OCR
RECOGNISING ACHIEVEMENT

Monday 25 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4773 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 8 page Answer Booklet (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

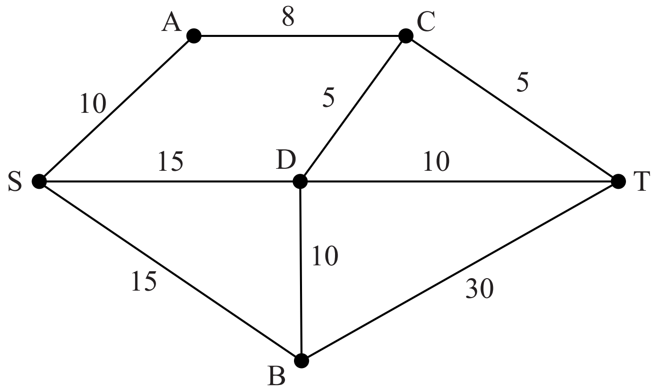
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 The diagram shows a network of pipes. Along pipes connected directly to the source, S, flows can only be away from S. Along pipes connected directly to the sink, T, flows can only be towards T. In all other pipes, flows can be in either direction. The number by each pipe is the flow capacity of that pipe.



(i) A flow has been established as follows: 15 from S to D, 5 from D to C, 5 from C to T, and 10 from D to T. Use the labelling algorithm to show this flow on a copy of the diagram, ensuring that all arcs are labelled. **[3]**

(ii) Find and identify a flow augmenting path. Augment the flow as much as is possible using this path, stating by how much you are augmenting. Show your augmented flow on a new diagram, with all arcs labelled. **[4]**

(iii) Repeat part **(ii)** as many times as is necessary to achieve a maximal flow. Produce a new, labelled, diagram for each flow augmentation. **[2]**

(iv) Prove that your final flow is maximal. **[2]**

(v) Formulate the maximal flow problem as an LP, ignoring the initial flow which was given in part **(i)**. Run your LP on LINDO, and interpret the results. **[7]**

2 The data in the table below relate to production possibilities for a mining company over a three-year planning cycle. Initially only mines A and B are open.

		Mine A	Mine B	Mine C	Mine D
Material availability (thousands of tonnes over the 3 years)		650	950	625	840
Material quality		66.0	64.1	63.0	62.1
Cost of mining (£ per tonne)	yr 1	30	28	–	–
	yr 2	33	31	27	28
	yr 3	35	33	29	30
Cost to open (£)		–	–	800 000	1 400 000

The company requires a production plan for the next 3 years satisfying the following:

- Requirements are for 600 000 tonnes of material in the first year, 850 000 tonnes of material in the second year, and 750 000 tonnes of material in the third year.
- The mean quality of the material mined in any year must not be less than 64.0.
- Mines C and/or D can be opened after the first year at the costs given. If either is opened for year 2, then it will be open for year 3 at no additional cost.
- Material cannot be stored from one year to the next.

Formulate, run and interpret a (mixed) integer LP to find the minimum cost 3-year production strategy.

[18]

3 A country's central bank has a committee which meets each month to set the bank rate, the interest rate at which the central bank lends to commercial banks. This is its tool for controlling the rate of inflation. The target is to achieve an inflation rate of 2.5%.

The committee does not realise that it takes two months for an adjustment to have an effect. Thus, if the committee puts up the bank rate by one percent in June, then the rate of inflation will rise by one percent, but not until August. By that time the committee will have had its July meeting, and will have made another change to the base rate.

(i) In June the rate of inflation was 3.74% and the committee put the bank rate down by 1.24% (since $2.5 - 3.74 = -1.24$). In May, the committee had put the bank rate down by 0.81%, so in July the rate of inflation fell from 3.74% to 2.93%. For the months up to and including December, give the rates of inflation, and the committee's actions.

Describe what will happen thereafter. **[3]**

(ii) The committee might have been better advised to be more cautious in its approach. Construct a spreadsheet column to show what would happen over a year, from the same starting values (inflation rate of 3.74% in June and 2.93% in July), if the committee subtracted the current rate of inflation from the target rate of inflation, and then applied an adjustment equal to half of this.

Produce another spreadsheet column to show what would happen over the year if, instead, the committee subtracted the current rate of inflation from the target rate of inflation, and then applied an adjustment equal to one eighth of this.

In each case you should describe the behaviour of your sequence of monthly inflation rates. **[4]**

(iii) Explain why, in the 'one eighth' case, the recurrence relation

$$u_{n+2} - u_{n+1} + 0.125u_n = 0.3125$$

models the sequence of inflation rates. **[1]**

(iv) Given that a particular solution is $u_n = 2.5$ (with $n = 0$ for June), solve the recurrence relation

$$u_{n+2} - u_{n+1} + 0.125u_n = 0.3125.$$

(If you compute numerical values, then give 4 decimal place accuracy.) **[5]**

(v) Use your spreadsheet to check your answer to part **(iv)**, and explain why your answer shows that u_n converges to 2.5. **[2]**

(vi) In more general terms, if instead of using a half or one eighth, the committee uses α, where $0 < \alpha < 1$, then the recurrence relation for u_n is $u_{n+2} - u_{n+1} + \alpha u_n = 2.5\alpha$. By considering the auxiliary equation, explain why $\alpha = 0.25$ might be a good value to use.

Construct a spreadsheet column to show what would happen over a year using $\alpha = 0.25$, and comment. **[3]**

4 Arnold and Juan are to play a round of golf. This consists of 18 holes. At each hole they each try to get their ball in the hole in the least number of strokes.

There are two methods of scoring. In strokeplay the player with the lower total number of strokes over the 18 holes is the winner of the round.

In matchplay each hole is competed for. If one player uses fewer strokes for a hole, then he wins the hole. If the number of strokes is equal then they 'halve' the hole. At the end of the 18 holes if either player has won more than 9 holes, counting each 'half' as 0.5 of a hole, then he wins the round, otherwise it is a draw.

Each hole has associated with it a 'par', the expected number of strokes needed for the hole. At the course where Arnold and Juan are playing there are 4 holes with a par of 3, 10 holes with a par of 4 and 4 holes with a par of 5.

Playing records show that Arnold and Juan play to the same standards – they both achieve par on average – but that Juan is more consistent than Arnold. An analysis of their records gives the following probability distributions for the number of strokes taken per hole.

	Arnold					Juan				
par 3	1	2	3	4	5	1	2	3	4	5
	0.01	0.19	0.62	0.15	0.03	0.00	0.12	0.80	0.04	0.04
par 4	2	3	4	5	6	2	3	4	5	6
	0.06	0.15	0.60	0.11	0.08	0.02	0.11	0.77	0.05	0.05
par 5	3	4	5	6	7	3	4	5	6	7
	0.09	0.14	0.55	0.12	0.10	0.05	0.10	0.71	0.08	0.06

(i) Construct a spreadsheet to simulate the outcomes of each of a par 3, a par 4 and a par 5 hole when Arnold and Juan are playing each other. Use the matchplay scoring method for each of the three holes, and summarise your results in a table. **[10]**

(ii) Simulate a round of golf played between Arnold and Juan, with 4 par 3 holes, 10 par 4 holes and 4 par 5 holes. Ensure that you print out a summary of your simulated round of golf, showing the results for each hole. Determine who is the winner using each of strokeplay and matchplay. **[4]**

(iii) Simulate a further 19 rounds using both scoring methods. Summarise your results in a table. You do not need to show the results for each hole, only for each round. **[2]**

(iv) By repeating your simulations, decide whether or not one player has an advantage over the other using strokeplay, and whether or not one player has an advantage over the other using matchplay. **[2]**

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