

**Wednesday 23 January 2013 – Morning**

**AS GCE MATHEMATICS (MEI)**

**4755/01 Further Concepts for Advanced Mathematics (FP1)**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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This paper has been pre modified for carrier language

## Section A (36 marks)

- 1 Transformation A is represented by matrix  $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  and transformation B is represented by matrix  $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ .
- (i) Describe transformations A and B. [3]
- (ii) Find the matrix for the composite transformation A followed by B. [2]
- 2 Given that  $z = a + bj$ , find  $\operatorname{Re}\left(\frac{z}{z^*}\right)$  and  $\operatorname{Im}\left(\frac{z}{z^*}\right)$ . [4]
- 3 You are given that  $z = 2 + j$  is a root of the cubic equation  $2z^3 + pz^2 + 22z - 15 = 0$ , where  $p$  is real. Find the other roots and the value of  $p$ . [6]
- 4 (i) Show that  $x^2 - x + 2 > 0$  for all real  $x$ . [2]
- (ii) Solve the inequality  $\frac{2x}{x^2 - x + 2} > x$ . [5]
- 5 You are given that  $\frac{3}{(5 + 3x)(2 + 3x)} \equiv \frac{1}{2 + 3x} - \frac{1}{5 + 3x}$ .
- (i) Use this result to find  $\sum_{r=1}^{100} \frac{1}{(5 + 3r)(2 + 3r)}$ , giving your answer as an exact fraction. [5]
- (ii) Write down the limit to which  $\sum_{r=1}^n \frac{1}{(5 + 3r)(2 + 3r)}$  converges as  $n$  tends to infinity. [1]
- 6 Prove by induction that  $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$ . [8]

## Section B (36 marks)

- 7 Fig. 7 shows a sketch of  $y = \frac{x-4}{(x-5)(x-8)}$ .

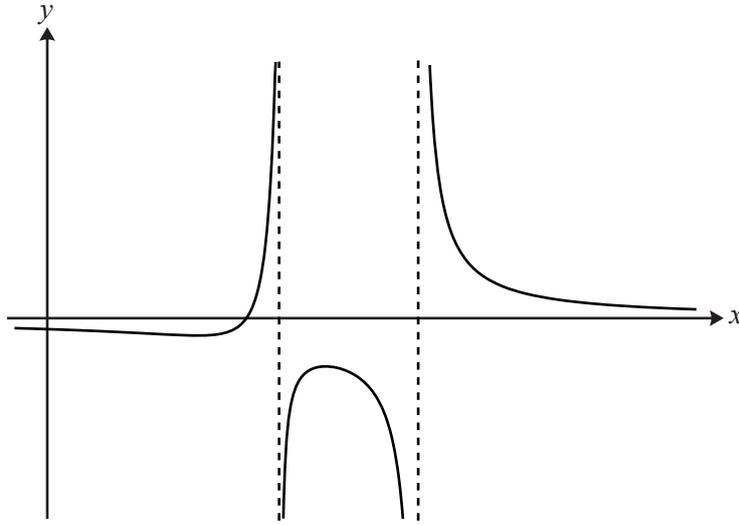


Fig. 7

- (i) Write down the equations of the three asymptotes and the coordinates of the points where the curve crosses the axes. Hence write down the solution of the inequality  $\frac{x-4}{(x-5)(x-8)} > 0$ . [6]
- (ii) The equation  $\frac{x-4}{(x-5)(x-8)} = k$  has no real solutions. Show that  $-1 < k < -\frac{1}{9}$ . Relate this result to the graph of  $y = \frac{x-4}{(x-5)(x-8)}$ . [7]
- 8 (i) Indicate on an Argand diagram the set of points  $z$  for which  $|z - (-8 + 15j)| < 10$ . [4]
- (ii) Using the diagram, show that  $7 < |z| < 27$ . [3]
- (iii) Mark on your Argand diagram the point,  $P$ , at which  $|z - (-8 + 15j)| = 10$  and  $\arg z$  takes its maximum value. Find the modulus and argument of  $z$  at  $P$ . [4]

9 You are given that  $\mathbf{A} = \begin{pmatrix} 8 & -7 & -12 \\ -10 & 5 & 15 \\ -9 & 6 & 6 \end{pmatrix}$  and  $\mathbf{A}^{-1} = k \begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$ .

(i) Find the exact value of  $k$ . [2]

(ii) Using your answer to part (i), solve the following simultaneous equations.

$$\begin{aligned} 8x - 7y - 12z &= 14 \\ -10x + 5y + 15z &= -25 \\ -9x + 6y + 6z &= 3 \end{aligned} \quad [4]$$

You are also given that  $\mathbf{B} = \begin{pmatrix} -7 & 5 & 15 \\ a & -8 & -21 \\ 2 & -1 & -3 \end{pmatrix}$  and  $\mathbf{B}^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 0 & 5 \\ -4 & -3 & 1 \\ 2 & 1 & b \end{pmatrix}$ .

(iii) Find the values of  $a$  and  $b$ . [2]

(iv) Write down an expression for  $(\mathbf{AB})^{-1}$  in terms of  $\mathbf{A}^{-1}$  and  $\mathbf{B}^{-1}$ . Hence find  $(\mathbf{AB})^{-1}$ . [4]

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