

**Thursday 31 May 2012 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4763**      Mechanics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4763
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The fixed point A is at a height  $4b$  above a smooth horizontal surface, and C is the point on the surface which is vertically below A. A light elastic string, of natural length  $3b$  and modulus of elasticity  $\lambda$ , has one end attached to A and the other end attached to a block of mass  $m$ . The block is released from rest at a point B on the surface where  $BC = 3b$ , as shown in Fig. 1. You are given that the block remains on the surface and moves along the line BC.

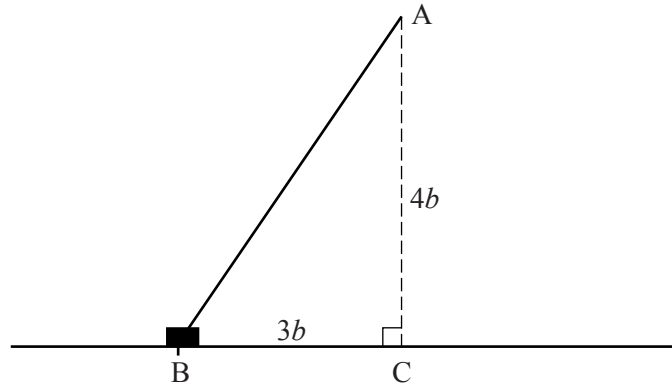


Fig. 1

- (i) Show that immediately after release the acceleration of the block is  $\frac{2\lambda}{5m}$ . [4]
- (ii) Show that, when the block reaches C, its speed  $v$  is given by  $v^2 = \frac{\lambda b}{m}$ . [4]
- (iii) Show that the equation  $v^2 = \frac{\lambda b}{m}$  is dimensionally consistent. [3]

The time taken for the block to move from B to C is given by  $km^\alpha b^\beta \lambda^\gamma$ , where  $k$  is a dimensionless constant.

- (iv) Use dimensional analysis to find  $\alpha$ ,  $\beta$  and  $\gamma$ . [4]

When the string has natural length 1.2 m and modulus of elasticity 125 N, and the block has mass 8 kg, the time taken for the block to move from B to C is 0.718 s.

- (v) Find the time taken for the block to move from B to C when the string has natural length 9 m and modulus of elasticity 20 N, and the block has mass 75 kg. [3]

- 2 (a) Fig. 2 shows a car of mass 800 kg moving at constant speed in a horizontal circle with centre C and radius 45 m, on a road which is banked at an angle of  $18^\circ$  to the horizontal. The forces shown are the weight  $W$  of the car, the normal reaction,  $R$ , of the road on the car and the frictional force  $F$ .

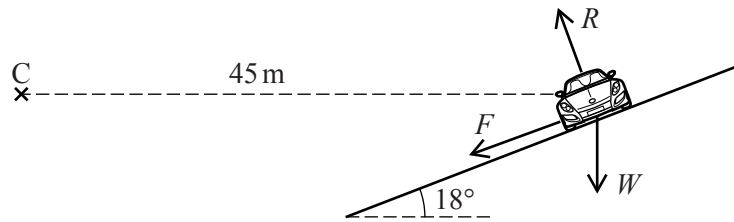


Fig. 2

- (i) Given that the frictional force is zero, find the speed of the car. [4]
- (ii) Given instead that the speed of the car is  $15 \text{ m s}^{-1}$ , find the frictional force and the normal reaction. [7]
- (b) One end of a light inextensible string is attached to a fixed point O, and the other end is attached to a particle P of mass  $m$  kg. Starting with the string taut and P vertically below O, P is set in motion with a horizontal velocity of  $7 \text{ m s}^{-1}$ . It then moves in part of a vertical circle with centre O. The string becomes slack when the speed of P is  $2.8 \text{ m s}^{-1}$ .

Find the length of the string. Find also the angle that OP makes with the upward vertical at the instant when the string becomes slack. [7]

- 3 A particle Q is performing simple harmonic motion in a vertical line. Its height,  $x$  metres, above a fixed level at time  $t$  seconds is given by

$$x = c + A\cos(\omega t - \phi)$$

where  $c$ ,  $A$ ,  $\omega$  and  $\phi$  are constants.

- (i) Show that  $\ddot{x} = -\omega^2(x - c)$ . [3]

Fig. 3 shows the displacement-time graph of Q for  $0 \leq t \leq 14$ .

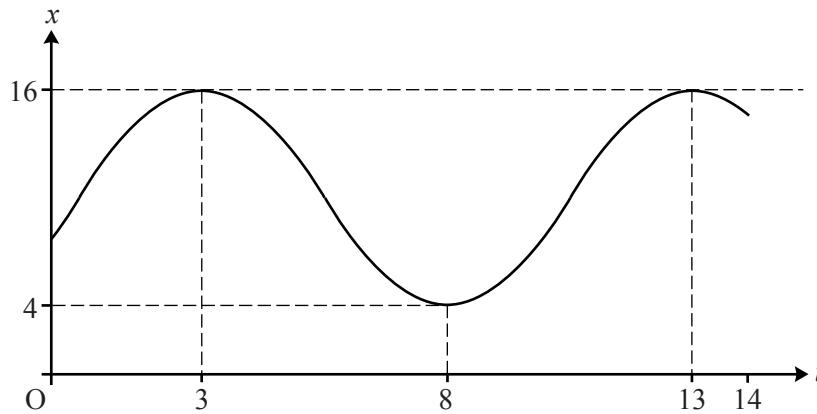


Fig. 3

- (ii) Find exact values for  $c$ ,  $A$ ,  $\omega$  and  $\phi$ . [6]
- (iii) Find the maximum speed of Q. [2]
- (iv) Find the height and the velocity of Q when  $t = 0$ . [3]
- (v) Find the distance travelled by Q between  $t = 0$  and  $t = 14$ . [4]

- 4 (a) A uniform lamina occupies the region bounded by the  $x$ -axis, the  $y$ -axis and the curve  $y = 3 - \sqrt{x}$  for  $0 \leq x \leq 9$ . Find the coordinates of the centre of mass of this lamina. [9]
- (b) Fig. 4.1 shows the region bounded by the line  $x = 2$  and the part of the circle  $y^2 = 25 - x^2$  for which  $2 \leq x \leq 5$ . This region is rotated about the  $x$ -axis to form a uniform solid of revolution  $S$ .

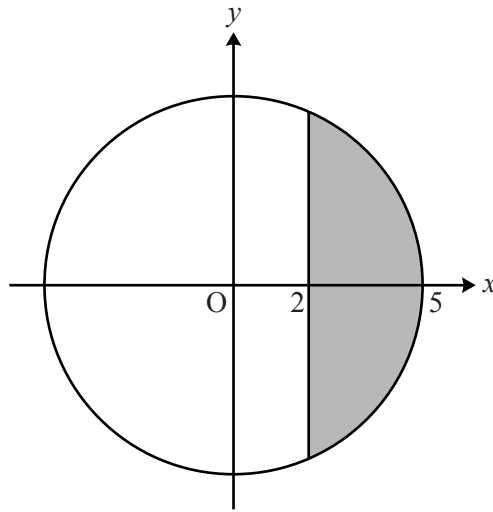


Fig. 4.1

- (i) Find the  $x$ -coordinate of the centre of mass of  $S$ . [5]

The solid  $S$  rests in equilibrium with its curved surface in contact with a rough plane inclined at  $25^\circ$  to the horizontal. Fig. 4.2 shows a vertical section containing  $AB$ , which is a diameter and also a line of greatest slope of the flat surface of  $S$ . This section also contains  $XY$ , which is a line of greatest slope of the plane.

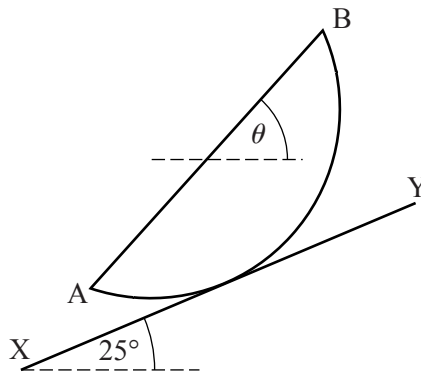


Fig. 4.2

- (ii) Find the angle  $\theta$  that  $AB$  makes with the horizontal. [4]

**BLANK PAGE**

**BLANK PAGE**

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.