OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4757:** Further Applications of Advanced Mathematics

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\begin{pmatrix}-1\\4\\1\end{pmatrix}\times\overrightarrow{BC}=\begin{pmatrix}-1\\4\\1\end{pmatrix}\times\begin{pmatrix}-6\\18\\3\end{pmatrix}=\begin{pmatrix}-6\\-3\\6\end{pmatrix}\quad[=3\begin{pmatrix}-2\\-1\\2\end{pmatrix}]$	M1* A1	Vector product of directions	*Intention sufficient*
			Shortest distance is $\dfrac{\overrightarrow{AB}.\mathbf{d}}{\lvert\mathbf{d}\rvert}=\dfrac{\begin{pmatrix}8\\-2\\-13\end{pmatrix}.\begin{pmatrix}-2\\-1\\2\end{pmatrix}}{\sqrt{2^2+1^2+2^2}}$	M1* M1	Appropriate scalar product Evaluation of $\lvert\mathbf{d}\rvert$	*Dep* * *Dep* **
			Shortest distance is $\dfrac{40}{3}$	A1 **[5]**		
		OR	$\left[\begin{pmatrix}11-6\lambda\\18\lambda\\-3+3\lambda\end{pmatrix}-\begin{pmatrix}3-\mu\\2+4\mu\\10+\mu\end{pmatrix}\right].\begin{pmatrix}-1\\4\\1\end{pmatrix}=0$	M1* Two appropriate scalar products		
			and $\begin{pmatrix}8-6\lambda+\mu\\-2+18\lambda-4\mu\\-13+3\lambda-\mu\end{pmatrix}.\begin{pmatrix}-6\\18\\3\end{pmatrix}=0$			
			$81\lambda-18\mu=29$, $123\lambda-27\mu=41$	A1 Two correct equations		
			$\lambda=-\dfrac{5}{3}$, $\mu=-\dfrac{82}{9}$, $\overrightarrow{XY}=\begin{pmatrix}80/9\\40/9\\-80/9\end{pmatrix}$	M1* Obtaining $\overrightarrow{XY}$	*Dep* *	
			Shortest distance is $\sqrt{\left(\dfrac{80}{9}\right)^2+\left(\dfrac{40}{9}\right)^2+\left(\dfrac{80}{9}\right)^2}$	M1	*Dep* **	
			Shortest distance is $\dfrac{40}{3}$	A1		

Question			Answer	Marks	Guidance	
1	(ii)		$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{pmatrix} 8 \\ -2 \\ -13 \end{pmatrix} \times \begin{pmatrix} -6 \\ 18 \\ 3 \end{pmatrix} = \begin{pmatrix} 228 \\ 54 \\ 132 \end{pmatrix}$	M1*	Appropriate vector product	
				A2	Give A1 if one error	
			$\left\lvert \overrightarrow{AB} \times \overrightarrow{BC} \right\rvert = \sqrt{228^2 + 54^2 + 132^2}$	M1*		*Dep* *
			$\left\lvert \overrightarrow{BC} \right\rvert = \sqrt{6^2 + 18^2 + 3^2}$			
			Shortest distance is $\dfrac{\left\lvert \overrightarrow{AB} \times \overrightarrow{BC} \right\rvert}{\left\lvert \overrightarrow{BC} \right\rvert} = \sqrt{\dfrac{72324}{369}}$	M1		*Dep* **
			Shortest distance is 14	A1		*Sign error in vector product can earn M1A1M1M1A1*
				[6]		
		OR	$\left[\begin{pmatrix} 11-6\lambda \\ 18\lambda \\ -3+3\lambda \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 10 \end{pmatrix} \right] . \begin{pmatrix} -6 \\ 18 \\ 3 \end{pmatrix} = 0$		M1* Allow one error A1	
			$\lambda = \dfrac{1}{3}$		M1* Obtaining a value of λ A1	*Dep* *
			Shortest distance is $\sqrt{(6)^2 + (4)^2 + (-12)^2}$		M1	*Dep* **
			Shortest distance is 14		A1	

Question			Answer	Marks	Guidance	
1	(iii)		$\begin{pmatrix}11\\0\\-3\end{pmatrix}+\lambda\begin{pmatrix}-6\\18\\k+3\end{pmatrix}=\begin{pmatrix}3\\2\\10\end{pmatrix}+\mu\begin{pmatrix}-1\\4\\1\end{pmatrix}$			
			$11-6\lambda=3-\mu$	M1	Allow one error	Must use different parameters
			$18\lambda=2+4\mu$	A1	Two correct equations	
			$\lambda=5,\ \ \mu=22$	A1		
			$-3+\lambda(k+3)=10+\mu$	M1	Obtaining a value of k	*Other methods possible*
			$k=4$	A1		*(e.g. distance between lines is 0)*
			Point of intersection is $\begin{pmatrix}3\\2\\10\end{pmatrix}+22\begin{pmatrix}-1\\4\\1\end{pmatrix}$	M1		
			Point of intersection is $(-19,\ 90,\ 32)$	A1		
				[7]		
1	(iv)		$\left\|\begin{pmatrix}-1\\4\\1\end{pmatrix}\right\|=\sqrt{18}$, so $\overrightarrow{AD}=(\pm)\frac{12}{\sqrt{18}}\begin{pmatrix}-1\\4\\1\end{pmatrix}=2\sqrt{2}\begin{pmatrix}-1\\4\\1\end{pmatrix}$	M1* A1	Obtaining $\overrightarrow{AD}$ or D	
			Volume is $\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC})\,.\,\overrightarrow{AD}$	M1*	Appropriate scalar triple product	
			$=\frac{1}{6}\left[\begin{pmatrix}8\\-2\\-13\end{pmatrix}\times\begin{pmatrix}2\\16\\-10\end{pmatrix}\right].(2\sqrt{2})\begin{pmatrix}-1\\4\\1\end{pmatrix}$	A1 ft	Correct expression	*Can be implied*
			$=\frac{\sqrt{2}}{3}\begin{pmatrix}228\\54\\132\end{pmatrix}.\begin{pmatrix}-1\\4\\1\end{pmatrix}=\frac{\sqrt{2}}{3}(120)$	M1	Evaluating scalar triple product	*Dep* **
			$=40\sqrt{2}$	A1	Accept 56.6	
				[6]		

Question			Answer	Marks	Guidance	
2	(i)		$\frac{\partial z}{\partial x} = 6x^2 + 6x + 12y$	B1		
			$\frac{\partial z}{\partial y} = 6y^2 + 6y + 12x$	B1		
			If $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$, $6x^2 + 6x + 12y = 6y^2 + 6y + 12x$			
			$x^2 - y^2 - x + y = 0$			
			$(x-y)(x+y-1) = 0$	M1	Identifying factor $(x-y)$	*SC* If M0, then give B1 for verifying $y = x$
			$y = x$ or $y = 1 - x$	E1E1		B1 for verifying $y = 1 - x$
				[5]		
2	(ii)		$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$	M1		Can be implied
			If $y = x$ then $6x^2 + 6x + 12x = 0$	M1	Obtaining quadratic in x (or y)	Or quartic, and factorising as x(linear)(quadratic)
			$x = 0, -3$	M1	Obtaining a non-zero value of x	
			Stationary points $(0, 0, 0)$ and $(-3, -3, 54)$	B1A1	Condone $(0, 0)$ for B1	
			If $y = 1 - x$ then $6x^2 + 6x + 12(1-x) = 0$			
			$x^2 - x + 2 = 0$	M1	Obtaining quadratic with no real roots	
			Which has no real roots ($D = -7 < 0$)	A1	Correctly shown	Just stating 'No real roots' M1A0
				[7]		
2	(iii)		At P, $\frac{\partial z}{\partial x} = \frac{21}{2}$, $\frac{\partial z}{\partial y} = \frac{21}{2}$	M1 A1	Substituting into $\frac{\partial z}{\partial x}$ or $\frac{\partial z}{\partial y}$	Correct value, or substitution seen
			$\delta z \approx \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$	M1		
			$w \approx \frac{21}{2}h + \frac{21}{2}h$	A1 ft		
			$h \approx \frac{w}{21}$	A1		

Question			Answer	Marks	Guidance	
				[5]		
2	(iv)		$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 24$	M1	Allow sign error	24λ is M0 unless $\lambda = \pm 1$ appears later
			If $y = x$ then $6x^2 + 6x + 12x = 24$ $x = 1,\ -4$	M1	Obtaining quadratic in x (or y)	Or quartic, and one linear factor
			Points $(1,\ 1,\ 22)$ and $(-4,\ -4,\ 32)$	A1A1	If neither correct, give A1 for $x = 1,\ -4$	
			If $y = 1 - x$ then $6x^2 + 6x + 12(1 - x) = 24$ $x = 2,\ -1$	M1	Obtaining quadratic in x (or y)	Or third linear factor of quartic
			Points $(2,\ -1,\ 5)$ and $(-1,\ 2,\ 5)$	A1A1 [7]	If neither correct, give A1 for $x = 2,\ -1$	
3	(a)		$r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 = a^2(1+\cos\theta)^2 + (-a\sin\theta)^2$	B1	Condone ... $+ (a\sin\theta)^2$	
			$= a^2(1 + 2\cos\theta + \cos^2\theta + \sin^2\theta) = 2a^2(1+\cos\theta)$		or $4a^2\cos^4\frac{1}{2}\theta + 4a^2\sin^2\frac{1}{2}\theta\cos^2\frac{1}{2}\theta$	
				M1	Using $1 + \cos\theta = 2\cos^2\frac{1}{2}\theta$	
			$= 4a^2\cos^2\frac{1}{2}\theta$	A1		
			Arc $\int\sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta = \int_0^{\frac{1}{2}\pi} 2a\cos\frac{1}{2}\theta\,\mathrm{d}\theta$	M1	For $\int\sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ in terms of θ	Limits not required
			$= \left[\ 4a\sin\frac{1}{2}\theta\ \right]_0^{\frac{1}{2}\pi}$	A1	For $4a\sin\frac{1}{2}\theta$	
			$= 2\sqrt{2}\ a$	A1 [6]		

Question			Answer	Marks	Guidance	
3	(b)	(i)	$1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2$	B1		
			$= \frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4}$	M1		
			$= \left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2$	A1		
			Area is $\int 2\pi y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$	M1*		
			$= \int_1^2 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)\mathrm{d}x$	A1 ft	Integral expression including limits	
			$= 2\pi\int_1^2\left(\frac{x^5}{12}+\frac{x}{3}+\frac{1}{4x^3}\right)\mathrm{d}x$	M1	Obtaining integrable form	*Dep* *
			$= 2\pi\left[\frac{x^6}{72}+\frac{x^2}{6}-\frac{1}{8x^2}\right]_1^2$	A1	Allow one error	
			$= \frac{47\pi}{16}$	A1		
				[8]		

Question			Answer	Marks	Guidance	
3	(b)	(ii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x + \frac{1}{x^3} \quad \left(= \frac{17}{8}\right)$	B1		
			$\rho = \frac{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^3}{x + \frac{1}{x^3}}$	M1	Using formula for ρ or κ	
				A1 ft	Correct expression for ρ or κ	
			$= \frac{\left(1 + \left(\frac{15}{8}\right)^2\right)^{\frac{3}{2}}}{2 + \frac{1}{8}} = \frac{\left(\frac{17}{8}\right)^3}{\frac{17}{8}}$	A1 ft	Correct numerical expression for ρ	
			$= \frac{289}{64}$	E1	Correctly shown	
				[5]		
3	(b)	(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{15}{8}$, so unit normal is $\frac{1}{17}\begin{pmatrix}-15\\8\end{pmatrix}$	M1 A1	Obtaining a normal vector Correct unit normal	Allow M1 for $\begin{pmatrix}\pm 8\\ \pm 15\end{pmatrix}$ or $\begin{pmatrix}\pm 15\\ \pm 8\end{pmatrix}$
			$\mathbf{c} = \begin{pmatrix}2\\ 19/12\end{pmatrix} + \frac{289}{64}\begin{pmatrix}-15/17\\ 8/17\end{pmatrix}$	M1	Allow sign errors	Must use a unit vector
			Centre of curvature is $\left(-\frac{127}{64}, \frac{89}{24}\right)$	A1A1		
				[5]		

Question			Answer	Marks	Guidance	
4	(a)	(i)	Identity is e Element: a b c d e f g h Inverse: b a c g e h d f	B1 B2 **[3]**	Give B1 for four correct	
4	(a)	(ii)	$d^2 = a,\ \ d^4 = c$ Hence d has order 8, and G is cyclic	M1 A1 A1 E1 **[4]**	Finding powers of an element Identifying d (or f or g or h) as a generator Or $f^2 = b,\ \ f^4 = c$ Or $g^2 = b,\ \ g^4 = c$ Or $h^2 = a,\ \ h^4 = c$ Correctly shown	At least fourth power *Implies previous M1*
4	(a)	(iii)	H: 0 2 4 6 8 10 12 14 G: e d a f c h b g or e f b d c g a h or e g b h c f a d or e h a g c d b f	B1 B1 B1 **[3]**	For $e \leftrightarrow 0$ and $c \leftrightarrow 8$ For $\{d, f, g, h\} \leftrightarrow \{2, 6, 10, 14\}$ For a fully correct isomorphism	In any order
4	(a)	(iv)	Rotations have order 2 or 4 Reflections have order 2 There is no element of order 8 Hence not isomorphic	B1 E1 **[2]**	Correct statement about rotations and/or reflections which implies non-IM *Or* More than one element of order 2 *Or* Not commutative Fully correct explanation	*Or* (4) reflections (and 180° rotation) have order 2 *Or* composition of reflections (or 90° rotation and reflection) is not commutative Dependent on previous B1

Question			Answer	Marks	Guidance	
4	(b)	(i)	$\mathrm{f}_m\mathrm{f}_n(x) = \dfrac{\dfrac{x}{1+nx}}{1+m\left(\dfrac{x}{1+nx}\right)}$	M1	Composition of functions	In either order
			$= \dfrac{x}{1+nx+mx} = \dfrac{x}{1+(m+n)x} = \mathrm{f}_{m+n}(x)$	E1 **[2]**	Correctly shown	E0 if in wrong order
4	(b)	(ii)	$(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n}\mathrm{f}_p = \mathrm{f}_{m+n+p}$ $\mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p) = \mathrm{f}_m\mathrm{f}_{n+p} = \mathrm{f}_{m+n+p}$ Hence S is associative	M1 E1 **[2]**	Combining three functions Correctly shown	M1E1 bod for $(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n+p} = \mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p)$
4	(b)	(iii)	For any f_m, f_n in S, $\mathrm{f}_m\mathrm{f}_n = \mathrm{f}_{m+n}$ $\mathrm{f}_m\mathrm{f}_n$ is in S (so S is closed) Identity is f_0 Inverse of f_n is f_{-n} since $\mathrm{f}_n\mathrm{f}_{-n} = \mathrm{f}_{n-n} = \mathrm{f}_0$ S is also associative, and hence is a group	M1 A1 B1 B1 B1 E1 **[6]**	Referring to this in context B0 for x B1 for $n = 0$ Closure, associativity, identity and inverses must all be mentioned in (iii)	Dependent on previous 5 marks
4	(b)	(iv)	$\{\mathrm{f}_{2n}\}$ for all integers n	B2 **[2]**	Or $\{\mathrm{f}_{3n}\}$, etc Give B1 for multiples of 2 (or 3, etc) but not completely correctly described	e.g. $\{\mathrm{f}_0, \mathrm{f}_2, \mathrm{f}_4, \mathrm{f}_6, \ldots\}$

Question			Answer	Marks	Guidance	
5	(i)		***Pre-multiplication by transition matrix*** $\mathbf{P} = \begin{pmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0.05 & 0.5 & 0 & 0 \\ 0 & 0.45 & 0.05 & 0.5 & 0 \\ 0 & 0 & 0.45 & 0.05 & 0 \\ 0 & 0 & 0 & 0.45 & 1 \end{pmatrix}$	B3 **[3]**	Allow tolerance of ± 0.0001 in probabilities throughout this question Give B2 for four columns correct Give B1 for two columns correct	
5	(ii)		$\mathbf{P}^8 \begin{pmatrix} 0 \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5042 \\ 0.0230 \\ 0.0278 \\ \mathbf{0.02071} \\ 0.4242 \end{pmatrix}$ P(3 lives) = 0.0207 (4 dp)	M1 E1 **[2]**	For $\mathbf{P}^8$ (allow $\mathbf{P}^7$ or $\mathbf{P}^9$) and initial column matrix Correctly shown	
5	(iii)		Let q(n) = P(not yet ended after n tasks) $= \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \end{pmatrix} \mathbf{P}^n \begin{pmatrix} 0 \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ 0 \end{pmatrix}$ q(10) = 0.0371	M1 M1 A1 **[3]**	Obtaining probabilities after 10 tasks Adding probabilities of 1, 2, 3 lives	Allow M1 for using $\mathbf{P}^9$ or $\mathbf{P}^{11}$

Question			Answer	Marks	Guidance	
5	(iv)		$q(9)-q(10)$ $=0.05072-0.03709$ $=0.0136$	M1 M1 A1 **[3]**	Using q(9) and q(10) Evaluating q(9)	
		OR	$\mathbf{P}^9\begin{pmatrix}0\\ \frac{1}{3}\\ \frac{1}{3}\\ \frac{1}{3}\\ 0\end{pmatrix}=\begin{pmatrix}.\\ 0.01506\\ .\\ 0.01355\\ .\end{pmatrix}$ $0.01506\times0.5+0.01355\times0.45$ $=0.0136$		M1 Probs of 1 and 3 lives after 9 tasks M1 A1	
5	(v)		$q(13)=0.01374$ $q(14)=0.00998$ Smallest N is 14	M1 M1 A1 **[3]**	Evaluating q(n) for some $n>10$ Consecutive values each side of 0.01 Must be clear that their answer is 14	Just $N=14$ www earns B3
5	(vi)		$\mathbf{P}^n\to\begin{pmatrix}1&0.7880&0.5525&0.2908&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0.2120&0.4475&0.7092&1\end{pmatrix}=\mathbf{L}$	B2 **[2]**	Give B1 for any element correct to 3 dp (other than 0 or 1)	
5	(vii)		$\mathbf{L}\begin{pmatrix}0\\ \frac{1}{3}\\ \frac{1}{3}\\ \frac{1}{3}\\ 0\end{pmatrix}=\begin{pmatrix}0.5438\\0\\0\\0\\0.4562\end{pmatrix}$ P(wins a prize) = 0.4562	M1M1 A1 **[3]**	Using **L** and the initial column matrix	

Question			Answer	Marks	Guidance	
5	(viii)		Maximum probability is 0.7092 Always start with 3 lives	B1 ft B1 **[2]**		
5	(ix)		$\mathbf{L}\begin{pmatrix}0\\0.1\\p\\q\\0\end{pmatrix}=\begin{pmatrix}0.4\\0\\0\\0\\0.6\end{pmatrix}$ $0.7880\times0.1+0.5525p+0.2908(0.9-p)=0.4$ P(2 lives) = 0.2273 , P(3 lives) = 0.6727	M1 M1 A1 **[3]**	Or $0.0212+0.4475p+0.7092(0.9-p)=0.6$ Obtaining a value for p or q Accept values rounding to 0.227, 0.673	Allow use of $p+q=1$
5			***Post-multiplication by transition matrix***		Allow tolerance of ±0.0001 in probabilities throughout this question	
5	(i)		$\mathbf{P}=\begin{pmatrix}1&0&0&0&0\\0.5&0.05&0.45&0&0\\0&0.5&0.05&0.45&0\\0&0&0.5&0.05&0.45\\0&0&0&0&1\end{pmatrix}$ **[3]**	B3	Give B2 for four rows correct Give B1 for two rows correct	
5	(ii)		$\begin{pmatrix}0&\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\end{pmatrix}\mathbf{P}^8$ $=\begin{pmatrix}0.5042&0.0230&0.0278&\mathbf{0.02071}&0.4242\end{pmatrix}$ P(3 lives) = 0.0207 (4 dp)	M1 E1 **[2]**	For $\mathbf{P}^8$ (allow $\mathbf{P}^7$ or $\mathbf{P}^9$) and initial row matrix Correctly shown	

Question			Answer	Marks	Guidance	
5	(iii)		Let q(n) = P(not yet ended after n tasks) $= \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \mathbf{P}^n \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ $q(10) = 0.0371$	M1 M1 A1 **[3]**	Obtaining probabilities after 10 tasks Adding probabilities of 1, 2, 3 lives	Allow M1 for using $\mathbf{P}^9$ or $\mathbf{P}^{11}$
5	(iv)		$q(9) - q(10)$ $= 0.05072 - 0.03709$ $= 0.0136$	M1 M1 A1 **[3]**	Using q(9) and q(10) Evaluating q(9)	
		OR	$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix} \mathbf{P}^9$ $= \begin{pmatrix} . & 0.01506 & . & 0.01355 & . \end{pmatrix}$ $0.01506 \times 0.5 + 0.01355 \times 0.45$ $= 0.0136$		M1 Probs of 1 and 3 lives after 9 tasks M1 A1	
5	(v)		$q(13) = 0.01374$ $q(14) = 0.00998$ Smallest N is 14	M1 M1 A1 **[3]**	Evaluating q(n) for some $n > 10$ Consecutive values each side of 0.01 Must be clear that their answer is 14	Just $N = 14$ www earns B3
5	(vi)		$\mathbf{P}^n \to \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.7880 & 0 & 0 & 0 & 0.2120 \\ 0.5525 & 0 & 0 & 0 & 0.4475 \\ 0.2908 & 0 & 0 & 0 & 0.7092 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{L}$	B2 **[2]**	Give B1 for any element correct to 3 dp (other than 0 or 1)	

Question			Answer	Marks	Guidance	
5	(vii)		$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}\mathbf{L}$ $= \begin{pmatrix} 0.5438 & 0 & 0 & 0 & 0.4562 \end{pmatrix}$ P(wins a prize) = 0.4562	M1M1 A1 **[3]**	Using **L** and the initial row matrix	
5	(viii)		Maximum probability is 0.7092 Always start with 3 lives	B1 ft B1 **[2]**		
5	(ix)		$\begin{pmatrix} 0 & 0.1 & p & q & 0 \end{pmatrix}\mathbf{L}$ $= \begin{pmatrix} 0.4 & 0 & 0 & 0 & 0.6 \end{pmatrix}$ $0.7880 \times 0.1 + 0.5525p + 0.2908(0.9 - p) = 0.4$ P(2 lives) = 0.2273 , P(3 lives) = 0.6727	 M1 M1 A1 **[3]**	 Or $0.0212 + 0.4475p + 0.7092(0.9 - p) = 0.6$ Accept values rounding to 0.227, 0.673	 Allow use of $p + q = 1$

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