GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

June 2013

This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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OCR REPORT TO CENTRES

4751 Introduction to Advanced Mathematics (C1)

General Comments

Candidates coped well with this question paper. As commented in previous reports, many candidates are well-drilled in many of the techniques required for this unit, but applying them in unfamiliar situations (such as question 7) is found to be much more difficult.

In general, the standard of algebra was quite good. However, candidates do not always see the need to use brackets correctly. Those who omitted brackets penalised themselves in some questions by then failing to expand brackets correctly or, in question 6, failing to cube the 4. Brackets were formally needed in the answer to question 3, for instance.

Candidates are far happier dealing with whole numbers and quadratic equations that factorise, than they are with fractions, decimals and the use of quadratic formula. In the latter case, when this was necessary, some candidates 'gave up'. Those attempting the completing the square method with fractions were rarely successful.

Comments on Individual Questions

Section A

1) In finding the equation of the line, most candidates obtained full marks. The main mistake was to use a gradient of 2, due to confusion between perpendicular and parallel. There was a significant number of arithmetic errors especially in coping with negative signs and the fraction $-\frac{1}{2}$.

2) In the main, this question was completed well. Some candidates found the arithmetic challenging, especially if rearranging $x + 3y = 1$ to substitute in for y, with the resulting need to cope with fractions. A slight majority choose the substitution method rather than elimination. A few neglected to find y having found x.

3) In evaluating $(0.2)^{-2}$, many stopped after evaluating $\frac{1}{0.2^2}$ as $\frac{1}{0.04}$ (or, sadly often, as $\frac{1}{0.4}$). Those who converted to fractions first were more successful in reaching 25.

   In the second part, the majority found the power of a correctly, but the $16^{\frac{3}{4}}$ proved more challenging. A surprising number did $\frac{3}{4} \times 16 = 12$ to obtain $12a^9$.

4) There were many good answers in rearranging the formula. Most candidates managed at least one mark; some triple-decker fractions or the use of ÷ signs were seen. The π and the $(a + b)$ sometimes became separated. The radius was sometimes considered to be ±, and the > sign was used on more than one occasion. It was encouraging to see very few penalties incurred due to a poor square root symbol.

5) Those using f(2) = 18 were usually successful but occasionally 2^5 was miscalculated. A few f(–2) were seen. Those attempting long division were rarely successful, often not knowing how to deal with the term that was to be equated to 18. A few used a grid or 'backwards division' method having realised the constant term in the quotient needed to be 19 to give the correct remainder.

6) Finding the binomial coefficient was done successfully by many candidates, but a surprising number omitted the negative sign in their answer. Virtually all the candidates managed to pick up at least one mark, usually for writing down the binomial coefficient either in Pascal's triangle or as part of an expression. Many candidates wrote down an expression involving the key elements 10, 2^2 and $(-4)^3$, though the brackets were often omitted. It was at this point that some arithmetical errors crept in, in the attempts to calculate $10 \times 4 \times -64$.

7) This question was found to be difficult by many candidates. In the first part, although the correct answer was seen fairly frequently, a significant number of candidates, having correctly shown 125 and $\sqrt{5}$ to be 5^3 and $5^{\frac{1}{2}}$ respectively, then multiplied the indices to give an answer of $5^{\frac{3}{2}}$. Others found one of the indices correctly, but not the other. Some candidates treated it as though the square root applied to 125 as well.

   Few correct answers were seen in the second part. Being in a different format from usual, many candidates did not know how to cope with the initial $10 + 7\sqrt{5}$. Many multiplied the '$10 + 7\sqrt{5}$' term by $2 + \sqrt{5}$, sometimes losing the denominator altogether. Those who knew they should rationalise the denominator of the fraction often made errors in multiplying the denominator, with 9, –9 or 19 often seen (19 often following the correct 1 – 20). Some who correctly reached this point then only divided the first term in the numerator by –19.

8) Some who completed the square correctly lost the final mark by giving the minimum point of (2, –7) rather than the minimum *y*-value. Most common part-correct answers were getting the values of *a* and *b* correct but ignoring the multiple of 3 in establishing any value of *c*. The most common wrong values of *b* were –6 (dividing the '$-12x$' by 2) and 4 (taking the 3 out as a common factor and forgetting to divide by 2).

9) A small number of candidates made no attempt to generalise in either part, and simply gave examples to demonstrate the properties, so, of course, gained no marks. Most earned the mark in the first part for adding the values. In the second part there were some errors in multiplying but many correctly reached $3n^2 + 2$. The last mark was often lost due to an incomplete explanation centred on the fact that 2 was not divisible by 3, without making any reference to the fact that $3n^2$ is always divisible by 3.

Section B

10) (i) Almost all candidates obtained both marks for this part. Some gave 20 for the radius. (–3, –2) was only very occasionally seen.

   (ii) Finding the intersections of the circle with the axes was often well done. Almost all candidates obtained the first mark for substituting $y = 0$ or $x = 0$ in the circle equation, although some then omitted the $(-2)^2$ and/or $(-3)^2$. Some, having correctly found the *x*-intersections, substituted those values instead of starting again by substituting 0 to find the *y* values. Since the correct *y* equation did not factorise, there was distinctly less success in finding the *y* values than the *x* values. Some good solutions using completing the square were seen, after reaching $(y - 2)^2 = 11$, for instance, although some omitted the negative square root and then gave just one value.

(iii) Almost all candidates were able to show that A and B lie on the circle, usually by substituting the coordinates or finding the distance between each point and the centre, though some used the longer method of substituting one coordinate and solving the resultant quadratic equation. A few candidates omitted to show that B, as well as A, lies on the circle. Almost all candidates obtained the coordinates of the midpoint of AB (4, 5) successfully, with a small minority subtracting rather than adding. Most candidates realised that the distance of the chord from the centre of the circle was the distance from (4,5) to (3,2) and obtained the correct answer of $\sqrt{10}$. Some calculated the length of AB and proceeded no further; some halved it and used Pythagoras but only a minority were successful with this approach.

11) (i) Most candidates were able to sketch the correct shape for the cubic (the correct way up) and the majority were also able to correctly label the interceptions on the x-axis, although some gave the positive x intercept as ½ or 2/3 or 3. A few candidates failed to label the y-intercept or gave a wrong value such as 12 or -12. Some candidates drew their graph stopping at one of the roots (usually when x = – 4) instead of crossing the x-axis. Only a small number of candidates drew the graph upside-down and a handful drew the wrong shape altogether.

(ii) Quite a few errors were seen here, although a minority knew what to do and wrote down the correct values. Some gave factors or coordinates instead of roots, some solved x – 2 = 0 to give x = 2 as the root, and some went back to the equation but made an algebraic error in replacing x with x – 2, reaching $2x – 5$ as a factor instead of $2x – 7$.

(iii) The first part was generally well done; most correctly expanded two brackets and continued to simplify and add 15 to get the required result. Common errors were: not dealing correctly with the 15 such as saying g(x) = –15 to get the result, and errors in expanding or collecting terms. There was some poor 'mathematical grammar' with the '+15' often appearing out of nowhere.

In part (*B*) most candidates correctly showed g(1) = 0 although some failed to show enough working. Candiates were well-versed, in general, with the techniques of long division or inspection so that most achieved the correct quadratic factor and were able to go on and factorise this to gain full marks. Some tried to use the quadratic formula and then only gave $(x + 1)(x + 4.5)$ oe as factors.

12) (i) Almost all candidates were able to draw the line accurately. Omission of one or both of the signs on the negative intersections was quite common; a few reversed the coordinates. A few just wrote the two x-values only.

(ii) Most were able to obtain the correct equation and many went on to solve it successfully, although as expected, there were some errors in using the formula, especially frequently in evaluating the discriminant after correct substitution.

(iii) After the previous part, most candidates realised that they had to equate the two expressions and manipulate the resulting equation, although many had problems dealing with the 'k' terms ('$kx + 2x = 2kx$' for instance). Most candidates stopped there, but some realised that they needed to use '$b^2 - 4ac = 0$' to establish the final values of k. Some were confused with the k and x terms and were unable to identify the coefficients correctly or made errors in simplifying the equation. A few candidates used their graphs to establish the results for k. A few tried to apply calculus but rarely with any success.

4752 Concepts for Advanced Mathematics (C2)

General Comments

Most candidates were well-prepared for the examination, and were able to demonstrate a good understanding of the specification content. However, even some high scoring candidates lost marks due to basic errors in routine algebra and arithmetic or poor notation, especially in calculus questions. For the most part, work was clearly presented, but in a few instances marks were lost because it was so badly set out that it was difficult for the examiner to decipher just what the candidate was trying to convey. Many candidates adopt the practice of working with calculator values and only rounding final answers to an appropriate accuracy when presenting the final answer. However, a significant proportion lost marks by working with rounded or truncated numbers at an early stage, and then presenting an over-specified answer which could not possibly be justified from the figures used.

Comments on Individual Questions

1) (i) The overwhelming majority of candidates scored full marks on this question. A few candidates omitted the minus sign, and others lost a mark because they calculated the power as – 5 – 1 = − 4. A small number of candidates integrated. Some of these did so incorrectly, obtaining the answer $\frac{-2x^{-6}}{-6}$.

1) (ii) Most candidates identified the correct power, and went on to differentiate correctly. However, a significant minority gave the new power as $-\frac{1}{3}$, and a few integrated instead of differentiating. In cases where candidates failed to identify $\frac{1}{3}$ as the power, − 3 and $^3/_2$ were the most common errors.

2) (i) Nearly all candidates spotted the algebraic definition and correctly found the required terms. A few lost a mark by calculating the first, second and fourth term, and a few thought it was an inductive definition and substituted u_1 in the formula instead of n = 2. The most common description was "arithmetic"; a few candidates also earned the mark with "divergent". However, a significant minority either omitted a description altogether, gave an incorrect answer (most commonly "convergent" or "geometric" and occasionally "periodic") or spoiled their correct answer by hedging their bets: for example, "converging arithmetic" was fairly common.

2) (ii) A little over half of candidates scored full marks on this question. A surprising number either specifically identified *d* as ½, or omitted the minus sign when calculating the sum of the A.P., and ended up with an answer of 562.5. Very few of these candidates had the sense that something must have gone wrong. A few others mistakenly identified *a* as 12, but were still able to score 2 marks. Some candidates did not use the formula, instead writing out all the terms and calculating the sum directly: as often as not the arithmetic went astray and so only the first mark was earned. Approximately one fifth of candidates made no headway. The sigma notation proved insurmountable for a few, and others used the formula for the sum of a geometric progression or simply attempted to find the *n*th term. Others confused $\sum u_n$ with $\sum n$, and thus failed to score.

3) Most candidates recognised this standard question and integrated successfully; the majority went on to score full marks. A few dropped the minus sign on the first term and ended up losing both A marks, a few made arithmetic or substitution errors: c = − 1 and c = 81 were the most common wrong answers. In a small number of cases the final mark was withheld because at no point did the candidate write "y =" in their solution. A small number of candidates spoiled fully correct answers by reverting to an answer based on y = m*x* + c and an equally small number integrated successfully but used the original expression to evaluate c. Some candidates were unable to deal with the negative power successfully: variations of $\dfrac{18x}{x^3/3}$ and $\dfrac{18x^{-4}}{-4}$ were the most common errors. A significant minority of candidates (approximately 20%) failed to score because they multiplied by *x* and added c.

4) (i) Over half of candidates failed to score on this question. A surprising number drew "equilateral" triangles with unequal angles or sides, defined the cosine ratio incorrectly or not at all, or were unable to use Pythagoras correctly to obtain the third side of their right angled triangle. Generally, candidates did not set out their work rigorously; even those who understood what was required were minimalist in their approach and missed out on both marks.

4) (ii) Almost half of candidates obtained full marks on this question. Most obtained $\pm\dfrac{\pi}{6}$ or $\pm 30°$ to earn the first mark; some obtained the correct angles and left their answers in degrees or only found one of the angles and a few lost a mark by adding extra values, usually $\dfrac{\pi}{6}$ and / or $\dfrac{5\pi}{6}$. Over a quarter of candidates failed to score: the usual mistake was a first move of $2\theta = \sin^{-1}(\pm 1)$.

5) (i) More than half the candidates earned full marks; only a small minority failed to score at all. Almost all candidates drew a reasonable tangent, though it was occasionally at (1, 2) instead of (2, 4). A few lines were not tangents at all, the normal being the usual error, although occasionally curves were seen. Most candidates knew that they should draw a right-angled triangle, but many were very small, leading to a gradient which was outside the acceptable range. Some candidates clearly used two points taken from the curve which did not score, others tried (vainly) to differentiate the function, perhaps not understanding the word 'hence' in the question.

5) (ii) Approximately one third of candidates scored full marks and nearly all knew what was required. However, marks were commonly lost because of premature approximation. Candidates whose values for 2^x were 3.5 and 4.6 calculated a gradient of 2.75, outside the range and earned no marks. Candidates who stated the values 3.48 and 4.59 earned the first mark, but lost the second if they left their gradient as 2.775 rather than correcting it to 2.8 or 2.78, all that their values were qualified to give. Candidates who gave more figures (up to ten) usually earned the second mark. A few candidates calculated $\dfrac{\Delta x}{\Delta y}$; a few calculated the midpoint or calculated the gradient using the point (2, 4).

6) (i) Most candidates did not earn this mark: in spite of $\frac{2a}{1-r}$ being commonly seen, candidates were unable to make the connection to "S". Those who did, often left their answer embedded in irrelevant working.

6) (ii) Approximately three quarters of candidates made the correct initial move of $\frac{a}{1-r^2}$. A few then recognised that factorising the denominator was relevant, but only a tiny minority went on to earn the second mark.

7) A majority of candidates scored full marks, nearly always through correct application of the formula; although a few successfully used individual trapezia (the majority of those who adopted the latter approach were unsuccessful). Some slipped up by omitting the outer brackets and taking 3.0 (or occasionally 9) as the final *y*-value or by using an incorrect value for *h* (usually 1, occasionally 9 and rarely from incorrectly calculating 9÷6). Only a very small minority failed to score at all.

8) (i) The majority of candidates scored full marks. A few lost a mark by extending their function to the left or the right or by misplacing (2, 2) or (3, 0). Approximately 30% of candidates failed to score. A translation of $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ or a stretch in the *x*-direction scale factor 2 were the most common errors; a few candidates gave the end point as (2, 0) and the adjacent vertex as (1.5, 2).

8) (ii) There was an even better response to this part with almost 70% of candidates obtaining full marks. As in part (i), a few lost a mark by extending their function to the left or the right or by misplacing (4, 6) or (more often) (6, 0). Approximately one quarter of candidates failed to score. A translation of $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ or a stretch in the *x*-direction scale factor 3 were common; occasionally (4, 6) and (6, 0) were correct, but the other two points were simply left unaltered.

9) (i) Nearly all candidates differentiated successfully and set their derivative to zero. Over 60% of candidates went on to score full marks, although a few candidates made an error (usually $2x^2$ but occasionally + 24 was retained). However, a significant minority attempted unsuccessfully to factorise the quadratic and then gave up and a surprising number were unable to use the quadratic formula correctly. Very few candidates appeared to check their answers. Some candidates lost an easy mark by leaving their answers in an exact form or by quoting a different precision. Occasionally, candidates found the second derivative and set this equal to zero. A significant minority wasted time either by finding the associated *y*-values or by determining the nature of the turning points, neither of which were required.

9) (ii) This was very well answered by most candidates. Well over 80% earned the first mark and most went on to score full marks. Occasionally, candidates slipped up when collecting like terms and a few made a sign error when factorising. The minority who failed to score either omitted the question altogether, or set $6x + 24$ equal to the derivative.

9)(iii) This question was accessible to most candidates, although a significant minority scored zero. Many candidates found the area of the triangle using ½×base×height. Most of those who used a base of −4 realised that a negative area was impossible and so removed the minus sign. Some used integration and more often than not were successful – sometimes after 'losing' a minus sign. Most candidates also integrated successfully, but some made no further progress, as they ignored the upper limit and then 'airbrushed' the minus sign. A good proportion of those who did integrate successfully then made errors with the arithmetic. Some candidates earned two marks by combining the equations and integrating correctly, but a similar proportion ignored the upper limit or made arithmetical slips.

10)(i)(A) Nearly all candidates used Pythagoras to obtain *AC* correctly. A few used the Cosine Rule instead: most were successful. However, those who did calculate arctan $\left(\frac{12.8}{7.5}\right)$ were in the minority; most used the Sine or Cosine Rule and often lost the A mark having worked with rounded or truncated values. A few used the Cosine Rule wrongly obtaining an answer close to 90° and yet failed to spot that something must be wrong. A small minority of candidates assumed that *AC* bisects angle *ACB* and a similar sized group stopped at this point. However, most went on successfully to use the Sine Rule and obtain a value within the specified range.

10)(i) (B) Most candidates successfully found the area of *ABC*, although some used convoluted methods and lost accuracy or made errors with the arithmetic. Many candidates adopted the anticipated approach of ½×*AC*×*AD*×sin*DAC* and went on to present a final answer within range. However, some candidates omitted to add the two areas together and some used angle *DCA* in the formula. A small minority used convoluted methods involving the vertical height of triangle *ADC* or calculated *DC* and worked with that length instead. Accuracy was often lost, but about half of these candidates were successful.

10(ii) Approximately a quarter of candidates failed to score on this question. Either no response was made, or initial assumptions such as *MH* = *MG* or *HMG* = 45° were made and no progress was made. However, most were able to obtain one of the required angles correctly and many went on to use this to find *MH* or *HG* successfully. Far too many candidates then worked with truncated values or values which were approximated too severely. The method mark for finding the area of the sector was often earned, although a few candidates used the formula for arc length, found the area of the segment or selected something more exotic from the formula booklet. A small minority converted to degrees: sometimes this was successful, but it was disappointing to see calculations such as $\frac{1}{2}\times 1.72^2\times 63$ on occasion. In some cases, this was added to a correct value for the triangular sections, apparently without any awareness that the numbers generated couldn't possibly match up. A number of candidates found *HG* successfully and then used Pythagoras incorrectly to obtain a value for *MH* which was smaller. Again, this was usually ignored. Approximately 20% of candidates scored full marks, but a further 7% or 8% lost the last mark either by combining their answers incorrectly or by working with rounded or truncated figures and then over-specifying their final answer.

11) (i) A surprising number of candidates failed to score any marks. Many of these candidates adopted a 'simple interest' approach and evaluated $65 - 3\times0.017\times65$. A few candidates evaluated $65 - 3\times0.017$ or wrote $0.017^3\times65 = 61.7$. About two thirds of candidates did understand what was required but failed to appreciate the need to show more than 3 significant figures in their answer to 'show that' the value is 61.7 to this precision. $65\times0.983^3 = 61.7$ was quite common. A significant minority of candidates adopted a long-winded approach, showing each stage of the change, and were no more successful.

11) (ii) Fewer than 40% of candidates earned this mark. $65\times0.983^{n-1}$ was quite common, but more often than not the response was either non-existent or irrelevant.

11) (iii) This was inaccessible to most candidates, at least partly due to lack of success in the first two parts. It was surprising how few took advantage of the mark for obtaining n = 180: this request was either ignored, or a decimal answer was presented – although a few wrote $n > 180$. Very few scored all 3 marks for finding the given result. Most who did, had a correct formula from (ii) but had the inequality sign incorrect or used "=". Very few started off correctly, of those who did start correctly, a high proportion lost the third mark for reversing the sign too early. $\log_{10}(65 \times 0.983^n) < \log_{l0}3$ very often incorrectly led straight to $\log_{10}(65) \times \log(0.983^n) < \log_{l0}3$ which then became $\log_{10}65 + \log_{10}0.983^n < \log_{10}3$. It was pleasing that many of the successful candidates who did score full marks were justifying the reversal of the inequality sign, even though this was not required.

11) (iv) This proved more accessible than part (iii). A little under half of candidates were able to correctly substitute the appropriate value for d in conjunction with $t = 1$. However, $63.895 = 65 \times10^{-k}$ leading to $\log 63.895 = \log65 \times \log10^{-k}$ was quite common, so the remaining marks were inaccessible. Some candidates went on to earn the method mark, but lost at least one of the accuracy marks due to premature approximation - some candidates lost a mark by omitting to give an explicit statement of the value of *k.* Some lost both A marks because they divided by log65 instead of subtracting. A significant minority omitted the question altogether. In cases where there was an attempt which scored zero, the most common error was to begin with $d = 1$.

4753 Methods for Advanced Mathematics (C3 Written Examination)

General Comments

This question paper proved to be accessible, and many candidates scored over 65 marks. There were few candidates who scored below 25 marks. Virtually all candidates had enough time to complete the paper, though some questions, especially question 5, rewarded candidates who selected efficient methods, and there were a lot of candidates who required additional sheets to make further attempts at questions. It should be pointed out to candidates that examiners mark the last of a number of equally complete attempts (unless instructed otherwise), and this is not always the solution which scores the most marks.

There is a lot of calculus in this question paper, and the standard methods for differentiating and integrating were usually applied well. Notation is, however, important and candidates who miss out dx's or du's (especially when integrating by substitution), or essential brackets, can lose marks. This aspect has improved over the years but there are still candidates who do not understand the need for accurate notation and lose marks as a consequence. Other questions with given answers, such as question 5, require enough working to be shown as evidence that a correct method is being used, and it is particularly important to emphasise this to more able candidates, who are capable of processing steps in their heads which are nevertheless required to be written down for complete solutions. Usually, candidates who try to 'fiddle' solutions lose marks – question 6 is a good example of this.

The presentation of solutions varies enormously from candidates who write fluent, logical mathematics to those who offer disjointed, unconnected statements which lack any logical coherence, and leave others to decide whether they constitute a correct solution. While this is often not penalised – see the example offered below for question 7(i) – it would be nice to see more evidence of mathematics perceived as a true language, with statements linked with appropriate logical connectives such as 'equals' and 'which implies'.

Comments on Individual Questions

1(i) Some candidates were able to write down the correct values of a and b. Those who chose to use transformation arguments sometimes confused the stretch (1/2 or 2) and the translation (+1 or −1). Others chose to substitute the coordinates of specific points, with variable success.

1(ii) Most candidates, who knew what they were doing here either used ½ (x + 1) = ± x or squared both sides to find a quadratic in x. In the latter approach, some forgot to square the ½ and got the wrong quadratic. Examiners followed through their values for a and b. Some candidates omitted the y-coordinates. Candidates who found (1, 1) without showing a valid method got no marks, and there was evidence of the usual mistakes in using modulus, such as $|x + 1| = |x| + 1$, etc.

2(i) Many candidates failed to factorise the $n^2 - 1$, leaving their answer as $n(n^2 - 1)$. This rendered the second part of the question very difficult.

2(ii) There were two ideas needed here, the realisation that $n - 1$, n and $n + 1$ were consecutive integers, and that the product contained factors 2 and 3. Many candidates argued that the product had to be even, but this was not enough to gain credit. Others, predictably, verified the result with a few values of n, often describing this as 'proof by exhaustion'.

3(i) This was generally well done, with one mark awarded for −1 and 3 seen, and one for the correct notation. Some used x instead of y or f(x), and others confused domain and range.

3(ii) Most candidates are well practiced at finding inverses, and were familiar with arcsine, gaining full marks here. Leaving the result as $y = \arcsin((x - 1)/-2)$ lost the final A1. Very occasionally, candidates gave the answer as 1/f(x) or f'(x).

3(iii) Nearly all candidates found f'(x) and f'(0) correctly. The gradient of the inverse function was less successful. Many confused this with the condition for perpendicularity and gave the answer ½ instead of - 1/2 . Those who tried to differentiate $f^{-1}(x)$ directly had little success.

4(i) This proved to be an accessible 5 marks, with many candidates getting the question fully correct. Of those who did not, dh/dt = 10 (instead of dV/dt) was quite a common misconception; some tried to find dh/dV but failed to handle the constant of $1/\sqrt{\pi}$ correctly; and a surprising number finished off by saying that $10/10\pi = \pi$ instead of $1/\pi$.

5 Some candidates spotted the trick of simplifying the given function to get $y = ½ \ln(2x - 1) - ½ \ln(2x + 1)$ before differentiating, and thereby made lives considerably easier for themselves! However, writing the answer down from here omitted the vital 2 x ½ working and lost two marks. Those who started differentiating from $y = \ln(\sqrt{(2x - 1)} - \ln(\sqrt{(2x + 1)})$ needed to convince that they were using a chain rule on $\sqrt{u}$, where $u = 2x - 1$. Some tenacious candidates even managed to differentiate the given function correctly without these preliminaries, but made life hard for themselves.

6 The error d/dx (cos 2x) = 2sin 2x proved costly here, earning only a consolation M1; many also wrote the limits the wrong way round on the integral, and scored 3 out of 5, unless they 'lost' the negative sign, and scored M1 only. Many candidates seem unaware that swapping limits dealt with the negative sign. We also needed to see some evidence of why ln 4 – ln 2 = 2 to score the final A1.

7(i) This was generally well answered, though the flow of the argument was not always apparent. Many candidates write down arguments such as:

$f(-x) = -f(x)$
$2(-x)/(1 - (-x)^2) = -2x/(1 - x^2)$,

rather than the more convincing:

$f(-x) = 2(-x)/(1 - (-x)^2)$
$= -2x/(1 - x^2) = -f(x)$

Examiners condone this sort of logical error where possible, but candidates should be encouraged to frame such arguments correctly, with the use of implication signs if possible. Using 'RTP' for 'required to prove' might help to prevent candidates from arguing from the result they are trying to establish.

Those candidates who failed to scored 2 marks here either made errors in writing f(−x) as $-2x/(1 - -(x)^2)$, wrote that for odd functions $f(-x) \neq f(x)$, or verified using one value of x.

7(ii) Many candidates scored full marks here. The asymptote need to be indicated for the A1, and occasionally the section of curve from $x = 0$ to $x = -1$ was omitted.

8(i) The points of intersection were a write-down for many candidates. Weaker attempts failed to solve $(1 - x)\, e^{2x} = 0$ convincingly.

8(ii) This proved to be an accessible 6 marks for candidates. The derivative of e^{2x} and the product rule were generally correct, and deriving $x = ½$ and $y = e^{1/2}$ was straightforward, though many did not simplify the derivative to $e^{2x} – 2xe^{2x}$ immediately. Some candidates approximated for $e^{1/2}$ and lost a mark.

8(iii) Most candidates applied integration by parts to either $\int(1 – x)\, e^{2x}dx$ or $\int x\, e^{2x}dx$, using appropriate u, v', u' and v. Sign and/or bracket errors sometimes meant they failed to derive the correct result, but many were fully correct.

8(iv) This part proved to be quite demanding. Deriving the formula for g(x) was rarely correctly done. Common errors were an extra factor of 3 and an incorrect exponent. Most graphs showed the correct points of intersection (0, 3) and (2, 0), but the turning point was quite often incorrect or missing, and the shape failed to convince.

8(v) Those, of the relatively few candidates, who got this correct just wrote down $2 \times 3 \times ¼ (e^2 – 3)$. Some tried to integrate g(x), with little success.

9(i) Nearly all candidates gained this mark for the asymptote.

9(ii) Candidates tended to score heavily on this part. The implicit differentiation of y^3 was usually correct (albeit introduced into solutions belatedly), and the quotient rule was done well, though occasionally omission of brackets was penalised. Those who cube rooted and differentiated often succeeded in arriving at the given derivative. Another approach was to multiplying across before differentiating implicitly, but with required candidates to substitute for y to deduce the required form for the derivative. Finding $x = ¾$ for the turning point from the given derivative was straightforward, but some failed to find the correct y-coordinate by omitting the necessary cube root.

9(iii) There were plenty of accessible marks here as well. The first three marks, for transforming the integral to the variable u, were usually negotiated successfully, although poor notation – omitting du's or brackets – was sometimes penalised in the A1 mark. The second half involved evaluating the given integral with the correct limits. Some calculated the correct limits, but made errors in the integral (or forgot to integrate altogether). However, a reasonable number of candidates managed to do this work without errors. A rather curious misconception was to cube the correct value of the integral, because the function was presented implicitly in terms of y^3.

4754 Applications of Advanced Mathematics (C4)

General Comments

This paper was of a similar standard to previous years.

The questions were accessible to candidates of all abilities who were able to demonstrate their skills. There were few very low scores and also few very high scores, with full marks obtained by a few candidates. The higher scoring candidates were able to show their skills - particularly in Paper A questions 3, 6(iii) and 7.

The comprehension, Paper B, was well understood and most candidates scored good marks here.

As in previous years, many candidates lost unnecessary marks through poor algebra. Some particularly common such examples being:

- $\dfrac{1}{3(1+x)} = 3(1+x)^{-1} = 3(1-x+x^2...)$
- $\left(e^{x/5} + e^{-x/5}\right)^2 = e^{2x/5} + e^{-2x/5}$
- $\operatorname{cosec} x + 5\cot x = 3\sin x \Rightarrow \operatorname{cosec}^2 x + 25\cot^2 x = 9\sin^2 x$

These, and other algebraic errors, are detailed later in this Report.

Sign errors also continue to be a common cause of an unnecessary loss of marks.

In contrast, however, it was very pleasing to note that, unlike in previous years, few candidates failed to put a constant of integration. Examiners would now like to encourage candidates to change the constant when say multiplying through by 2 rather than renaming their constant as *c* at every stage.

Candidates should be reminded that when they are asked to 'Show' they need to show all stages of working. This is improving, but it is disappointing when marks are lost in this way.

Centres should be reminded that Papers A and B are marked separately and so supplementary sheets should be attached to the appropriate paper.

Comments on Individual Questions

Paper A

1)(i) 1)(ii) Whilst almost all candidates knew the general method for expressing the given fraction in partial fractions, there were a surprising number of numerical errors.

Most candidates were able to use the binomial expansion correctly although there were sign errors - often from using (-2*x*) as (2*x*).

The most common error-which was **very** common- was using $\dfrac{1}{3(1+x)} = 3(1+x)^{-1} = 3(1-x+x^2...) = 3-3x+3x^2$ and similarly for $\dfrac{1}{3(1-2x)}$.

The other frequent error was in the validity. Some candidates omitted this completely but many others failed to combine the validities from the two expansions, or failed to choose the more restrictive option.

2) Many candidates scored full marks when showing that the trigonometric equation could be rearranged as a quadratic and then solving it.

Where there were errors, these were usually in the first part when trying to establish the given result. Errors included failing to use the correct trigonometric identities, failing to use $\sin^2\theta + \cos^2\theta = 1$ or squaring the original expression term by term. Few candidates would say $x+3=7$ so $x^2+9=49$ and yet they happily square $\operatorname{cosec} x + 5\cot x = 3\sin x$ term by term.

Those who were unable to complete the first part sensibly then proceeded to solve the quadratic equation. Few errors were seen here. Occasionally the final solution was incorrect and few candidates offered additional incorrect solutions.

3) There were some good explanations with appropriate triangles in the first part.

However, too many candidates felt it was enough to only give the information given in the question and this was not sufficient. More was needed than, for example, a right-angled triangle with lengths of 1, 1 and 45º to show that tan 45º=1. It was necessary to clearly show the triangle was isosceles by giving the other angle or showing that the hypotenuse was $\sqrt{2}$, or equivalent. Some made errors when calculating the other lengths in both triangles. Some good candidates failed to score here seemingly being unfamiliar with where these identities came from.

The second part started well for most candidates, who usually used the correct compound angle formula, (although there were a few who thought that tan75º=tan45º+tan30º) and made the first substitution. Thereafter, this question gave the opportunity for candidates to show that they could eliminate fractions within fractions and rationalise the denominator. This was a good discriminator for the higher scoring candidates. A few candidates abandoned their attempt at half way and equated $\dfrac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$ at that stage to the given answer $2+\sqrt{3}$.

4) Most candidates scored high marks throughout this question.

In (i) the most common error was to omit ***r*** = at the start. Few candidates would write, for instance, $y = x+3$ without the y but the ***r*** = is too often omitted from vector equations.

In (ii) errors were usually numerical and in (iii) they were either numerical errors or the wrong vectors.

5) Most candidates scored the first four marks by forming the equations and solving them.

Marks were usually lost both when candidates failed to show their solutions worked in all three equations or failed to realise that O, A, B and C must all lie on the same plane for the final mark.

6)(i) Most candidates scored all four marks when solving the differential equation. It was pleasing to see so few candidates failing to include the constant of integration. Some candidates, however, tried to work backwards from the answer, or wrote $v^2= -4x^2+c$ without showing from where it came. The answer was given in this case so stages of working were needed.

Whilst, on this occasion, examiners condoned the change of constant candidates should be encouraged to change their constant when appropriate in the future and not use c twice to mean different things within the same question.

6)(ii) Most candidates obtained the mark for verifying that x =1. Many others also scored the following three marks but some had the incorrect coefficients when differentiating and only had the correct coefficient in the second term when working backwards from the answer, 4.

6)(iii) This part was rarely answered completely successfully. Most candidates understood that the '*R*' method was needed and scored the first three marks. This was the modal mark.

Calculus was needed in this question. The candidates were asked to find the constant α, and t is a time to be combined in ($2t$-α) so answers given in radians were required. The use of degrees here was a very common mistake. Many candidates then differentiated their angles in degrees and obtained no marks for v.

The last two marks were obtained only rarely but they were a good differentiator for the able candidates. Use of unspecified α, or in degrees or radians was allowed in this last part. Some candidates had difficulty as they had found both x and v separately using the '*R*' method and so had different values for their angles. Others realised the problem and were able to use trigonometric identities to change their v (or x) to have the same angle.

6)(iv) The majority of candidates scored the first two marks.. It was disappointing that candidates did not realise their mistake in part (ii) when they obtained an answer of time, t= 31.7 degrees.

7)(i) Some candidates were able to score full marks here with ease. Some candidates gave their answers as coordinates instead of lengths and others found OC instead of AC. There were also, however, some very confused and unclear methods used and many candidates lost marks having failed to use u =1 and u =10 or equivalent.

7)(ii) Most candidates understood that they needed to find and divide dy/du by dx/du. There was some very poor algebra when attempting to simplify $\frac{1-1/u^{-2}}{5/u}$. The derivatives were also frequently wrong-often including ln u. Many candidates stopped at this stage or substituted u =10 in their derivative and then stopped. Some candidates, who were able to score marks in the following stages, failed to realise that they could invert the derivative and quickly find the answer. Some used the gradient, 1.98, to form an equation of a straight line and find its intercept with the x axis. Other candidates, unfortunately, felt they could use 1.98 as a hypotenuse in the triangle with AC-with no success.

7)(iii) This was well understood but candidates lost marks by giving insufficient working when establishing a given result.

7)(iv) There were some excellent solutions here but the majority only scored one mark. This was awarded to those who correctly showed us their intention to find $V = \int \pi (e^{x/5} + e^{-x/5})^2 \, dx$. However, the majority could not expand this bracket. Usually it was thought to equal $e^{2x/5} + e^{-2x/5}$ but other incorrect options were seen, including powers such as x²/25. For those who did expand the bracket correctly, other errors followed-either using the wrong upper limit, failing to substitute the lower limit or, more commonly integrating either 2 as 0, or more particularly $e^{2x/5} as \frac{2e^{2x/5}}{5}$ and similarly for the other term.

Paper B

1) Answers were often correct but surprisingly many, and various, incorrect positions were seen. A number of candidates only used the letter *R* or *M* to indicate their points where the addition of a cross or dot would have made their position clearer.

2) Most candidates had the right idea but some were inaccurate with 10, 6, 4 being the most common alternative solution.

3) The graphs were usually identified correctly although there were also many guesses. The response tended to be either fully correct or all wrong.

4) Most candidates substituted α =60º and found *t*=9.2449 or similar and then the majority multiplied it by 2 to compare it with18. A few worked backwards from t=9 to reach approximately 60º when substituting in the appropriate equation, and were given full credit.

5(i) Most candidates found α = -17.31 as required. A few chose to use the number of days in January as 30 and lost one mark. Those who thought February was the first or third month received no credit.

5)(ii) Some candidates carelessly lost the negative sign in their angle or the negative sign in the formula and so lost unnecessary marks.

Many correctly obtained *t*=4.37 but not all converted this correctly to the 24 hour clock. 16:37 was commonly seen.

Candidates were able to follow through for full marks from the Special Case in part(ii)

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments

There were more candidates for this examination than has been usual for summer session; this may be the result of the winter session no longer being available. There were, as usual, many very good scripts, where the candidates produced accurate work which was well expressed, and scored high marks. There seemed to be a greater proportion of candidates who were less confident, and who may have found that they had insufficient time to do themselves justice. As has happened in the past, there were scripts which could have showed more attention to presentation. In many cases, marks were forfeited through insufficient algebraic competence, from simple mistakes over signs to more fundamental errors, such as in finding factors. There were many cases of wishful thinking in dealing with some of the lengthy expressions.

Comments on Individual Questions

1) This question provided a straightforward beginning to the question paper in which nearly all candidates did well. Any errors were mostly due to inattention either to signs or to the coefficients of x being matched, with C and D in error as a result. Finding D by setting $x = 2$ was not often used, and this could have provided a quick check on accuracy.

2) This question was also well done with the vast majority of candidates earning at least the first four marks. Most chose to use direct factorisation by inspection or to use division, and surprisingly, most used the linear factor $(z - \frac{3}{2})$ instead of $(2z - 3)$.

Candidates who used the root relationships were also frequently successful, but more often made an error with the signs in the resulting quadratic factor. The few candidates who assumed at the outset that the roots would be complex failed to justify this.

3) (i) Nearly all candidates were able to show a valid row by column multiplication leading to the correct value of p.

(ii) Most candidates used the inverse matrix successfully to solve the equation. Some chose to solve three simultaneous equations, and not many managed to do this without error.

4) (i) There was a good response from most candidates but a surprising number believed that z_2 was either $3 + 4j$ or $4 + 3j$. Some candidates forgot that exact expressions were requested.

(ii) An incorrect z_2 allowed the method mark to be earned but as the position of z_2 could be shown from the information given, the remaining marks were easily lost. $z_1 + z_2$ was usually well positioned, $z_1 - z_2$ was often seen in a strange place. Candidates who worked the sum and difference in terms of the exact expressions did not always appreciate the size and sign of the real and imaginary parts.

5) This question was well done by many candidates, but there were also many instances of poor written presentation. Summation sigmas could usefully be employed to make sense of the work. It was asked that a given result be <u>shown</u>: this indicates that a thorough and complete solution is necessary for full marks. In particular the provenance of the factor ¼ should be made clear from the outset or by demonstration at the end of the series summation. Responses where this factor appeared at a seemingly random place, or as an afterthought, lost a mark.

6) There was an almost equal split between those who tackled this question by substitution and those who used root relationships. In the former case there were some erroneous substitutions, usually $(3w-1)$, but also $(3w+3)$, $(3w+1)$ and $(\frac{w}{3}+1)$. Both methods required careful algebraic work that was not always forthcoming, in particular in developing the sum of products of the new roots, taken two at a time.

7) (i) Without giving a method in every case, most candidates showed the insight necessary to achieve full marks.

(ii) A few candidates convincingly argued this from an algebraic viewpoint. Most substituted a large number for x. This needed evaluation, at least to the point where the relative sizes of numerator and denominator could be seen. It is insufficient to discuss the signs of the constituent parts of the expression for y in the case when the asymptote is other than $y=0$. The sketches were mostly carefully drawn, but some candidates believe that a sketch can be a rough one, and fail to indicate clearly the salient features of the curve.

(iii) Many candidates found the correct intersections with $y=1$ and wrote down the relevant inequality, but many forgot the obvious inequality arising from the given graph. Some candidates initially tried to solve $\frac{3x^2}{(2x-1)(x+2)}<1$, which was unnecessary given the wording of the question, and lost marks by proceeding to multiply by $(2x-1)(x+2)$ without justifying that this was a positive quantity.

8) (i) This was usually answered well. The most common error was to write $\sum_{r=1}^{n} 1=1$ and this led to difficulty with earning the next mark, especially when compounded by trying to work back from the given result. This question was also subject to some careless notation; too few sigmas and missing brackets.

(ii) Most candidates knew how to earn the first three marks, but again the written work was frequently scruffy with missing sigmas in particular. It is nonsense to write the sum of a series as equal to its last term. In some scripts, the added term in the series was the k th, not the $(k+1)$ th. The following algebra proved too much for quite a few candidates, again not helped by missing brackets. It was just about possible to believe that the correct four term cubic could be instantly factorised, and some benefit of the doubt was given here.

Inevitably, marks were lost in the details of the induction argument. Initially, "assume $n=k$ " does not state what is being assumed. "True for $n=1$ and $n=k$" is not so, when the latter is conditional. The language must be precise, and many candidates displayed only half remembered sentences, indicating that they did not fully understand the induction argument.

9) This question was one in which many candidates gave no response to some or all sections, whether through lack of confidence or from running out of time.

(i) There were several reflections mentioned, but the most common error was to omit to give the centre of the rotation.

(ii) This was done fairly well, most used a point on $y = 2,$ usually $(0, 2)$.

(iii) Many instances of confused notation were seen. When $\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ is evaluated the result is not $\begin{pmatrix} x \\ y \end{pmatrix}$ but $\begin{pmatrix} -y \\ y \end{pmatrix}$. The first use of this style of notation was penalised, but not subsequently. A minority of candidates did distinguish between the original point and its transform, usually as $\begin{pmatrix} x' \\ y' \end{pmatrix}$. A safer route was to transform particular points and to recognise the relation between the x and y co-ordinates.

(iv) Some candidates confused the object and image, writing $\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -6 \\ 6 \end{pmatrix}$ which lost the mark for method, even if $y = 6$ was recovered.

(v) Many candidates found that the determinant was zero, although not always through a correct expression. Not many were able to give a coherent interpretation, without describing what was already given in the question. Several confused this with the role played by a zero determinant in solving a set of equations, which was not relevant here.

(vi) Ignoring unfortunate notation, many candidates scored both marks for combining the matrices in the correct sequence and for deducing the equation of the line. It was pleasing that a few did consider the clockwise rotation of the line $y = -x$ through 90° about the origin.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

Candidates performed very well on this question paper with a little over one-third of the entry scoring at least 60 marks and only about 5% scoring 20 marks or fewer. Question 1 (Maclaurin series, polar curves) was the best done question, followed by Question 3 (matrices), Question 4 (hyperbolic functions) and Question 2 (complex numbers).

A lot of the scripts were well presented and there seemed to be less use of supplementary sheets than in the past.

Candidates might have done even better if they had:

- read some of the questions more carefully, especially if they asked for more than one thing, such as Q3(iii) and Q4(ii);
- taken more care with 'elementary' algebra and numerical work, e.g. in Q3;
- written down more of their working, e.g. in Q3(iii) where it is inadvisable to do so much of the elimination 'in the head' and in Q4(iii) where a given answer had to be shown;
- taken more care with drawing diagrams, e.g. in Q1(b)(i) and Q2(b)(i);
- taken a little time to resolve the ambiguities of sign in Q4(ii): these are standard results and it is not inconceivable that they, or their close relatives, will reappear in a future series;
- improved their judgement in choosing an appropriate method for differentiation: it is much easier to use the Chain Rule in Q1(a) rather than the quotient rule;
- understood better what the inverse of a matrix does, i.e. that it can be used to solve systems of linear equations, such as those in Q3(ii); it was not necessary to start again from scratch;
- realised that parts labelled (i), (ii) etc. are linked, so Q3(i) was meant to be helpful in solving Q3(ii).

Comments on Individual Questions

1) **Maclaurin series, polar curves**

In part (a), candidates were asked to differentiate $\frac{1}{(1-2x)^2}$ repeatedly with respect to *x*, obtain the Maclaurin series, and give the domain of validity by considering the equivalent binomial expansion. The majority of candidates did all this as required, and very accurately. When differentiating, some used the quotient rule, which led to much unpleasantness if expressions were not simplified. Multiplying out the brackets was seen occasionally. A substantial number, having obtained the Maclaurin series, also obtained the binomial expansion, which was not always the same. The domain of validity was often correct but was sometimes omitted or not strict.

Part (b) was about the polar curve $r = a\sin 3\theta$. The sketches in (i) were often correct; a few candidates had a pointed extreme at $\theta = \frac{\pi}{6}$ or an incorrect form at the origin. Drawing further loops (beyond $0 \le \theta \le \frac{\pi}{3}$) was not penalised. The unfamiliar nature of (ii) put off some candidates: what was expected was that they would use their knowledge of the sine function and/or their sketches to conclude that the maximum value of *r* would occur when

$\theta = \frac{\pi}{6}$. Many did this, but some confused *x* and *y*. Others deployed all the formulae to do with cartesian and polar coordinates that they knew in futile attempts to reach an answer. Differentiating the given polar equation with respect to θ was fairly common. The area of the loop in (iii) was frequently correct: the overwhelming majority of candidates knew exactly what to do and had good ideas about how to do it, although sign and factor errors in the trigonometric identity and the integration were fairly common.

2) **Complex numbers**

Part (a) first asked candidates to produce a given expression for cos 5θ in terms of powers of cos θ. The technique was well known and usually carried out extremely accurately. The examiners ignored work which led to imaginary terms in the expansion of $(\cos\theta + j\sin\theta)^5$; had we not, maybe a couple more marks would have been lost. A small number of candidates went into $\left(z + \frac{1}{z}\right)^5$ mode: this can produce the required answer, but extremely few gave a complete correct argument by this method. Then candidates were asked to find the two possible values for $\cos^2\theta$ given that cos 5θ = 0 and cos $\theta \neq 0$, and go on to produce a given expression for cos 18°, and find a similar one for sin 18°. Most saw that they could use their quintic expression from part (i) to derive a quadratic equation in $\cos^2\theta$, which they solved generally accurately give or take a number of careless errors in applying the quadratic formula, and scored the first three marks. The fourth mark, for showing the given expression was cos 18°, was awarded very infrequently; most were content to ignore where the 18° had come from, while very, very few considered other possibilities such as 54°. Some used their calculators to find the inverse cosine of the given expression. For sin 18°, a substantial number of candidates went back to the imaginary parts in (i): again, this can produce a correct answer (via a quintic in sin θ, one of whose roots is 1 and which has a repeated quadratic factor) but this was never seen, and fortunately most attempts involved the use of $\cos^2\theta + \sin^2\theta = 1$. The final answer was expected to be given “in similar form”, i.e. simplified.

Part (b) (i), requiring the cube roots of $4\left(\sqrt{3} + j\right)$ to be obtained and plotted on an Argand diagram, met with widespread approval and many fully correct answers were seen. Errors, where they occurred, usually involved taking the modulus of $4\left(\sqrt{3} + j\right)$ to be 4 or 7. Most candidates knew that the cube roots occurred every $\frac{2\pi}{3}$ but the Argand diagram sometimes stretched the definition of rotational symmetry.

Part (ii) was less well done, with some candidates taking the wrong two points; the question stated that values of arguments in part (i) should be in the interval $0 < \theta < 2\pi$. *n* was sometimes not an integer and was less frequently correct than the argument.

3) **Matrices and linear equations**

Finding the inverse of a 3 × 3 matrix is a familiar process for almost all candidates, and nearly all the marks lost in part (i) were as a result of arithmetical or simple algebraic slips, for example, $13k - 52 - 13 = 13k - 39$. One or two multiplied their cofactors by the elements of the original matrix. A variety of methods were employed for finding the determinant, including Sarrus’ method.
In part (ii), although most candidates realised that they could use the inverse matrix they had found in (i) with $k = 4$, many others started again with algebra, wasting much time and making many errors. Some who used the matrix ‘lost’ the determinant.

Part (iii) caused the most trouble. An efficient method to find p is to pick one of the unknowns, eliminate it in two different ways obtaining equations which are not independent, and then find p. Once again, this was much more accurately done when candidates wrote down organised working, rather than trying to do the manipulation in their heads. Also once again, there was much 'tail-chasing' as some candidates eliminated first x, then y, and then z, filling the whole answer space (and sometimes supplementary sheets) with futile algebra. A neat method is to observe that 2 × equation (2) + equation (3) gives $5x - 7y + 4z = 4$, so $p = 4$; this was very rarely seen. Having failed to find p, many candidates gave up and did not try to find the general solution: those who did sometimes found a factor of $\frac{1}{13}$ unappealing and multiplied everything by 13 to remove it, thereby changing the 'point' on the solution line as well as the 'direction'. The geometric description of the general solution was frequently correct but was sometimes omitted.

4) **Hyperbolic functions**

In part (i), a proof that $\cosh^2 u - \sinh^2 u = 1$ was wanted. Most candidates knew what to do, but this part was perhaps less well done than expected, with too many careless errors. e^{-2u} was liable to become e^{2u} or even u^{-2u} and there were various sign slips.

Part (ii) asked for the proofs of two given answers; the derivative with respect to x of arsinh x, and its logarithmic form. Both seemed familiar and very many candidates were able to score 7/9, losing the marks given for resolving the ambiguity of sign in both expressions. For the logarithmic form, many spurious arguments involving gradient, "ln cannot be negative", $\left(x+\sqrt{1+x^2}\right)\left(x-\sqrt{1+x^2}\right)=1$ (sic) and "principle (sic) value" were seen.

Part (iii), involving an arsinh integral, was very well done. The final mark was withheld from candidates who omitted essential working required to show the given answer, and there were many of these.

Part (iv) was a good discriminator. Parts was probably the most common method employed, but candidates often could not make progress beyond the 'first line', not realising that their integral expressions on both sides could be combined. Substitution of u = arsinh x often ensured better progress and produced a correct indefinite integral. A few candidates recognised the standard form $\int f(x)f'(x)\,\mathrm{d}x = \frac{1}{2}\left(f(x)\right)^2 + c$. Many candidates lost the final mark through sloppy use of brackets or the invention of new 'log laws' which, for instance, caused $\frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)\right)^2$ to become $\ln\left(1+\sqrt{2}\right)$.

4757 Further Applications of Advanced Mathematics (FP3)

General Comments

Each of the five questions contained parts which proved to be accessible to the great majority of candidates, as well as parts which turned out to be more challenging. The candidates performed similarly in all questions, and the average mark for each question was about 17 out of 24. Candidates had to choose three questions to answer; questions 1 and 2 were the most popular, and question 3 was the least popular.

Comments on Individual Questions

1) ***Vectors***

In part (i) most candidates knew the standard formula for the shortest distance between skew lines and could apply it accurately. Throughout this question sign errors in answers were surprisingly common, both in the evaluation of vector products and in copying a vector from one line to the next.

In part (ii), the formula for the shortest distance from a point to a line was less well known. Many candidates completed the work confidently and efficiently, but common mistakes included using a scalar product instead of a vector product, as well as the usual sign errors.

In part (iii) the method for finding the point of intersection of two lines was well understood, and this was very often answered correctly.

In part (iv) most candidates gave the volume of the tetrahedron correctly as a scalar triple product. To make further progress it was necessary to find the vector **AD** of length 12. Very many candidates were unable to do this, often confusing **AD** with the position vector of D. Some just substituted the scalar 12 for the vector **AD**.

2) ***Multi-variable calculus***

In part (i) the partial differentiation was done well; then most candidates verified that $y = x$ and $y = 1 - x$ both made the two derivatives equal. This did not rule out other possibilities, and to earn full marks candidates needed to do more. The usual way to do this was to identify a factor $(y - x)$; some found an alternative method applying the quadratic formula to $y^2 - y + (x - x^2) = 0$.

In part (ii) almost every candidate knew that stationary points occurred when the partial derivatives were both zero. Most candidates then used the results of part (i) to obtain two quadratic equations and hence find the stationary points. Marks were sometimes lost through arithmetic slips, and especially for not showing convincingly that one of the quadratic equations had no real roots. Several candidates did not use the results of part (i) and obtained a quartic equation, some managing to factorise this correctly.

In part (iii), the application of partial derivatives to small changes was quite well understood. Having obtained $w \approx 21h$ many candidates lost the final mark by failing to give h in terms of w.

In part (iv), most candidates stated that the partial derivatives were both equal to 24, and proceeded in a similar way to part (ii). This was often completed successfully, but sometimes spoilt by careless slips. Those who formed a quartic equation here were rarely able to solve it.

3) ***Differential geometry***

In part (a) most candidates could write down a correct integral expression for the arc length of the polar curve. Further progress required the use of half-angle formulae; while some could do this confidently, many others did not know how to proceed.

In part (b) (i) it was essential to express $1 + (\mathrm{d}y/\mathrm{d}x)^2$ as a perfect square. Many candidates were able to do this, and often went on to obtain the correct value for the curved surface area.

In parts (b)(ii) and (b)(iii) the concepts of radius of curvature and centre of curvature were well understood, and many candidates answered both parts correctly.

4) ***Groups***

Most candidates gave the identity and inverses correctly in part (a) (i) and established a generator for *G* in part (a)(ii).

In part (a)(iii) candidates who considered powers of one generator from each group were able to obtain an isomorphism easily. Some thought it necessary to write out the composition table for *H*, and many matched up elements of the same order without further consideration.

Part (a) (iv) turned out to be the most difficult item on this question paper, and most candidates did not score any marks. A statement such as 'The group of symmetries of a square is not cyclic' must be justified by something like 'The rotations have orders 1, 2 or 4 and the reflections have order 2'. Similarly 'The group of symmetries is not abelian' must be justified by giving an example of two transformations which do not commute. Nevertheless, there were some excellent explanations. Quite a few nearly correct answers were spoilt by confusing elements of order 2 with self-inverse elements, in statements such as '*G* has only one self-inverse element' when there are in fact two (*c* and *e*).

In part (b) (i) most candidates established the result correctly, although some combined the functions in the wrong order. A few candidates simply multiplied $\mathrm{f}_\mathrm{m}(x)$ by $\mathrm{f}_n(x)$.

Most candidates could prove associativity in part (b)(ii), although many confused it with commutativity.

In part (b)(iii) most were able to complete the proof that *S* is a group.

In part (b)(iv) the (correct) subgroup most often given was the functions f_n for all even integers *n*. Most candidates did not score any marks in this part, with many thinking that $\{ \mathrm{f}_0 \;\; \mathrm{f}_1 \;\; \mathrm{f}_{-1} \}$ was a subgroup.

5) ***Markov chains***

Almost every candidate who attempted this question demonstrated competence in using their calculator to handle the matrices. In parts (i), (ii) and (iii) the great majority wrote down a correct transition matrix and used it convincingly to obtain the required probabilities. The only common error was the omission of the two 1's in the transition matrix.

Part (iv) turned out to be quite challenging; most candidates started correctly by finding probabilities after 9 tasks. However, these probabilities were often used wrongly, for example multiplying the probability that the game has not ended after 9 tasks by the probability that it has ended after 10 tasks.

Part (v) was answered well; the probability that the game has not ended after 14 tasks is only just less than 0.01, and rounding errors very often led to a wrong answer of 15.

In parts (vi) and (vii) most candidates obtained the limiting matrix and used it correctly to find the probability of winning. Some did not write down the limit of $\mathbf{P}^n$ but gave instead the limiting state probabilities.

The situation in part (viii) was well understood, with almost every candidate realising that the maximum probability of winning would occur when the contestant always starts with three lives, and most gave the corresponding probability correctly. A common wrong answer was 0.45 (which is the probability of winning after the first task).

In part (ix) most candidates started with a correct matrix equation, and a good number obtained the correct starting probabilities. A fairly common error was to use **P** instead of the limiting matrix.

4758 Differential Equations (Written Examination)

General Comments

The quality of the work offered by most candidates was of a high standard. The methods and approaches which needed to be applied in different situations were identified, understood and executed successfully by the majority of candidates. The reasons for less than full marks being awarded were often due to arithmetic or algebraic inaccuracies.

One particular point which arose in this series was in the sketching of graphs of solutions. It should be noted that when the behaviour of a graph for large values is required, then candidates must show a sufficient portion of the graph for it to be clear that this behaviour continues indefinitely. For example, when a curve becomes oscillatory with constant amplitude, then there must be at least two oscillations to demonstrate this. It is not sufficient either to show just one oscillation or to show several oscillations of varying amplitude from which the examiner is left to deduce the candidate's intention.

Most candidates attempted Questions 1 and 4, with either Question 2 or 3. It was notable that few candidates attempted all four questions.

Comments on Individual Questions

1) **Second order linear differential equation**

Candidates are very familiar with the method of solution of a second order linear differential equation and the majority of responses were well-presented and accurate. The main challenge in this example proved to be in appreciating the link between the two parts of the motion of the particle. The solution in part (i) was valid only for values of *t* between 0 and 10π and the solution in part (v) was valid only for values of *t* greater than 10π.

(i) Almost all candidates showed that they understood the method of finding the general solution of the given equation and many worked accurately. A minority of candidates made numerical errors when solving the simultaneous equations to find the particular solution.

(ii) Candidates were asked to sketch the graph of the particular solution for large positive values of *t.* Many candidates recognised that the exponential terms in the solution became negligible, leaving only oscillatory terms, but then sketched just one oscillation. This does not provide sufficient evidence of the nature of the solution. In particular, as in this case, at least two oscillations are required to show that the oscillations have constant amplitude.

(iii) Most candidates realised that the exponential terms became negligible and earned full marks in this part of the question. Full credit was given to correct follow-through values from incorrect solutions obtained in part (i).

(iv) The most common approach was to consider the discriminant arising from the auxiliary equation. Having found its value to be equal to one, many candidates concluded that overdamping was present without mentioning the significant reason for their conclusion, namely that the discriminant was positive.

(v) Only one-quarter of the candidates scored full marks in this part. The majority of candidates used the values of the displacement and the velocity found in part (iii) as values at time $t = 0$ instead of at time $t = 10\pi$. Those candidates who explained that they were re-setting the clock and measuring from time $t = 0$ again were given credit.

2) **First order linear differential equations**

This question was chosen by just over a quarter of the candidates.

(i) Almost all candidates scored full marks on this part, with the majority opting for a solution using the separation of variables. It is worth noting that both: the integrating factor method and the method using a complementary function and a particular integral provided very neat concise solutions to this request.

(ii) This part was usually answered correctly. The most common cause of error was the omission of a constant of integration.

(iii) It was pleasing that the majority of candidates realised the significance of the instruction to verify the value of k as 2.5, and substituted $k = 2.5$ and $t = 5$ into their result from part (ii). The small number of candidates who tried to solve to find k were always unsuccessful.

(iv) This request presented no problems to the candidates.

(v) The method of separation of variables, favoured by candidates, served them well here. The majority successfully negotiated the integration and the rearrangement to find v in terms of x, with accuracy. A common error was in determining the initial condition to apply to the solution. Most candidates measured the distance from zero, without stating or perhaps without realising, that their solution was only valid from the surface of the sea, where $x = 124$. The sketch graphs of the solution were usually of the correct shape, with the correct asymptote, but with an incorrect starting-point.

3) **First order linear differential equations**

This was a popular choice of question and most candidates earned the majority of the available marks.

(a) (i) The method required here was clearly understood and applied with success. The only errors were in the solution of the linear simultaneous equations for finding the particular integral.

(a) (ii) When sketching graphs, detailed algebraic analysis is not required. However, features of the solution which are given in the question or can easily be identified are expected to be seen on the sketch. In this case, the initial condition $y = 2$, $x = 0$ needed to be clearly identified. In addition, by using this condition with the given differential equation, it can easily be seen that the gradient of the curve when $x = 0$ is - 4. A sketch graph which started at the point (0, 2) with a negative gradient was required for the first of the two available marks. The second mark was awarded to a graph that showed oscillations of constant amplitude for larger values of x. As in Question 1, at least two complete oscillations with constant amplitude were needed.

(b) (i) The standard of the responses to this part was very high, with almost all candidates scoring full marks.

(b) (ii) Again, this part posed very few problems to candidates.

(c) Almost all candidates found and used the integrating factor successfully to find the expression $ye^{2x} = \int e^{2x} \tan x \; dx$. The limits $x = 0$ and $x = 1$ then needed to be applied to both sides of this expression, using the approximation given in the question, for $\int_0^1 e^{2x} \tan x \; dx$ to evaluate the right hand side. A very common error was to apply the limits to the right hand side of the expression but only to the exponential term on the left hand side.

4) **Simultaneous linear differential equations**

Simultaneous linear differential equations is a topic which is attractive to most candidates and although the beginning of the question was slightly different to usual, the pleasingly good standard of solutions was again evident.

(i) The vast majority of candidates solved the equation in z to find z in terms of t. A few candidates did not find the constant of integration and continued through the remainder of the question with an extra constant.

(ii) The method of solution was understood by almost all candidates and was executed accurately by the majority. Others made numerical or algebraic slips during the process of solution.

(iii) Almost all candidates scored at least two of the three available marks, demonstrating clearly that they knew what to do. Often, more arithmetical errors crept in.

(iv) Just under 50% of candidates gave the correct expressions for the particular solutions for x and y. The remaining candidates were still successfully applying the correct techniques but accuracy errors prevented the award of full marks.

(v) The first mark in this final part of the question was only available to those who had worked accurately throughout. The result was given in the question, partly to encourage those who had reached this stage without error, but also so that the last two marks were available to all candidates. Only a minority of candidates made any progress that could be rewarded. The most common response was to ignore the exponential term and state that because sin t was an oscillatory function, there were an infinite number times when x and y were equal. The candidates who were successful were those who drew a sketch of the exponential function and the sine function and noted that they crossed infinitely often.

4761 Mechanics 1

General Comments

This paper produced a satisfactory mark distribution. Candidates of all abilities were able to show what they could do but there were places where even the most able were challenged.

Several questions on this paper required candidates to work in vectors, which were in various formats. It was very pleasing to note that this caused no problems to candidates; they were all entirely comfortable working with them.

Comments on Individual Questions

1) This question, about drawing a force diagram, was not well answered. Candidates were expected to identify the three forces acting on a block and to mark each of them on a given diagram. Many tried to combine two of them, even though they were quite different forces; other answers can only be described as chaotic.

2) This question was about a projectile (a golf ball). The horizontal and vertical components of its initial velocity were given. Nearly all candidates were able to find the initial speed in part (i) and the flight time and range in part (ii). Common mistakes were to interchange the vertical and horizontal components, and, for those who used the method of finding the time to maximum height, to fail to double it for the flight time.

   In part (iii) (*A*) candidates were asked to show that the range was the same if the components of the initial speed were interchanged; most did this by repeating the calculation from part (ii) but a few saw that this result could be deduced from the form of the expression for the range. Candidates went into part (iii) (*B*) having just met an example where the same initial speed but a different angle of projection produced the same range; they were asked whether this was generally true. Many candidates saw the point of the question and gave a counter-example (commonly the ball being projected vertically upwards). However, others incorrectly thought that the statement was generally true. There were also many answers which gave an inadequate explanation of the correct result.

3) In part (i) of this question candidates were asked to find which of three forces, given as 3-dimensional column vectors, had the greatest magnitude. Almost all candidates got this right.

   Part (ii) of this question was about the application of Newton's second law to an object subject to the same three forces and its weight. Candidates needed to write the weight of the object in vector form. Many candidates got this completely right but others made mistakes with the weight, some applying it in the wrong direction or all three directions.

4) In this question, candidates were given the velocity of a particle using **i**, **j** notation to denote east and north, and they were asked to find when it was travelling on a compass bearing of 045^{o} and its speed at that time. This involved equating the components of **v**; this gave a quadratic equation, leading to two possible times. Candidates then had to recognise that at one of these times the bearing was 225^{o} not 45^{o}.

   Many candidates obtained full marks on this question. A few made the mistake of trying to work with position vector instead of the velocity. A common mistake was to fail to eliminate the 225^{o} case.

   A small number of candidates set out to answer this question using a trial and error method and some credit was given for this.

5) This question was about connected particles, in the form of two blocks on a table.

Part (i) was best answered treating the system as a whole; part (ii) asked for the tension in the connecting string and so required candidates to work with one of the blocks.

Both parts were correctly answered by many candidates. However, a few candidates did not realise that an acceleration of magnitude 2 could be in either direction, to the left or to the right. A not uncommon mistake, particularly in part (ii), was to introduce extra forces into the equations of motion.

6) This question was about motion with non-constant acceleration along a straight line. It was very well answered with many candidates obtaining full marks.

In part (i) candidates used a given equation for v to find when the particle is stationary. In part (ii) they had to integrate to find an expression for the position and substitute in the two times they had found in part (i). It was pleasing to note an almost complete absence of attempts using constant acceleration formulae.

7) This question was about forces in equilibrium. It was set in the context of two people hoisting an object towards the top of a building.

In part (i) candidates were asked to draw a triangle of forces. While there were plenty of correct answers many marks were lost through incorrect or missing labelling and the absence of arrows. A lot of candidates drew a force diagram instead and so could only obtain 2 out of the 3 marks.

Candidates who had drawn a correct triangle of forces in part (i) were usually successful in part (ii), which asked for information that could easily be obtained from it. Those who had drawn force diagrams in part (i) could still answer part (ii) and many did so successfully but usually after a little more work.

In part (iii) the situation had changed and many of those candidates who had made mistakes in the earlier parts were able to recover. The question asked for the vertical and horizontal equilibrium equations and there were many correct answers. Common errors involved incorrect signs or the omission of one of the forces.

In part (iv) candidates were asked to solve the equations they had obtained in part (iii) with particular values given for the two angles. There were many right answers but also many careless mistakes.

In part (v) candidates were presented with another situation and asked to explain why it was impossible. This was probably the most challenging question on the paper. There were a few excellent answers but many candidates did not present a coherent argument.

8) This question involved a sledge being pulled, initially horizontally and then up a slope.

Part (i) asked for the resistance to motion and required the use of a constant acceleration formula and then Newton's 2^{nd} Law. It was very well answered. A few candidates lost marks by using the given final answer in an argument that was less than a valid verification.

In part (ii) the situation changed because the rope pulling the sledge broke. In part (*A*) candidates were asked to find the speed of the sledge at a time when it was still moving and in part (*B*) at a later time when it would have come to a halt. Most candidates obtained the right answers to both parts. However, a few did not recognise that the acceleration changed when the rope broke and continued with the same value as they had in part (i). A more common mistake was to give a negative speed in part (*B*) rather than zero.

In part (iii) the sledge was being pulled up a smooth slope. There were many correct answers to this part but a few candidates were unable to use the component of the weight down the slope.

In part (iv), there was no longer a pulling force (the rope had broken again) and the sledge started moving up the slope, came to a stop and then slid down to the bottom of the slope. Candidates were asked to find how long this took. Many candidates knew what they had to do and there were plenty of correct answers; however, there were also many sign errors. This was the last question on the paper and several low-scoring candidates did not get started on it. There were also those who substituted completely wrong numbers into their constant acceleration formulae, indicating incorrect analysis of the situation.

4762 Mechanics 2

General Comments

The performance of candidates on this paper was variable. On certain topics, such as centres of mass and collisions, solutions were of a high standard, demonstrating candidates' familiarity with the subject matter and ability to apply their knowledge to good effect. In the parts of questions that seemed less familiar, candidates fared less well. In many cases, a good clear diagram would have been an invaluable aid to potentially making meaningful progress, and would certainly have helped to eliminate sign errors.

There was evidence that candidates were finding the paper rather long and some of the more straightforward parts of Question 4 were omitted by a significant number of candidates.

Comments on Individual Questions

1) **Momentum and Impulse**

Part (a) was approached with confidence, and the majority of candidates showed that they knew the principles involved. Those who drew a clear diagram in part (iii) usually scored full marks, whereas those who did not make a clear statement, in a diagram or in words, frequently made sign and arithmetical errors. Part (b) was tackled with much less confidence. There were a pleasing number of excellent solutions, but about half of the candidates demonstrated a wide range of misconceptions about impulse when a smooth inclined plane is involved.

(a)(i) The vast majority of candidates used the principle of conservation of linear momentum together with the formula 'Impulse = Force x Time' and earned full marks. A minority of candidates opted to use Newton's second law and *suvat* as an alternative approach, and again were successful.

(a)(ii) Most candidates earned both marks and it was pleasing that they showed sufficient working to support the given answer.

(a)(iii) Almost all candidates knew that they were required to apply the principle of conservation of linear momentum and Newton's experimental law to the collision. However, many attempted to proceed without a diagram or a clear statement about which was the positive direction, and many sign errors appeared, particularly in the equation resulting from Newton's experimental law. Of those candidates who did have two correct linear simultaneous equations, many made arithmetic errors in solving them.

(b)(i) There were some excellent, concise solutions to this part, from about half of the candidates. The other half of the candidates seemed to have little idea about how to make any creditable progress. There were several common errors, with any individual candidate making one, some or all of them. Some did not take into account the motion of the particle before it collided with the plane and used the initial speed as the speed of contact. Some attempted to consider horizontal and vertical motion at the collision, rather than motion parallel and perpendicular to the plane. Some seemed confused about the direction in which momentum was conserved. Some brought the coefficient of restitution into the motion parallel to the plane. Some did not appreciate the vector nature of momentum and impulse and worked with 15 and 13 instead of the components of the velocities, with $e = \frac{13}{15}$ being a common incorrect answer.

(b)(ii) Again, candidates did not recognise the vector nature of the problem and it was rare to see a correct solution. The vast majority gave the impulse as $0.2(15 - 13)$, rather than considering the change in momentum perpendicular to the plane.

2) **Work, energy and power**

On the whole, candidates demonstrated that they have a grasp of the principles of work and energy, and most secured more than half of the available marks. The most common errors in each part were to omit one of the necessary terms and to work in an unstructured way that led to sign errors.

(i) The majority of candidates scored full marks, although a significant minority seemed not to realise that there were two separate parts to the request.

(ii) Again, many fully correct solutions. The most common error was to omit the weight and find the power required to overcome the resistance only.

(iii) Most candidates demonstrated that they understood how to set up a work-energy equation and many scored full marks. Common errors were to omit the resistance, even though it had been included correctly in part (ii) of the question, and sign errors.

(iv) The majority of candidates knew that the energy equation needed to involve kinetic energy terms and work done terms, but only a minority had a systematic approach that led them to the correct answer. Often, the terms were evaluated separately, and then put together into an equation in what appeared a random fashion, with sign errors prevalent. The work done against the resistance was omitted by many candidates.

3) **Forces and equilibrium**

The majority of candidates did not perform well on this question and did not appreciate the help that was available; for example, they did not realise that the given answer in part (i) was significant for a solution to part (ii). It was surprising, and disappointing, that so many candidates attempted to solve part (iii) without drawing a diagram. A clear diagram with all the relevant forces labelled is key to solving problems on equilibrium.

(i) Those candidates who split the 60N weight into two components were usually successful in achieving the given answer. Those who tried to find the perpendicular distance from A to the weight force often made trigonometrical mistakes and had to make dubious adjustments to reach the given answer. Candidates should be reminded that examiners check through all the working for consistency and, in the case of a given answer, need to be totally convinced by the candidate's working.

(ii) There were some good solutions to this part, but many candidates either did not see the relevance of part (i) or, having failed to complete part (i) successfully, did not even attempt part (ii).

(iii) This was the least well-attempted part of any question on this paper, with less than 10% of candidates scoring any marks at all. There were two common invalid assumptions: firstly, the assumption that the reaction at B was vertical and secondly, the assumption that there was only a vertical reaction force at A and no horizontal component of the force. It cannot be stressed enough that candidates need to draw a diagram when attempting equilibrium questions. A significant number of candidates proceeded without a diagram and most of these scored zero marks.

(iv) Most candidates were back on familiar territory here and knew the method of approach. The most common error was to use the weight as the mass in the equation for Newton's second law. Another common error was to ignore the component of the 200 N force when considering equilibrium perpendicular to the slope. Other candidates confused themselves by using the notation F to mean both the force in $F = ma$ and the frictional force.

4) **Centre of mass and light framework**

Candidates performed strongly in part (a) of this question, with some good well-presented solutions. Performance in part (b) was quite patchy, with evidence that some candidates were short of time and writing down what they could in the hope of securing some marks. Again, in part (iii) of part (b), candidates seemed reluctant to label a force diagram with all the relevant forces.

(a)(i) The majority of candidates made a good attempt at this part, with clearly presented solutions, identifying the masses and centres of mass of the individual rods before taking moments. Errors were usually due to an incorrect calculation of distances parallel to the x-axis or an omission of minus signs in some of the *y* distances.

(a)(ii) Most candidates had the idea that they needed to take moments, but there were many errors, either in identifying distances or in the omission of *g* from one side of the moments equation.

(b)(i) Only about one-quarter of the candidates were able to offer an acceptable explanation as to why the internal forces in OR and RQ must be zero. The most common attempts suggested simply that because the system was in equilibrium, the internal forces had to be zero. Those who resolved horizontally and vertically at R and wrote down equations were almost always successful.

(b)(ii) Candidates who took moments about O usually earned full marks, while those who did not, rarely made any progress.

(b)(iii) Few diagrams were completely labelled, showing both the external and internal forces.

(b)(iv) Many candidates did not attempt this part. Some appeared to have run out of time, others seemed to have given up because they could not do the earlier parts of the question. However, those who did attempt it usually did so with some success. Most were able to write down two relevant equilibrium equations, usually at P. Any errors were due to sign errors and/or a sine/cosine confusion.

4763 Mechanics 3

General Comments

The work on this question paper was generally of a very high standard, with most candidates demonstrating a sound understanding of the topics being examined. The questions on dimensional analysis and centre of mass of a solid of revolution were particularly well answered; but the questions on circular motion and simple harmonic motion did present difficulties for a significant number of candidates. Almost all candidates appeared to have sufficient time in which to complete the question paper.

Comments on Individual Questions

1) In part (a), many candidates were unable to solve this problem about a conical pendulum. The difficulty lay in the horizontal equation of motion, where the length of the string (3.2 m) was often taken to be the radius of the circle. Some candidates confused angular speed with speed. Almost all candidates resolved vertically, but those who had not found the tension were then also unable to calculate the angle.

   Part (b), on dimensional analysis, was answered extremely well, with candidates applying the techniques accurately and confidently. About half the candidates used the given equation for *u* to establish the dimensions of *k*, rather than the much simpler (force)/(length), so they did essentially the same work in parts (b) (i) and (b) (ii). Hardly any mistakes were made; there were just a few sign errors, in the calculation of the powers in part (b) (iii), and especially in the energy equation in part (b) (iv).

2) In part (i), the majority of candidates used conservation of energy to find an expression for v^2, and then used the radial equation of motion to obtain the given equation for T. The potential energy term was sometimes incorrect, and the weight was often omitted or wrongly resolved in the radial equation. A few attempts did not involve any consideration of energy, and some candidates tried to resolve vertically.

   In part (ii), the maximum and minimum tensions were usually found correctly, although some did use $\theta = \pi/2$ to find one of the extremes. It was fairly common for proof of motion in a complete circle to be based on positive kinetic energy at the highest point rather than positive tension.

   Again, in part (iii), many candidates assumed that the velocity would be zero at the highest point.

   In part (iv), most candidates realised that the string becomes slack when $T = 0$, and were able to use their previous results to find the speed of P at this point.

3) This was found to be the most difficult question, but even so, about one quarter of the candidates scored full marks on it.

   In part (i), almost every candidate used Hooke's law to find the natural length correctly.

   In part (ii), many candidates stated that the tension in AP is $mg + (\lambda/l)x$, which clearly yields the given result. Some did explain this satisfactorily, as (tension in equilibrium position) plus (stiffness) times (*extra* extension), but as the answer is given it is much more convincing to say (stiffness) times (total extension $0.45 + x$). The thrust in BP was usually given correctly, although some candidates added an *mg* term to this.

In part (iii), most candidates realised that they were expected to set up an equation of motion using the results from part (ii). The terms and signs usually appeared correctly, possibly helped by the displayed result.

In part (iv), the great majority of candidates recognised that this was simple harmonic motion, and gave the period correctly. The amplitude caused some difficulty, with many candidates appearing not to realise that 3.4 ms^{-1} is the maximum speed in the motion. Some candidates became confused between ω and the period, in this and the subsequent parts.

In part (v), those candidates who used $v = 3.4 \cos 5t$ were usually successful, although some then assumed that the positive direction was upwards. Those who found the displacement and then used $v^2 = \omega^2(A^2 - x^2)$ were very rarely able to determine the direction.

In part (vi), most candidates found the displacement when $t = 2.4$, but using this to obtain the actual distance travelled presented a considerable challenge.

4) In part (a) (i) the techniques for finding the centre of mass of a solid of revolution were very well understood, and usually applied accurately.

However, part (a) (ii) was often omitted or poorly attempted, even though it is a simple application of moments using the centre of mass found in the previous part.

In part (b), most candidates treated this as a lamina between a curve and the *y*-axis, using appropriate formulae and integrating with respect to *y*, and this was very often carried out correctly. A common error was expanding $(2 + y^{1/3})^2$ as $4 + 2y^{1/3} + y^{2/3}$, and the factor ½ was sometimes missing from the *x*-coordinate. Many candidates integrated with respect to *x* instead; some of these did not appear to realise that they had found the centre of mass of the wrong lamina, but others went on to apply the composite body formulae correctly.

4764 Mechanics 4

General Comments

The standard of performance was very high. As usual, candidates showed their skills in solving differential equations and manipulating complicated expressions, and most of them demonstrated a solid knowledge of the techniques and concepts required.

Comments on Individual Questions

1) ***Variable mass***

(i) Most candidates found the given answer correctly, though many did far more work than required. Examiners do not require the *δt* derivation of the equation of motion in variable mass questions of this type where in the direction of motion of the truck there is no external force and *mv* is the total linear momentum of the system. In such a case we would accept $\frac{d(mv)}{dt}=0$ without justification. The best candidates having started by deriving or stating $\frac{d(mv)}{dt}=0$ or equivalent then went on to say $mv = m_0v_0$ or equivalent. Some candidates passed through a stage with $m\frac{dv}{dt}+v\frac{dm}{dt}=0$, or equivalent, without recognising that this gives $\frac{d(mv)}{dt}=0$ and went on unnecessarily to solve a differential equation involving *v* and *t*.

Many candidates did not give enough justification for the expression for mass in terms of *t*, or enough steps in their approach to the final given answer.

(ii) A large majority of candidates misinterpreted this request as asking for the two values at the moment when $m = 2m_0$ rather than $3m_0$. A special case was added to the mark scheme to allow 1 out of 2, but candidates are reminded of the need to read stems carefully to avoid this sort of error.

2) ***Equilibrium***

(i) This was done well by most candidates. It is difficult to give a good justification for this given answer purely symbolically. The best solutions included a diagram to show where the various factors of ½ came from.

(ii) The general approach was well understood by most candidates. The derivation of an expression for BD was done very well, with most using the hint from part (i) effectively. Many candidates chose to expand their expressions for the GPE terms before differentiating rather than use the chain rule; this led, in some cases, to copying and sign errors that might otherwise have been avoided.

(iii) Most candidates found the values of *θ* at the potential points of stability from the graph and correctly determined whether or not they were stable. However, many could not provide justification, or justified their choice by evaluating the second differential of *V* at each value rather than using the graph as directed. For a solution to be awarded full marks it had to include a clear use of information from the graph, an explicit relationship between f(*θ*) and *V* and a brief mention of the conditions for stability.

3) ***Variable force***

Parts (i) (ii) and (iii) were done very well by the majority of candidates, with careful and precise work.

(iv) Most candidates integrated a correct Newton's second law equation to obtain an equation in *v* and *t*. However, many then used incorrect conditions, usually *t* = 0 rather than ln 3 when *v* = 0.8. Some did define a new variable for the time at constant power, for example *T* = *t* – ln 3, and this was awarded full marks when done carefully.
Many candidates did not derive an equation for *v* in terms of *t* as requested, instead finding the limiting value of *v* by consideration of their implicit equation.
This last step of finding the limiting value was performed very well by the majority.

4) ***Rotation***

(i) This proof was done very well by most candidates. Some chose to take the mass per unit area to be 1, but only those that did so explicitly were awarded full marks.
A few candidates used the standard bookwork to derive the moment of inertia of a disc and then argued from symmetry to get the given answer. This method was only awarded full marks when carefully justified.

(ii) Most candidates used the formula in MF1 correctly. Values of $\frac{a}{\pi}$ and $\frac{2a}{3}$ were followed through for all but given answers.

(iii) This was well answered by the majority of candidates. To obtain full marks needed careful manipulation to the given answer without leaving out too many steps towards the end.

(iv) This was generally well done with only a few candidates stopping once they had a value for $\dot{\theta}$.

(v) Two approaches were seen each in approximately half the scripts.

1. Differentiation of the expression in (iii). This was often done well, but many candidates left out the $\dot{\theta}$ term on the RHS.
2. From $C = I\ddot{\theta}$. Many candidates had the sign of the couple as positive rather than negative, but otherwise this was done well.

(vi) Most candidates who attempted this could set up an equation of the form $Jx = I\Delta\omega$, but many errors were made, usually in evaluating *ω* or taking both values of *ω* as having the same sign.

(vii) Very few correct solutions were seen to this question. Many candidates who attempted it did not include both GPE terms and therefore scored zero. Those who included all three terms often had an error, for example the wrong value of *ω*, sign errors in the GPE terms or manipulation errors in the solution of their equation,

4766 Statistics 1

General Comments

On the whole candidates coped well with this paper. A good number of candidates scored 60 marks or more out of 72. A considerable number of candidates scored the majority of their marks on topics which overlap with Higher Tier GCSE; however, Question 3 on the binomial distribution was well answered. Most candidates supported their numerical answers with appropriate working. However, when written explanations were required, the poor handwriting and use of English of some candidates made it difficult to determine what they were trying to say.

There was no evidence of candidates being unable to complete the paper in the allocated time. As last year only a small minority of candidates attempted parts of questions in answer sections intended for a different question/part and most candidates had adequate space in the answer booklet without having to use additional sheets. Those candidates who overwrote pencil working in ink, even if they made an attempt to rub out the pencil, made the work very difficult to read. Candidates should be advised to refrain from doing this.

Unfortunately, as in recent series, most candidates lost marks due to over specification of some of their answers, despite recent examiners' reports warning against this. The worst cases of this were in both parts of Question 1 and in Question 4(ii), where the vast majority of candidates gave the variance to 8 significant figures. It is possible that they thought that as it was a sum of money it should be exact, but of course the units of the variance would be pounds^2.

Comments on Individual Questions

1)(i) The vast majority of candidates answered this part correctly, though many lost marks for over-specification of the standard deviation (often given as 14.475). A small minority managed to over-specify the mean, giving it as 249.40. Only a few candidates found the root mean standard deviation instead of the standard deviation.

1)(ii) The mean was usually tackled correctly, but then the mark sometimes lost was for over-specification. Calculating the standard deviation seemed to cause more problems, with attempts made to 'start again' or comments such as 'it remains the same'. Candidates were not penalised a second time if they over-specified again – many in fact gave 6 or 7 significant figures in their (correct) answer.

2)(i) Candidates using the ${}^nC_r$ method tended to be more successful, as when using the product of 3 fractions method many did not realise that they needed to multiply the final product by 3. A small minority of candidates did not follow instructions and either left a fraction in unsimplified form (usually 15/36) or gave the answer as a decimal.

2)(ii) Most candidates made a reasonable start in this part, using their answer from part (i). However, many only calculated one probability, or missed the coefficient of 4 when calculating the probability of 3 evenings, not realising this was a binomial situation. Some candidates calculated the probability of 3, rather than *at least* 3, and thus only gained 1 mark. A small minority of candidates used statistical functions on graphical calculators to just write down an answer – this was a risky strategy, as a slip in copying the answer was heavily penalised, since no method was shown.

3)(i) This question was very well answered, with most candidates scoring all 3 marks. However, a few candidates seemed to have no idea about the binomial distribution.

3)(ii) Again another well answered question, although occasionally candidates did not read the question carefully and continued to use $n = 50$ in their calculation.

3)(iii) Full marks were available here for a correct follow through from part (ii), so many candidates managed to recover from an incorrect answer. However a large proportion of candidates rounded their answer to the nearest whole number, thus losing a mark. Others over-specified their final answer, again losing a mark. Other common errors were to use $p = 0.1$, rather than their answer to part (ii), or to use $n = 48 \times 20$.

4)(i) This was well answered by the majority of candidates with most of them using the product of 3 fractions method. A few successfully used $1/({}^{20}C_3)$. There were a few candidates who used the probabilities in the table to give 1-(0.45+0.45+0.05), for which of course no credit was available.

4)(ii) This was very well answered, with nearly all candidates picking up 4 marks out of 5. Very few candidates gained the final mark, due to over-specification of the variance, usually giving an answer of 445511.25. A minority of candidates made the usual errors in this type of question such as: squaring the probabilities when finding $E(X^2)$, subtracting $E(X)$ rather than $[E(X)]^2$ or introducing spurious multipliers or dividers. Candidates should be advised to check carefully the figures which they enter into their calculator, as although the written down calculation was usually correct, sometimes the answer written was not.

5) The wording of the researcher's theory appeared to cause confusion for some of the candidates throughout the question. This was translated into some poorly worded explanations and conclusions in all three parts of the question. Good comprehension skills are required in this type of question and, unfortunately, these skills were not always in evidence.

5)(i) Many candidates scored both marks. Unfortunately a good proportion lost either the first or the second mark by not mentioning 'guess' or only including it when they quoted the question or not mentioning, in any form, the idea of the two possible outcomes. Some candidates simply just re-stated the null hypothesis in words.

5)(ii) This was not as well answered as part (i). There was a failure to distinguish between guessing and being able to identify between the two types of water. A lot of candidates lost the mark because they gave the reason for the alternative hypothesis as '13 people out of 20 in the researcher's sample identified correctly' which of course is not a valid reason.

5)(iii) The most successful way of approaching this hypothesis test was to compare $P(X \geq 13)$ with the significance level. Several of the candidates, who used this method failed to gain the final mark due to not putting the explanation in the context of the question. Other candidates used incorrect probabilities, usually $P(X \geq 12)$ or $P(X \geq 14)$.
Candidates who used the critical region method normally gained the first two marks but then many of them failed to gain any more marks – usually because they had included 14 in the critical region. Unfortunately some candidates started looking at the two probabilities necessary for the critical region but made no mention of the critical region, or critical value, so did not gain any marks.
It is pleasing to report, on the other hand, that very few candidates tried to use point probabilities. However, although full marks could be obtained by comparing 0.8684 with 95%, many candidates either compared with 5% or made no explicit comparison at all - such candidates were unable to gain any credit.

6)(i) Most candidates successfully found the median, although instead of the 13th value some found average of the 12^{th} and 13^{th} values. However, candidates were less successful in finding the interquartile range. The lower quartile was usually found correctly, but the upper quartile was more frequently wrong, with an answer of 3.665 being the most common error. Occasionally candidates did not subtract to find the interquartile range, but instead some found the midpoint of their quartiles.

6)(ii) The response to this question was very disappointing. Perhaps because they were faced with a blank space rather than graph paper, most candidates thought that accuracy was not required. Very few had a scale and some of those that did failed to make it linear. Some candidates simply sketched a box and whisker plot and then labelled the diagram with the relevant values. This did not gain marks as the question clearly instructs candidates to 'Draw a box and whisker plot…'. It seems likely that many candidates either did not have, or did not think to use a ruler. Far too many freehand diagrams were seen, with the sizes of the box and whiskers and the position of the median not in proportion.

6)(iii) Many candidates correctly found the upper and lower limits for the outliers. The most common misconception was that outliers were calculated using median $\pm$ 1.5×IQR, although many other errors were also seen. A few candidates attempted to use the mean and standard deviation, and if they got both of these correct, full marks were available, but unfortunately one or other of the two statistics was usually incorrect.
It was necessary to check both limits to show that there was only one outlier, but some candidates ignored the upper limit. Many candidates failed to give an explanation in context regarding the outlier, though those that did often made a valid point about premature babies.

6)(iv) As in part (i), the median was usually found correctly, but some candidates lost a mark due to inaccurate reading of the scales in finding the quartiles.

6)(v) Only about one third of candidates scored both marks. Credit was given to those candidates who could only compare medians and interquartile ranges without an explanation of what they meant. Candidates who just said 'boys are heavier' failed to get credit without a comment such as 'generally' or 'on average' or 'tend to be'. Similarly 'more consistent' or 'vary less' or 'less spread' gained credit for interquartile range – 'smaller range' was not awarded credit.

6)(vi) This part discriminated very well between the higher-scoring candidates. Many candidates realised that approximately 10 male babies weighed more than 4.34 kg. Unfortunately many then did not know how to proceed, often squaring 0.05 (10/200) rather than multiplying by 9/199. Those candidates who misread the scale but knew how to proceed could gain a Special Case mark. A significant number of candidates missed out this part altogether.

7)(i) The majority of tree diagrams were well constructed with correct labelling. Weaker candidates sometimes became confused and made errors in the 2nd and/or 3rd branch.

7)(ii)*A* Many candidates employed the 1 – P(misses with all) method, usually successfully, but a significant number used the protracted method of listing all 7 triplets associated with at least one hit. Usually errors were made using such an approach.

7)(ii)*B* Most candidates found the correct three products and calculated them correctly. A small number failed to find all three. For those who got the tree diagram wrong, follow through marks were available.

7)(iii) Many of those who reached this part were successful. However, there was considerable confusion in finding the conditional probability, often with a correct denominator but a wrong numerator of P(at least one)×P(exactly one). Some candidates inverted the fraction.

7)(iv) Approximately one third of candidates were successful in this part. However many were confused. Many candidates successfully found the first product but then failed to find the second, or found additional products. Those who attempted the second product often made errors. The last three probabilities were often $0.1\times0.2\times0.2$ rather than $0.05\times0.2\times0.2$.

4767 Statistics 2

General Comments

Candidates appeared to have enough time to complete the question paper and seemed comfortable with the level of difficulty. The majority attempted all parts of all questions. A very small number made little or no attempt at any of the questions. In general, solutions were well-presented and easy to follow. Candidates continue to improve their techniques in tackling hypothesis tests and the use of non-assertive conclusions is becoming quite commonplace. Over-specified answers are also becoming less prevalent.

Comments on Individual Questions

1(i) Most candidates coped very well with this question. A few candidates did not calculate intermediate values and ran the risk of losing marks if they made an error in calculating the final value. Some candidates used a rounded value for the mean of y throughout, showing little awareness of rounding error.

1(ii) This part was well-answered by many, with the customary form of the hypotheses using the appropriate symbol, ρ, being seen. Having successfully looked up the appropriate critical value, many went on to demonstrate the comparison with the sample value before rejecting the null hypothesis and concluding in an appropriate, *non-assertive* manner.

1(iii) Many candidates recognised the requirement for the underlying bivariate Normal distribution and knew what to look for in the shape of the scatter diagram. The main confusion concerned the words "bivariate Normal", with "normal bivariate" regularly seen along with a host of poor spellings.

1(iv) Many candidates found this difficult. In trying to explain that the significance level was the probability of rejecting the null hypothesis when it is true, many ended-up writing meaningless phrases.

1(v) Most coped well with this part of the question. Even the few who were confused as to which point needed changing gained credit for realising there would be no change in Σxy.

2(i) Generally well done. The most common error was to use a Poisson distribution either directly or from tables. Another common error involved calculating the probability of 'one person' having red hair, rather than 'at least one'.

2(ii) Most responses seen were entirely correct, with occasional omissions of one or more of the three requirements. Some candidates reeled out general reasons why a Poisson distribution might apply rather than explaining why the binomial distribution could be approximated using a Poisson distribution. Ambiguous phrases such as "the probability is small" were not given credit.

2(iii)*A* Very few incorrect answers seen.

2(iii)*B* Again, very few incorrect answers seen. Some candidates calculated $P(X \geq 2)$ rather than $P(X > 2)$.

2(iv) The least well answered part of the question. Many candidates chose to discuss, individually, the size of n and p rather than the size of np, failing to note that np was too small to use a normal approximation.

2(v) Whilst this was mostly done correctly, many candidates mistook this either for a Poisson distribution with mean 150, leading them to lose marks in part 2(vi) with the wrong variance, or for a N(150, 145.5) distribution.

2(vi) A variety of errors cropped up amongst the many completely correct answers to this part. These included using the wrong variance, not using a continuity correction, using a wrong continuity correction and using the wrong tail of the distribution (i.e. giving $\Phi(0.7876)$ as the answer).

3(i) Most candidates obtained a correct answer. A small but significant number did not use one or both of the continuity corrections. Most used the difference column of the Standard Normal table correctly to provide suitably accurate answers. A relatively small number struggled with the structure of the calculation.

3(ii) Many candidates answered correctly, but a common mistake was to omit or to provide an incorrect continuity correction. A relatively small number did not use the difference column correctly.

3(iii) Most candidates knew and applied the method correctly but many were dependent on the FT to gain the 2 marks. A small number omitted the x3 from their binomial calculation.

3(iv) 1.282 was identified by the majority of candidates who went on to set up a correct equation and arrive at 91.38 or 91.88. Many of these gave 92% as the final answer but many others gave 91%. Others rearranged incorrectly and arrived at 60.6 for the first calculation. Few candidates demonstrated a proper understanding of the requirement of this question.

3(v) In this part, the continuity correction was omitted, or an incorrect value was used, by many candidates. Many used +0.8416 leading to 60.0992 which was a common answer. The issue of over-specification was most apparent in this part of the question.

4(i) Most candidates were able to give two correct hypotheses. Mistakes were rare, but included reversed hypotheses, hypotheses with no context and hypotheses which included correlation. Most candidates knew how to calculate the expected values, but a significant number did so with insufficient accuracy, either by rounding inaccurately or simply by getting one or two of the calculations wrong. The same was also true of the table of contributions, although there were a few more candidates who did not know how to calculate contributions. Almost all candidates knew to calculate χ^2, and that there were 3 degrees of freedom. Some candidates obtained the critical value for the wrong significance level or used the wrong row in the table. Some candidates thought that, as the test statistic was greater than the critical value, this was not a significant result and so accepted H_0. In the conclusion, many candidates knew what was required, but there were two reasonably common errors. The first of these was an over-assertive conclusion such as “there is some association between sex and artist preferred”. The second main error was a lack of context in the conclusion.

4(ii) This was poorly answered by most candidates. Very few seemed to understand the importance of the contributions and did not comment on them. Some candidates mentioned the contribution, but did not comment on the magnitude. For example "More females than expected preferred Monet, as indicated by a contribution of 1.408". The word 'large' was required to indicate the special feature of the contribution. A similar problem occurred with Renoir and Cezanne where the important feature of the contributions was that they were small. With small contributions the response should be that the observed frequencies are much as expected. Many candidates tried to 'sit on the fence' and state that the frequencies were much as expected but slightly more (or less). This led to a loss of marks. Poorer responses were characterised by two features. One was a lack of clarity. It was often unclear whether the candidate was saying the observed frequency was greater than the expected frequency, or vice-versa. The second feature was a comparison of male and female figures rather than of observed and expected values.

4768 Statistics 3

General Comments

As might be expected on a question paper at this level, the scripts indicated that most candidates knew what they were doing most of the time. In addition, there were very few scripts which showed evidence of candidates running out of time. In general, candidates seemed to be far more comfortable carrying out calculations than with the other requirements of the paper such as producing hypotheses and conclusions, interpreting results and providing definitions. In addition, as in January, many scripts suffered from a lack of precision. This manifested itself in many ways; inadequate hypotheses, over-assertive conclusions, over-specified final answers (yet too little accuracy carried forward in calculations), inaccurate reading of tables, and a large number of scripts which were very difficult to read.

Comments on Individual Questions

1 **Task in a factory – Wilcoxon single sample test**

1(i) Most candidates were able to score well on this part, with full marks being the most common outcome. However, a large number of candidates lost marks through a lack of precision in the presentation of the hypotheses and conclusion. The hypotheses should concern the population parameter m, which should then be defined as the *population* median time. Most candidates knew to subtract 7.4 and to rank the absolute values. Only a few ranked from largest to smallest. Most candidates were able to obtain the correct critical value from the relevant table, and only a few thought the test value to be significant. Many conclusions were either too assertive or lacked context.

1(ii) Most candidates are able to construct a confidence interval correctly. A few candidates used 1.645 instead of 1.96 and many gave anything up to 10 significant figures in their interval. . It was pleasing to see the overwhelming majority of candidates correctly opted to use s_{n-1} rather than s_n, but a surprising number were unable to use the standard deviation formula correctly. To justify the use of the Normal distribution many candidates simply stated that n was large, which was insufficient. The use of the Central Limit Theorem was required.
Ans: (6.859, 7.021)

1(iii) This part question produced one of the weakest responses from the candidates. Some candidates were able to give succinct responses about the probability of capturing the population mean and the width of the interval; others tended to give much longer responses which often included a discussion of imaginary hypotheses.

2 **Milk yields – paired *t* test**

2(i) About half of the candidates realised that the elimination of the differences between cows was the key point. Others wanted to eliminate the difference between feeds and many felt that a paired test was appropriate simply because there were two sets of data.

2(ii) The great majority of candidates were able to score at least one mark here, usually for the normality of the population. However, many did not mention that it was the normality of the differences that was important, and many that did failed to mention that it was the population of differences.

2(iii) This part was answered very well by most candidates, with full marks being the most common score. Again, the most common causes of lost marks were inadequate hypotheses and over-assertive conclusions. Most candidates were able to find the differences and then calculate the values of $\bar{x}$ and s_{n-1} correctly, with very few using s_n. The test statistic was almost invariably calculated correctly and compared with the correct value of *t*. Occasionally the degrees of freedom were incorrectly stated, with 8, 10 and 11 all seen. A few candidates also used the two-tailed 5% point rather than the single-tailed. A few candidates felt the result was not significant and some gave conclusions without context.

2(iv) Again, most candidates showed that they were able to construct a confidence interval. There were a small minority of candidates who switched to the Normal distribution and this was a costly error. A significant number of candidates gave too many significant figures in their answer. A few candidates appeared to misread the questions as the *mean increased yield*.
Ans: (4.948, 10.05)

3 **Stoppages during a football match – probability density function and goodness of fit test**

3(i) Most candidates were able to produce a reasonable sketch. The most common error by far was a lack of a stationary point at $x = 5$. Only a few sketches involved values of x outside the range $0 \le x \le 5$. Very few sketches had the wrong basic shape.

3 (ii) Virtually all candidates knew that they had to integrate $\mathrm{f}(x)$, but many produced an integral with no limits at all, or even with limits of 0 and 5. A smaller minority of candidates claimed that $\mathrm{F}(x) = \int_0^t \mathrm{f}(x)dx$. Not surprisingly, virtually all candidates were able to integrate $\mathrm{f}(x)$.

3(iii) This part was very well done with virtually all candidates stating F(5) = 1. Candidates do need to be aware that full working needs to be shown when the final answer has been given in the question.

3(iv) This part was extremely well done.
Ans: 10.848, 20.64

3(v) Most candidates knew how to carry out this test with full marks often seen. Most candidates were able to give satisfactory hypotheses, although some gave too little detail with hypotheses like “fits” and “does not fit”. Candidates were expected to merge the last two cells and most did so. A small minority also merged the first two cells. The calculation of X^2 was, in general, correctly done and compared with the correct value of χ_3^2. Candidates who had not merged cells as expected were given credit for the work that followed. A few candidates felt that a value of X^2 in excess of the critical value was not a significant result.

4 **Meat pies – linear combinations of Normal distributions**

4(i) Many candidates misinterpreted the information given in this question. This affected all parts of the question. The information given was variance $21\,\mathrm{g}^2$, but many interpreted this to mean that the variance was 21^2.
In general candidates were comfortable with this part of the question and scored well.
Ans: 0.7133

4(ii) The vast majority of candidates knew what to do here and only a few candidates read the tables inaccurately or chose the wrong tail.
Ans: 0.0882

4(iii) Most candidates were able to find the mean and variance of the total weight of 4 pies, although a few multiplied the variance of 1 pie by 16. The great majority gave the correct value of 1.645, although 1.96 was occasionally seen. A few candidates then gave a value in the wrong tail.
Ans: 592.5

4(iv) This part led to a wide range of responses from candidates. The demonstration that $0.65M - 0.35C \geq 0$ led to a large number of vague attempts at justification. Many included a mixture of equalities and inequalities. Others revolved around equating the weight of a pie to 1. Many candidates made no serious attempt.

In the calculation, most candidates were able to find the mean of the distribution, but many made errors in the variance. The most common errors were the use of 0.65 and 0.35 instead of their squares, subtracting the two variances instead of adding and using the multipliers the wrong way round.
Ans: 0.8818

4769 Statistics 4

General Comments

The majority of candidates for this paper were very well prepared. There were instances where candidates attempted more than the three questions demanded by the rubric, but on the whole this was not to the detriment of the questions that would count. As in previous years, question 4 on Design and Analysis of Experiments was the least popular, attempted by only half the candidates, with Question 3 on Inference marginally more popular than the other two.

Comments on Individual Questions

1) ***Estimation***

The first part was, on the whole, very well done with many scripts scoring full marks. Ideally candidates should show explicitly that the derivative of the log-likelihood function is set to zero to obtain the estimator, but the lack of this was not penalised. The final step of demonstrating that the Likelihood function achieved a maximum was not always fully explained.

Part (ii) was mostly well done but precision in the expressions employed is expected. The expression for $\mathrm{P}(X = 0)$ should employ the theoretical parameter θ, it is an estimator of this probability that uses $\hat{\theta}$.

In part (iii), most candidates made sensible use of the figures given, 1000 trials and sample mean 5, commenting that the observed number of minutes reported, in which no cars were seen, was well short of the expected number they found, predicted by the model. Some candidates attempted to calculate the probability of observing no minutes without cars but did not always turn the event into "$X > 0$" in each minute observed. Some numerical justification was expected. The use of the Normal approximation was of doubtful validity in this borderline situation.

In part (iv), many candidates failed to give a convincing argument deriving the probability distribution, with several not attempting this at all. Most of the successful answers used the idea of scaling the Poisson probabilities so that the sum from 1 to infinity was one, and most achieved the right scale factor. With undue haste to get to the required result, some algebraic mistakes did occur. There were some odd notations used seen. Candidates who used the idea of conditioning on $X \neq 0$ did not always express the argument coherently, but this was a quick and effective method to use. The final section of the question was mostly well done, where candidates could proceed with the quoted distribution, having earned the method points in part (i).

2) ***Generating Functions***

In part (i), three marks were almost always earned, as was the case for moment generating function in part (ii).

Part (iii) required some clear thinking and careful algebra. Not all candidates started by stating the results requested. When these were given, initially or later on, the need for the independence of the random variables in the sum was mostly ignored or forgotten.

The complicating factor of $e^{\frac{-\mu\sqrt{n}}{\sigma}}$ could have been removed by using $\mu = 0$ early on. The most successful and convincing arguments defined Z in terms of $\sum_{i=1}^{n} X_i$ rather than $\bar{X}$ which resolved the problem of where $\sqrt{n}$ appeared in the exponents. It was not always clear that candidates knew how to deal with all the steps required. Some of this was due to confused notation, between $\mathrm{M}_X(\theta)$, $\mathrm{M}_{\bar{X}}(\theta)$, and $\mathrm{M}_{\sum X}(\theta)$.

Part (iv) was mostly well done. The most common mistake was to omit explicit reference to the effect of large n on the expansion as a power series, or mentioning powers of θ instead of n^{-1} .

Part (v) was also well done, with most candidates recognising the mgf of the standard Normal distribution and quoting the uniqueness property.

3) ***Inference***

Part (i) well answered but for the few candidates who wanted to define the types of error as probabilities. The Operating Characteristic and the Power were sometimes not always described clearly as being functions of the parameter in question.
Power = 1 – Operating Characteristic is not acceptable as a definition of the power function.
Apart from some instances of strange labelling in (ii), the required extreme operating characteristic of the perfect test was well drawn.

In part (iii), most candidates were able to find the Type I error probability correctly. The Type II error probability was occasionally erroneous. Symmetrically placed bounds on the acceptance region were sometimes used; incorrect bounds and incorrect probability calculations were also seen. Most candidates said that their Type II error probability was high, not many made sensible comments on why this was so.

Part (iv) was usually well done, but for a few incomplete curves and instances of inadequate or incorrect labelling.

4) ***Design and analysis of experiments***

Part (i) produced responses which tended to lack concisely expressed relevant points. Where examples of designs were given the descriptions could be helpful, but could also be vague. Randomisation, to counter sources of bias, was on the whole more successfully described than the merits of replication. Not many candidates explicitly stated that the latter made possible the estimation of experimental error variance.

Part (ii) was mostly well answered. The definitions should be well known and likewise the emphasis on the parameters being those of the *population* under consideration.

In part (iv,) the hypotheses were carefully stated and analysis was nearly always successful. There were some instances of over-assertive conclusions and some where the context, about the mean yields from different fertilizers, forgotten.

4771 Decision Mathematics 1

General Comments

More than ever on this question paper, difficulties with the mathematics coupled with difficulties in written expression, were intertwined. Clarity of thought and clarity of expression go 'hand in glove'. Parts 3 (ii) and 4 (iii) were found by the candidates to be part particularly testing. Conversely, the responses to 6(vii) and 6(viii) were encouraging, showing that a good number of candidates were thinking well, right up to the end of the question paper. Some candidates were stretched for time, but it is difficult to disentangle that from inefficiencies of approach – see the comments below on Q2 parts (ii) and (iii).

Comments on Individual Questions

1) This question differentiated very well. Lower attaining candidates found difficulty getting to grips with the instructions. Higher attaining candidates breezed through it. There were some instances of rubbing lines out and trying again. If candidates do this then they should take very great care to ensure that their final answer is clear. It is the candidate's responsibility to communicate clearly.

2) Candidates are required to know bubble sort. Those who recognised it had an easier found the question easier to deal with, but were at risk of losing marks by not continuing to the end, as the algorithm specifies. Leaving the last line blank cost one of the three marks for the procedure, the mark for the numbers of comparisons and the mark for the numbers of swaps.

   Some candidates clearly spent an inordinately long time on parts (ii) and (iii), with only two marks and one mark available respectively. Candidates need to move on if their approach is taking longer than is commensurate with the marks allocated.

3) Part (i) of this question was answered well by candidates at all attainment levels. Everyone understood what is required in an application of Dijkstra.

   Part (ii) turned out to be very challenging. Most candidates scored zero, a few scored one, and very few completely correct solutions were seen. It had been expected that most candidates would score at least one, since the first mark was for realising that the network needed adapting to show times rather than distances. The majority of candidates did not try to modify their networks. They attempted to find the new solution – emphatically not what had been asked. Many who did attempt an adaptation turned the delay into an equivalent extra mileage, but then gave no subsequent indication that they needed to convert their shortest route into a fastest journey time, as had been requested.

   Dealing with the delay was a challenge. Adding 20 minutes to each arc through C gives a 40 minute delay on routes through C. One cannot add 20 to the "ins" or 20 to the "outs" since one does not know which will be which. 10 minutes extra on each arc incident on C is what is required.
   .
   There is an alternative - add 20 minutes on the time taken for the shortest route from part (i), and then apply Dijkstra to the time-weighted network with C and its arcs deleted. Then choose the better of the two.

   It was distressing to read, so many times, candidates who wished to add 20 minutes or 10 miles to C, thus showing a complete lack of understanding of the nature of a network.

4) Parts (i) and (ii) produced good scores for nearly all candidates, although there were more examples of activity-on-node than have been seen of late. One or two candidates had networks with arcs labelled with activities, but then with two unspecified times attached to each activity, rather than two times attached to each event. Their times earned no marks. Activity “L” was often omitted.

There was only one “burst” at which the method mark for the backward pass could be awarded, this also accessing an accuracy mark. This was at A’s “j” event / H’s “i” event. The late event time should have been 30, but 85 was often seen.

There were many instances of candidates inserting unnecessary dummies, e.g. between the end of G and the beginning of L. This does not incur any penalty, but there is then an extra event needed, with its early time and its late time. The late time should be the same as the late time for L’s “i” event, but many candidates lost the backward pass accuracy mark by subtracting the activity duration, i.e. by subtracting G’s duration in this example.

Whilst some good scores were seen, the quality seen differed greatly. The worst case scenarios had diagrams wandering over the page with connections which looked like tangled fishing lines. The best were neat, crisp and clear – the above point about clarity of thought and clarity of expression being clear to see.

The mark in part (iii) was earned by few candidates. The vast majority of answers could be paraphrased either by “Because he won’t complete within the minimum time without help” or by “Because two activities both have to be done at the same time”. Both leave the question unanswered. Arguably the best answer, that there are 160 minutes of work to be done in 100 minutes of elapsed time, was seen from only a very few candidates.

There were candidates, when answering part (iv,) who tried to schedule all non-critical activities to start at time 0. There were those who, whilst scheduling correctly, failed to show who was doing what.

5) It would be unusual if the report on the LP question did not start by stressing the importance of variable definition. The usual requirement for ’number of’ is not an arbitrary ‘hobby-horse’, but reflects the need that variables be unambiguously defined. This was well illustrated in this paper where there was the added complication of having both snowboards and **pairs** of skis. Suppose we had a report giving information about Marco's purchase of snowboards ... we are told how many, the cost and the delivered weight. So "x=snowboards" is not sufficiently specific ... it could be the number, the cost or the weight. The same report also has information about skis ... number of pairs, the cost and the delivered weight. In this case "y=pairs of skis" is unambiguous, and it was therefore allowed.

The ’10% more’ constraint posed some difficulties, though not so many as might have been expected.

The graph was often drawn well, although the gradient of the “10% more” constraint did not always match the candidate’s algebra.

Part (ii), the optimisation, was done well. Parts (iii) and (iv), the post-optimal analysis, proved to be more of a challenge. In particular, in part (iii) very many candidates produced the answer €6 from the computation (29000 –27500)/250 ... the reader might try to see why.

6) No matter how candidates twisted and turned in their definitions of their random variables, and there were many twists and turns, they all ended up being tested on the same issues – and most did very well indeed. The quality of answers was high. The denouement in parts (vii) and (viii) was pleasingly well handled by many candidates, even though it had been expected that it would be testing.

4772 Decision Mathematics 2

General Comments

Most candidates did well with some of the modelling in Q2, but less well in Q4, where the detail involved in the computational aspects seemed to mitigate against higher level thinking.

Question 1(a) caused specific and significant difficulties – see below.

Question 3 was found to be routine.

Comments on Individual Questions

1) (a) Candidates were asked to consider simulated responses to a logic question. Very few candidates were able to cut through the convolutions of the given answers. Part (iv) illustrates this well. Candidates had only to note that "route" had been used instead of "arc". There was, of course, a massive temptation to consider the ramifications of using "route", but for only one mark, which was not required.

(b) Answered well.

(c) Answered very well

2) It was gratifying to see a large proportion of candidates answering part (i) well, and to see some answering part (ii) well. It is, of course, crucial in part (ii) to identify correctly the order of decisions and chances. Those who did not succeed usually had the wrong ordering, or had omitted the consult/don't consult decision. The examiners are convinced of the value of this modelling, hoping that candidates who succeed with it now will find it useful in the future.

3) Most candidates had covered the network material well, and scored well throughout this question. If there was a weakness, it was in the final part, where few candidates considered all three pairings of odd nodes.

4) Whilst candidates generally ploughed through this question mechanistically, there was a strong developmental thread to it. The problem was degenerate - there were two vertices of the three-dimensional feasible region which were jointly optimal (as was any point on the line joining the two vertices). Candidates following through the given instructions should have first found, in part (iii), a non-integer solution. They were then led, in part (iv) to the adjacent solution, which was integer. Some candidates shortcut this process, but were credited appropriately.

In part (v), the set-up requires two new inequalities, that the number of bowls should be both less than or equal to 4 and greater than or equal to 4. Many candidates missed the first of these constraints.

A substantial minority of candidates lost marks and time in earlier parts of the question by formulating and maximising a profit function, whereas the question was quite specific in referring to income. In the final parts they and most other candidates made the error of assuming that part (vi) required the answer "4 bowls and 6 candle holders", with part (vii) demanding "4 bowls, 6 candle holders and 2 key fobs". In fact, part (vi) asked for the best integer solution, which was the latter of those two, and part (vii) was asking about using profit rather than income.

Some candidates gave answers which sensibly involved a three-month operational horizon, but this question had been clearly formulated in terms of the one-month problem.

4773 Decision Mathematics Computation

General Comments

It is disappointing that such a small number of candidates enter for this unit. The maths is realistic and powerful. Candidates develop good modelling skills.

Comments on Individual Questions

Question 1

Candidates were comfortable with most aspects of the simulation in part (i), except that few made provision to prevent Joanna's stocks of bread going negative. One candidate said that he/she was going to regard a negative stock as indicating that Joanna would be borrowing bread from other villagers. He/she would have been credited with that, had he/she modelled paying it back.

The teaching point from this is that candidates' models need to mirror reality. Negative amounts must be considered and dealt with accordingly.

In part (ii), candidates were required to define measures of system performance. Few did this well with. Many parroted back system requirements. For instance, "Number of loaves must be ≥ 0" ... marked wrong ... as against "Number of times there is a shortage of bread" ... marked correct.

This was a high level requirement, exactly the type of thinking skills targeted by this unit. Students with no suitable measures of performance were hamstrung in subsequent parts of the question.

Some students slipped up with the 0.75 in the stem before part (iv). The distribution being modelled is discrete, so there is a (big) difference between ≥0.75 and >0.75. In LP work on this paper the variables are often continuous, and then there is no such difference.

Question 2

The main teaching point from candidate performances on this question is the overriding importance of getting the recurrence relation correct.

Errors made by candidates in formulating the recurrence relations dramatically increased the level of difficulty of other parts of the question.

Question 3

Teachers are invited to consider and to emphasise to candidates the subtle but significant differences between the structures of the constraints in the model formulations in parts (i) and (iii). In part (i) the person constraints are equalities – everybody must compete exactly once. In part (ii) the corresponding constraints are inequalities – everybody is in team 1 at most once.

Students are often very poor in their use of language. Seldom are they able to say what they mean, though given the symbiosis of thought and expression, it may be that they are unable to reason adequately because they cannot express themselves adequately. Thus questions such as that in part (ii) always cause difficulties.

Question 4

The “clarity of expression” theme resurfaces in parts (iii) and (v) of this question. Part (iii) is difficult. The published answer shows the way to proceed ... by considering the costs of the two alternatives, not just the costs of parts of the alternatives. This could be an important lesson. Many errors are committed by failing to consider the whole picture.

Candidates need to be very explicit when answering questions in order to avoid confusion. It is not the role of the examiner to interpret what the candidate may have meant to say.

4776 Numerical Methods (Written Examination)

General Comments

The purely computational parts of this question paper were found straightforward by most candidates, though closer attention needs to be paid to the accuracy with which answers are given. The interpretation of results was less good this session than in recent years.

The standard of presentation of work, and in particular the systematic setting out of numerical algorithms, continues to be better than it was a few years ago. However some candidates are still resort to scattering calculations haphazardly on the page, making it difficult for examiners to detect and reward any correct work.

Comments on Individual Questions

1) **Solution of an equation, fixed point iteration**
This proved to be a very straightforward question for the majority of candidates. In part (i), some of the graphs were very inaccurate, and it was common to have no indication of scale on one or both of the axes. In part (ii), almost everyone made the obvious rearrangement using the square root.

2) **Absolute and relative errors**
This question, too, was straightforward for most candidates. The calculations in part (i) were generally done well and the comments were accurate. In part (ii), a surprising number of candidates did not see (or did not use) the information given that k is an integer. Indeed, many of those who did give k as an integer appeared to be doing so as a way of avoiding being too precise rather than because of the given information.

3) **Newton's forward difference interpolation formula**
The difference table in part (i) was generally done well, though inevitably some candidates made sign errors. A few omitted to comment on the near equality of the second differences. Part (ii) was a little more testing, but a majority gained full marks.

4) **Solution of an equation, false position method**
This question proved rather challenging for many candidates. The sketches of the graph in part (i) were often of poor quality and unconvincing: an increasing function of vague shape was as much as many candidates could come up with. However, showing that the root lies in the given interval was easy. The method of false position, required in part (ii), was sometimes confused with the secant method. It was good to see more candidates laying out their work in tabular form.

5) **Numerical differentiation, central difference method**
Most candidates knew which formula to use but did not always do so on a sensible set of three values of h. Given the need to estimate the likely accuracy in part (ii), the best values to use are 0.2, 0.1, 0.05 so that the rate of convergence can be judged as h is halved. In part (ii), credit was given for either 0.36 or 0.361 with appropriate justification.

6) **Numerical integration**

Part (i) was almost invariably done well. In part (ii), however, errors did build up. Candidates were told to give answers to 6 decimal places, but this instruction was frequently ignored. Consideration of the Simpson's rule values in part (ii) shows that the integral is known accurately to 6 decimal places. Part (iii) required a clear understanding of the difference between the order of a method and its accuracy. The mid-point and trapezium rules are both second order (as indicated by a ratio of differences approximating to 0.25) but the mid-point rule is more accurate (as indicated by the differences being smaller in magnitude). Many candidates carried out the calculations correctly but were unable to give full and clear comments on what the calculations show.

7) **Errors and approximations; computer representation of numbers**

This was the most challenging question on the paper – perhaps surprisingly so, as questions exploring similar ground have been set before. In part (i), candidates were required to deduce that in S_{100} there must be more rounding up than rounding down, with the opposite happening in S_{200}. The explanations offered frequently missed the point. In part (ii), very few candidates could calculate correctly the likely effect of chopping. In part (iii), many were able to come up with the left hand side of the equation by using the mid-point rule as directed, but very few realised that the right hand side simply came from doing the integration. In part (iv), most candidates picked up on the given formula and carried out the calculations correctly. Applying what they had done to part (v) was straightforward, though many candidates gave the final answer to an unjustified number of significant figures.

4798 Further Pure Mathematics with Technology

General Comments

There were some excellent scripts showing a strong understanding of the content of the unit.

There were parts, or occasionally whole questions, left blank on some scripts. Candidates should always present any information they have from using the technology, such as programs, results from using CAS or graphing, as they might be able to gain partial credit for this.

Candidates' explanations were difficult to follow in some cases. This unit is designed so that candidates are able to explain the results of using software and they are advised to practise this. This unit contains a greater requirement to write explanations than other units in the scheme.

Comments on Individual Questions

1(i) This part was generally well done. Sketches of graphs should always include some evidence of scale, such as important points on the axes.

1(ii) Appropriate use of CAS to solve the equations was seen on almost all papers.

1(iii) Clearly evaluating the limit of the derivative from both directions and also giving evidence the curve is defined at the point was necessary to obtain full marks.

1(iv) Many candidates obtained the correct value of k but struggled with obtaining a Cartesian equation of the curve. An explicit equation in the form y=f(x) was seen in a number of cases; however, an equation that is clearly multi-valued for both x and y was required for full marks.

2(i) This part was generally well done but there were some basic errors made on what was meant to be a straightforward first part to this question.

2(ii) (iii) Using a spreadsheet to demonstrate how a function behaves as it approaches a limit is explicitly stated in the specification but a number of candidates were not able to present a convincing explanation of how they would do this. The last part of these questions could have been interpreted as "the largest number that has 1 significant figure" or "the largest number, rounded to 1 significant figure": either of these was acceptable.

2(iv) Where this was attempted, there were some clear explanations though some candidates were not able to give a convincing argument about why sin and cos cannot both be 0.

3)(i) Where this was attempted, the programs given were generally correct, with the most common error being loops that went from 0 or1 to 17, instead of 16.

This question was left blank on a number of scripts. Candidates should be encouraged to write down their attempts at programs even if they don't think they are obtaining the correct numbers.

3(ii) Where this was attempted, it was generally well done.

3(iii) (iv) There is an expectation that candidates are familiar with modular arithmetic and able to manipulate congruences. Some candidates did not link the result in part (iv) to the result in i) which made this part more difficult.

Coursework

Administration

Most centres adhered very well to the deadline set by OCR and, if the first despatch was only the MS1, they responded rapidly to the sample request. A small minority of centres, however, cause problems with the moderation process by being late with the coursework despatch. All centres should heed the deadlines published by OCR and organise their own processes of assessment, internal moderation and administration to enable these deadlines to be met.

Centres should also check that the correct postage is added to letters and parcels in order to prevent difficulties with regard to delivery. Centres are also reminded that they need to include a copy of form CCS160 and send the top copy of form MS1 directly to OCR.

The marks of most centres were appropriate and acknowledgement is made of the amount of work that this involves to mark and internally moderate. The unit-specific comments are offered for the sake of centres that have had their marks adjusted for some reason. This relates to rather fewer centres than has previously been the case and this is encouraging. Furthermore, the number of samples where the moderator was unable to agree with the order of merit submitted by the centre was very small this session.

Moderators have commented before that some assessors give only domain marks. This might be acceptable if the candidate deserves full marks or zero for a domain, but it makes it very difficult for l moderators to understand the marking if the domain mark is neither of these – in this case moderators do not know which of the criteria have, in the opinion of the assessor, not been adequately met.

Teachers should note that all the comments offered have been made before. These reports should provide a valuable aid to the marking process and we would urge all centres to ensure that these reports are read by all those involved in the assessment of coursework.

Core, C3 – 4753/02

The marking scheme for this component is very prescriptive. However, there are a number of centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance.

Domain 1

Most quoted the midpoint of the interval with correct error bounds. Occasionally, the root is quoted to an accuracy not justified by the working in the tables or error bounds given which did not match the working. Some centres are still accepting general graphs as an illustration of the method.

Domain 2

The roots should be given to 5 significant figures, or better, and sometimes the accuracy is not justified. In some cases there was an over-reliance on Autograph; the purpose of this task is to demonstrate the ability to solve equations numerically, not to demonstrate the ability to use a particular piece of software. In this domain, in particular, in order to work the method the gradient function needs to be found; where it is not evident then a penalty should be applied.

There are two other problems associated with this comment, particularly evident in this domain. An illustration of the method working requires candidates to show how the use of tangents results in convergence. In many cases those tangents were not evident because the scale of the graph was not appropriate and the point at which the tangent touches the curve is not on the graph. Secondly, the software requires iterates to be within the scale of the graph and if it is not then the process stops with an 'overflow'. This does not mean that the method has failed but simply that the scales on the graph are not appropriate. A further common misunderstanding is that if x_2 is further away from the root than x_1 then divergence is happening; this is not necessarily true. Statements such as these should be penalised.

Domain 3

It is the discussion of convergence with reference to $g'(x)$ where the greatest problems arise. It is acceptable to differentiate the function, substitute a value close to the root (this is not usually the initial value) and to check that it satisfied the criterion for convergence. It was not the intention, however, to demand differentiation here but rather that comments be made based on geometric considerations at the point of intersection of $y = g(x)$ and $y = x$. Many of the comments made in this latter process were insufficient. In the former process there are candidates who differentiate incorrectly without penalty.

Domain 4

The overmarking of incorrect or insufficient work in this domain still occurs. Many candidates make general comments about the use of Excel or calculators without actually stating what they have themselves used and how they found their use.

Centres are also reminded that in order to make a meaningful comparison of the Newton-Raphson and rearrangement methods the same root of the same equation should be found to the same level of accuracy starting from the same initial value.

Domain 5

The major error in the written communication is the failure to write equations. Persistently calling an expression or a function an equation should be penalised. There were numerous scripts where the assessor had written on the cover sheet "all fine" and then on the first page the candidate states "I am going to solve the equation $y = x^3 - x^2 + 5$".

Differential Equations – 4758/02

Although there were various problems, which were commented upon in individual reports to centres, the following general points are worth emphasising.

The marking in the first four domains applies only to the initial model. It may well be that a variation of parameters, for instance, has been considered for the revised model but this cannot be used to meet any of the criteria in these first four domains. It is worth noting, once again, that a requirement of these four domains is that a model is set up and solved for the whole motion. It is not sufficient to deal only with the first 9 seconds in the Aeroplane Landing task. The differential equations set up should be for the whole motion, both of them solved to give a set of predictions and then for them to be compared to the given data. Unless both phases are dealt with there is no complete set of predictions to compare.

When comparing the predicted and the collected or experimental data both tables and graphs are expected whenever this is possible.

There seemed to be an increase in curve fitting, particularly when revising the initial model. For example in 'Cascades' , assuming that the flow is proportional to x^n and then finding the value of *n* which produced the best fit, is curve fitting and not modelling. A similar, but less common case was to assume, when investigating air resistance, that the resistance is proportional to v^n and again finding the optimum value of *n.*

Finally, in connection with the last paragraph, if resistance is assumed proportional to say v^2, some reason for this assumption is expected. Too often candidates simply make the statement or just write down the model without explanation; this is equivalent to the problem alluded to above.

If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed.

Numerical Methods – 4776/02

There continue to be several cases where incorrect work had been ticked. Assessors are requested not to tick work unless it has been checked thoroughly.

The most popular task is to find the value of an integral numerically.

In domain 1 the basic requirement of a formal statement of the problem is often not met.

In domain 2 candidates describe what method they are to use and how it works. There is no credit for this explanation however, but rather that they explain why they are going to use the chosen methods. For instance, it is perfectly possible to work a numerical integration using only the Midpoint rule. The process can be helped by one or other, or both, of the other methods. If candidates do so, however, it is necessary to say why they have chosen to do so.

In domain 3, a 'substantial' application is to find values of M_n, T_n or S_n up to at least n = 64.

In domain 4 we often see a general printout of the formulae being used being given full credit. A clear description of how the algorithm has been implemented is required, usually by presenting an annotated spreadsheet printout.

In domain 5 it is not appropriate to compare values obtained with 'the real value', for example
Additionally, it is accepted that candidates will use a function that they are unable to integrate (because of where they are in the course) but which is integrable. However, it is not then appropriate to state a value found by direct integration either from their own working or from the internet.

Many candidates, as a result of their insubstantial application, will state the value to which the ratio of differences is converging without justification from their values. This can of course lead to inaccuracy, and the failure to provide an "improved solution". Indeed, some candidates use the 'theoretical' value regardless of the values they are getting (or not if they do not work the ratio of differences) far too early giving inaccurate solutions. These are often credited, leading to some very generous marking.

In domain 6 most of the marks are dependent on satisfactory work in the error analysis domain and so often a rather generous assessment of that domain led also to a rather generous assessment here as well. Teachers should note that comments justifying the accuracy of the solution are appropriate here, but comments on the limitations of Excel are not usually creditworthy. Assessors should also note that the criteria for this task demand a solution to 'at least 6 figure accuracy'. Candidates should be finding a solution to an accuracy which they can justify and that should be least 6 significant figures. For example, if their working can justify 9 significant figures then they should give that level of accuracy with justification.

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