR Publications PO Box 5050 Annesley NOTTINGHAM NG15 0DL

Telephone:0870 770 6622Facsimile:01223 552610E-mail:publications@ocr.org.uk

Annotations and abbreviations

Annotation in scoris	Meaning
√and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations	Meaning
in mark scheme	
E1	Mark for explaining
M1 dep*	Method mark dependent on a previous mark, indicated by *
сао	Correct answer only
oe	Or equivalent
soi	Seen or implied
WWW	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Decision strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Mark Scheme

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

G	uestion	Answer	Marks	Guidance			
1	(i)	$ \begin{array}{c} J \\ K \\ L \\ M \\ N \end{array} $ $ \begin{array}{c} S \\ T \\ U \\ V \\ W \end{array} $	B1	Bipartite graph correct May show evidence of film types as well	Ignore any extra thickening of lines, labels attached for alternating paths or an additional arc to <i>X</i> for alternating path		
1	(ii)	N = S - J = V $J = V, K = U, L = T, N = S$	M1 A1	This alternating path written down, not just read off from labels on graph, do NOT condone if done in part (iii) This matching written down in words or letters	Allow <i>NS SJ JV</i> as path May be implied if seen in a longer list and matching is correct cao		
1	(iii)	W = K - U = M J = V, K = W, L = T, M = U, N = S	B1 B1	This alternating path written down, not just read off from labels on graph This matching written down in words or symbols	Allow <i>WK KU UM</i> as path Not implied from matching Not ft cao		
1	(iv)	<i>N</i> can only pair with <i>S</i> so <i>J</i> must pair with <i>V</i> Hence <i>L</i> can only pair with <i>T</i> and <i>M</i> with <i>U</i> So <i>K</i> pairs with <i>W</i>	B1	Evidence of $N - S - J - V$ and $W - K - U - M - T - L$, but NOT just repeating the complete matching already found	W can only pair with K so U must pair with M, hence T must pair with L, leaving V to pair with J and S to pair with N W can only pair with K and N with S, so J must pair with V, L with T and M with U		

Q	uestio	n Answer	Marks	Guida	ince	
2	(i)	$\begin{array}{c} C(60) & F(30) \\ \hline 20 & 20 \\ A(20) & B(100) & 120 \\ \hline 120 & 180 \\ $		Correct structure, even with unnecessary dummies (ie precedences are correct) even with directions implied. Arcs must be labelled <i>A</i> , <i>B</i> , <i>C</i> , Correct structure with no unnecessary dummies, using directed arcs labelled <i>A</i> , <i>B</i> , <i>C</i> ,		
2	(ii)	Min completion time = 360 minutes Critical activities: <i>A</i> , <i>B</i> , <i>D</i> , <i>E</i> , <i>H</i> , <i>I</i>	M1 M1 A1 B1 B1	Forward pass correct Backward pass correct Both passes correct 360 mins or 6 hours <i>A</i> , <i>B</i> , <i>D</i> , <i>E</i> , <i>H</i> , <i>I</i>	ft their network if possible ft their network if possible cao from a correct structure (even with unnecessary dummies) cao, <u>with units</u> cao	
2	(iii)	0 60 120 180 240 300 360	M1 A1	Critical activities correct All correct	Duration = 360 with one worker needed throughout and two for the last 20 mins cao Scale on vertical axis may be implied	
2	(iv)	Move <i>C</i> to start at 150 mins, hence 150 Painting the ceiling (<i>G</i>) could happen at the same time as hang wallpaper (<i>C</i>) and paint feature wall (<i>D</i>) or at the same time as paint woodwork (<i>E</i>)	B1 B1	150 C, D, E	cao Need all three, and no others	

Q	uestion	Answer		Guidance		
3	(i) Add a dummy column of equal values		B1	Using a dummy column of equal values	May be implied from initial matrix	
		Subtract all values from a constant (eg 100) P R S X B 20 0 80 100 C 60 65 40 100 G 40 55 70 100 H 80 0 20 100	B1	Subtracting all values from a constant and achieving an initial matrix with no negative values Dummy column in initial matrix can have any non-negative value, provided that it is the same throughout	May be implied from initial matrix	
		20 0 80 100 20 25 0 60 0 15 30 60 80 0 20 100	M1	Reducing rows (method correct) (may be implied by achieving a correct reduced cost matrix before augmenting) No errors for <u>their</u> matrix		
		2008040 20 25 00 0 15 30 0 80 0 20 40	M1	Reducing columns (method correct) (may be implied by achieving a correct reduced cost matrix before augmenting) No errors for <u>their</u> matrix	If columns are reduced first then rows are now already reduced so M1 for rows is automatic	
		Cross through zeros 20 0 80 40 20 25 0 0 0 15 30 0 80 0 20 40	A1 M1	A correct reduced cost matrix (cao) Crossing through 0's using min number of lines and augmenting by min uncovered value (method correct, including entries crossed out twice) Condone at most one wrong value	One of these two reduced matrices Cross through zeros $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
		0 0 60 20 20 45 0 0 0 35 30 0	Correct augmented matrix (may be achieved after recovery from earlier errors but must only have augmented once)	Then as other solution cao		

Questio	on Answer	Marks	Guid	idance		
	EitherOr $B = P$ $B = R$ $C = S$ C is innocent G is innocent $G = P$ $H = R$ $H = S$	B1 B1	First solution (may imply innocent by default) Second solution (may imply innocent by default)	Using words or symbols (cao) Using words or symbols (cao) Allow B for sapphire bracelet and N for pearl necklace, for example		
3 (ii)	Remove row for <i>B</i> and column for <i>S</i> P R X $C 60 65 100$ $G 40 55 100$ $H 80 0 100$ Using least values in <i>P</i> and <i>R</i> columns G = P and H = R So <i>C</i> is innocent	B1 B1	A valid explanation (eg achieving this matching) However, 'without <i>S</i> column, Cook's only 0 is in dummy' is not enough for first B1 <u>unless</u> <i>G</i> and <i>H</i> are also considered Cook is innocent	Or work through by reducing rows and columns (no augmentation should be needed): 60 65 40 40 55 70 80 0 20 80 0 20 0 15 30 80 0 20 0 15 30 0 55 0 0 55 0 40 0 0		

⁴⁷³⁷

(Question		Answer	Marks	Guidance				
4	(i)	(a)	20 + 25 + 45 = 90 cars per minute	B1	90				
4	(i)	(b)	25 + 20 + 25 + 30 = 100 cars per minute	B1	100				
4	(i)	(c)	Maximum flow ≤ 90 cars per minute	B1		Strict ft from their (a) and (b)			
4	(ii)		S and T are in the same set	B1	<i>S</i> must be in one set and <i>T</i> in the other	Need to separate <i>S</i> and <i>T</i> (oe)			
4	(iii)		Can travel either way	B1 Two-way flow 'Can travel D to E or E to D'					
4	(iv)		The maximum that can travel along BC is 30 so this is the max that can go along CF	B1	BC = 30 (or $AB = 40$) specifically referred to	Allow 'max into $C = 30$ '			
4	(v)	(a)	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	M1 A1	BC = 30, $CF = 30$ and $FT = 60This flow shown on diagram (cao)$	Condone labelling procedure without flow only if correct and easy to interpret Assume blanks mean zero			
4	(v)	(b)	Cut {S, A}, {B, C, D, E, F, T} Maximum flow = minimum cut Flow = 60 and cut = 60 (so this is max flow) Max flow \geq this flow = 60 = this cut \geq min cut 60 = this flow \leq max flow \leq min cut \leq this cut = 60	M1 A1	This cut identified or drawn (on (a)) or equivalent in a description Cut through AD , AB Allow finding cut then saying that (in the flow) arcs AD , AB are saturated (so no more can flow) Allow 'cut = 60 and max flow = min cut so flow shown is max	Allow 'only 20+40 can leave <i>A</i> ' Allow 'no more can flow out of <i>A</i> ' Allow finding cut and stating that a flow of 60 has been found			
4	(vi)		For $0 \le x \le 30$, max flow = $60 + x$ For $x > 30$, max flow = 90	M1 A1 B1	60 + x $x \le 30 \text{ (or } x < 30\text{)}$ 90 is max	Seen (even if only on arc <i>FT</i>), or at least three numerical examples Identifying $x = 30$ as critical value Max = 90, even without $x > 30$			

G	Question		Answer	Marks	Guidance		
5	(i)		$(0; 0) \underbrace{\begin{array}{c} (1; 0) \\ 30 \\ 45 \\ (1; 1) \\ 47 \\ (2; 1) \end{array}}_{(2; 0)} \underbrace{\begin{array}{c} (2; 0) \\ 15 \\ (3; 0) \\ 35 \\ (3; 0) \\ 36 \\ (3; 0) \\ 36 \\ (3; 0) \\ 36 \\ (3; 0) \\ (3$	M1	May be drawn in reverse (Stage; state) labels used correctly Graph structure correct (ignore whether arcs are directed or undirected)	cao May also show letters, but must have (stage; state) as well Condone (0, 0) etc. An arc within stage 1 or within stage $2 \Rightarrow M0$	
				A1	Arcs weighted correctly	cao	
5	(ii)	(a)	This row represents the transition from $(0; 0)$ to $(1; 1)$	B1	State 1 in next stage	Interpretation of table	
5	(ii)	(b)	45 = weight of the transition from (0; 0) to (1; 1)35 is the suboptimal minimax value from (1; 1)	B1 B1	(0; 0) to (1; 1) minimax from (1; 1)	Do not accept A to C NOT $(0;0)$ - $(1;0)$ - $(1;1)$ Do not accept 'arc weight $(1;1)$ – (2;0)', must refer to minimax (oe)	
5	(iii)		C and D or (1; 1) and (2; 0)	B1	<i>C</i> and <i>D</i> or (1; 1) and (2; 0)	Condone C, D, F or A, C, D, F or using (stage; state) labels	
			From stage (0; 0), the suboptimal minimax corresponds to action $1 \Rightarrow (1; 1)$ From stage (1; 1), the suboptimal minimax corresponds to action $0 \Rightarrow (2; 0)$	B1	Explaining the connection between <u>action for minimax</u> and <u>state for next stage</u> Need both cases	Must refer to actions	
5	(iv)	(a)	J	B1	Accept 'J or K'		
5	(iv)	(b)	Friday = G $Thursday = D$ $Wednesday = B$	M1 A1	Accept 'at least <i>G</i> '	G D, B	

Ģ	Question		Answer					Marks	Guidance			
5	(v)	Wednesday $B = (1; 0)$ or $C = (1; 1)$ Thursday $D = (2; 0), E = (2; 1)$ or $F = (2; 2)$						M1	Wednesday and Thursday correct May imply days from stage labels	B, C D, E, F (shown as a new set)		
		Frid Satu			0) or <i>H</i> 0) or <i>K</i>	f = (3; 1) = (4; 1)		A1	All correct and <i>I</i> excluded May imply days from stage labels	G, H (shown as a new set) J, K (shown as a new set)		
								A1	Appropriate allocation of (stage; state) labels, or clear statement of stage =, state =	Correct or with states swapped within a stage, condone <i>I</i> also labelled in stage 3		
5	(vi)		Stage	State	Action	Working	Suboptimal minimax		Follow through from their	If <i>I</i> is included in stage 3 or stage		
			4	0	0	15	15		appropriate labelling	4, follow through until no effect		
				1	0	5	5	M1	Subortimal minimar values	15 and 5. 45 and 25 in appropriate		
				0	0	max(45, 15)	45	IVI I	Suboptimal minimax values	15 and 5, 45 and 35 in appropriate		
			3	1	0	$\max(35, 15)$	35		correct for stages 4 and 3	(stage; states)		
				0	0	max(45, 5) max(45, 45)	45					
				1	0	$\max(33, 45)$	45	M1	Suboptimal minimax values	Not dependent on previous M		
					2	-	1	$\max(43, 35)$	43		correct for stage 2	mark
				2	0	max(30, 45)				(45, 43, 40 in appropriate places,		
					1	max(40, 35)	40			with no extras)		
				0	0	max(50, 45)	50					
			1		0	max(35, 45)		A1	Suboptimal minimax values	(50, 45, 45)		
				1	1	max(47,43)	45		correct for stages 1 and 0	Dependent on both previous M		
			0	0	2	max(50, 40)			C	marks		
			0	0	0	max(30, 50) max(45, 45)	45					
		45 miles					10	M1	45 (cao)	Stated or read from table		
			at <i>C</i> , <i>D</i> ,	, G and .	I			A1	C, D, G, J - not as (stage; state)	(cao), with no extras		

⁴⁷³⁷

C	Question	Answer	Marks	Guidance		
6	(i)	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1 A1 B1	Column maxima (or their negatives) shown <i>T</i> written down (cao) 2 (www)	All correct, or equivalent if done convincingly ft their play-safe:	
6	(ii)	W is dominated by Y Rowena always wins more by choosing Y	B1	<i>Y</i> dominates or <i>Y</i> is <u>always</u> better	N = 0, P = 1, Q = 1 5 > 4, 1 > -1, 2 > 1, 1 > -2 all correct	
6	(iii)	Remove row for W, then add 1 throughout to remove negative values 2 402 6 230 1 022 If Colin chooses N then Rowena can expect to win $2x + 6y + z$ points	B1 B1	Remove W row (first row) and add 1 to values This table written out \Rightarrow B1 'Add 1 throughout' \Rightarrow B1 N column (first column) identified eg 1, 5, 0	May imply removing W row Not $(1+1)x + (5+1)y + (0+1)z$ N	
6	(iv)	$2x + 6y + z = 2.3$ $4x + 2y = 1.6$ $3y + 2z = 1.6$ $2x + 2z = 1.6$ $m \le 2.3 \text{ and } m \le 1.6 \Rightarrow \text{ maximum for } m \text{ is } 1.6$ and so maximum for $M \text{ is } 0.6$	M1 A1	Achieving 2.3 and 1.6 three times Correct explanation of why maximum value for <i>m</i> is 1.6 (0.6 is given in the question)	A statement explaining why 1.6 is used rather than 2.3, ('minimum of 2.3, 1.6', or seeing $m \le 2.3$ and $m \le 1.6$ with 1.6 chosen) Or equivalent for <i>M</i> May imply 'so maximum for <i>M</i> '	

C	Question	Answer	Marks	Guidance		
6	(v)	Colin wins the negatives of the values in the table, so, removing row <i>W</i> gives	B1	Colin's winnings are negatives of values in table	Values in table × -1	
		$\begin{array}{ c c c c c c c c c }\hline -1 & -3 & 1 & -1 \\ \hline -5 & -1 & -2 & 1 \\ \hline 0 & 1 & -1 & -1 \\ \hline \end{array}$		May also remove column N		
		If Rowena chooses row X then Colin can expect to win $-3p + q - t$ Rowena's optimum is to win 0.6 points so Colin's is to lose 0.6 points	M1 A1	Relating $-3p + r - t$ to the X row Explaining why expression equals -0.6	Enough to identify row X 'Rowena wins 0.6'	
6	(vi)	p = 0.2 q = 0.4 and $t = 0.4$	M1 A1	cao cao		

OCR (Oxford Cambridge and RSA Examinations) 1 Hills Road Cambridge CB1 2EU

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998 Facsimile: 01223 552627 Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee Registered in England Registered Office; 1 Hills Road, Cambridge, CB1 2EU Registered Company Number: 3484466 OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations) Head office Telephone: 01223 552552 Facsimile: 01223 552553



