OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4754/01A

MATHEMATICS (MEI)
Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

FRIDAY 18 JANUARY 2013: Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED 18pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4754/01A
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (LK/SW) 1570372336 72506/1

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**
- **The Printed Answer Book consists of 16 pages.**
- **This paper will be followed by PAPER B: COMPREHENSION.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

SECTION A (36 marks)

1 Solve the equation $\dfrac{2x}{x+1} - \dfrac{1}{x-1} = 1$. [4]

2 Find the first four terms of the binomial expansion of $\sqrt[3]{1-2x}$. State the set of values of x for which the expansion is valid. [6]

3 The parametric equations of a curve are

$x = \sin\theta, \quad y = \sin 2\theta, \quad$ for $0 \leqslant \theta \leqslant 2\pi$.

(i) Find the exact value of the gradient of the curve at the point where $\theta = \frac{1}{6}\pi$. [4]

(ii) Show that the cartesian equation of the curve is $y^2 = 4x^2 - 4x^4$. [3]

4 Fig. 4 below shows the curve $y = \sqrt{1 + e^{2x}}$, and the region between the curve, the x-axis, the y-axis and the line $x = 2$.

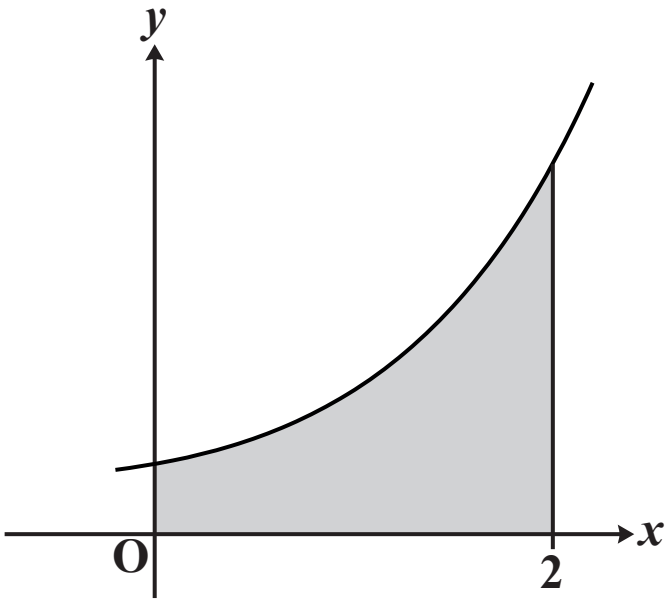

FIG. 4

(a) Find the exact volume of revolution when the shaded region is rotated through 360° about the x-axis. [4]

(b) (i) Complete the table of values, and use the trapezium rule with 4 strips to estimate the area of the shaded region. [3]

x	0	0.5	1	1.5	2
y		1.9283	2.8964	4.5919	

(ii) The trapezium rule for $\int_0^2 \sqrt{1 + e^{2x}}\,dx$ with 8 and 16 strips gives 6.797 and 6.823, although not necessarily in that order. Without doing the calculations, say which result is which, explaining your reasoning. [1]

5 Solve the equation $2\sec^2\theta = 5\tan\theta$, for $0 \leqslant \theta \leqslant \pi$. [6]

6 In Fig. 6 below, ABC, ACD and AED are right-angled triangles and BC = 1 unit. Angles CAB and CAD are θ and ϕ respectively.

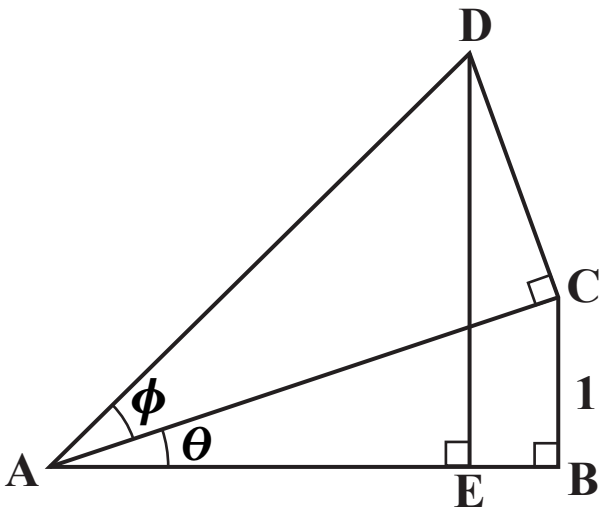

FIG. 6

(i) Find AC and AD in terms of θ and ϕ. [2]

(ii) Hence show that $DE = 1 + \dfrac{\tan\phi}{\tan\theta}$. [3]

BLANK PAGE

SECTION B (36 marks)

7 **A tent has vertices ABCDEF with coordinates as shown in Fig. 7 below. Lengths are in metres. The O*xy* plane is horizontal. The following points are shown on the diagram:**

A (0, –4, 0)
B (8, –a, 0)
C (8, a, 0)
D (6, 0, 2)
E (1, 0, 3)
F (0, 4, 0)

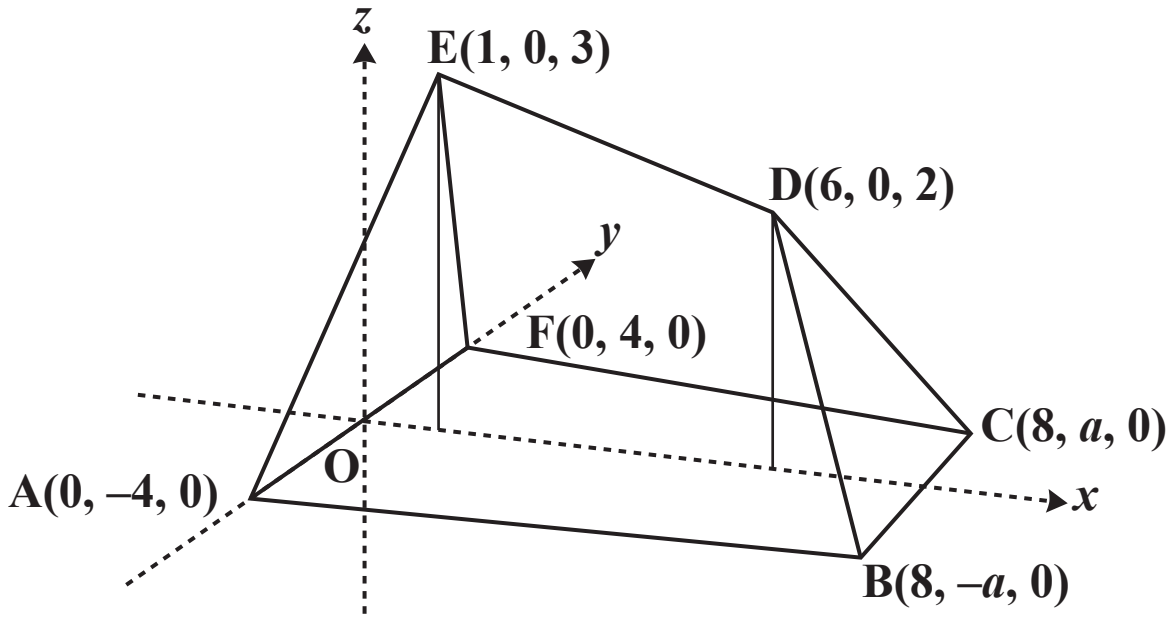

Fig. 7

(i) **Find the length of the ridge of the tent DE, and the angle this makes with the horizontal. [4]**

(ii) **Show that the vector $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal to the plane through A, D and E.**

Hence find the equation of this plane. Given that B lies in this plane, find a. [7]

(iii) **Verify that the equation of the plane BCD is $x + z = 8$.**

Hence find the acute angle between the planes ABDE and BCD. [6]

8 The growth of a tree is modelled by the differential equation

$$10\frac{\mathrm{d}h}{\mathrm{d}t} = 20 - h,$$

where h is its height in metres and the time t is in years. It is assumed that the tree is grown from seed, so that $h = 0$ when $t = 0$.

(i) Write down the value of h for which $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$, and interpret this in terms of the growth of the tree. [1]

(ii) Verify that $h = 20\left(1 - \mathrm{e}^{-0.1t}\right)$ satisfies this differential equation and its initial condition. [5]

The alternative differential equation

$$200\frac{\mathrm{d}h}{\mathrm{d}t} = 400 - h^2$$

is proposed to model the growth of the tree. As before, $h = 0$ when $t = 0$.

(iii) Using partial fractions, show by integration that the solution to the alternative differential equation is $h = \dfrac{20\left(1 - \mathrm{e}^{-0.2t}\right)}{1 + \mathrm{e}^{-0.2t}}$. [9]

(iv) What does this solution indicate about the long-term height of the tree? [1]

(v) After a year, the tree has grown to a height of 2 m. Which model fits this information better? [3]

BLANK PAGE