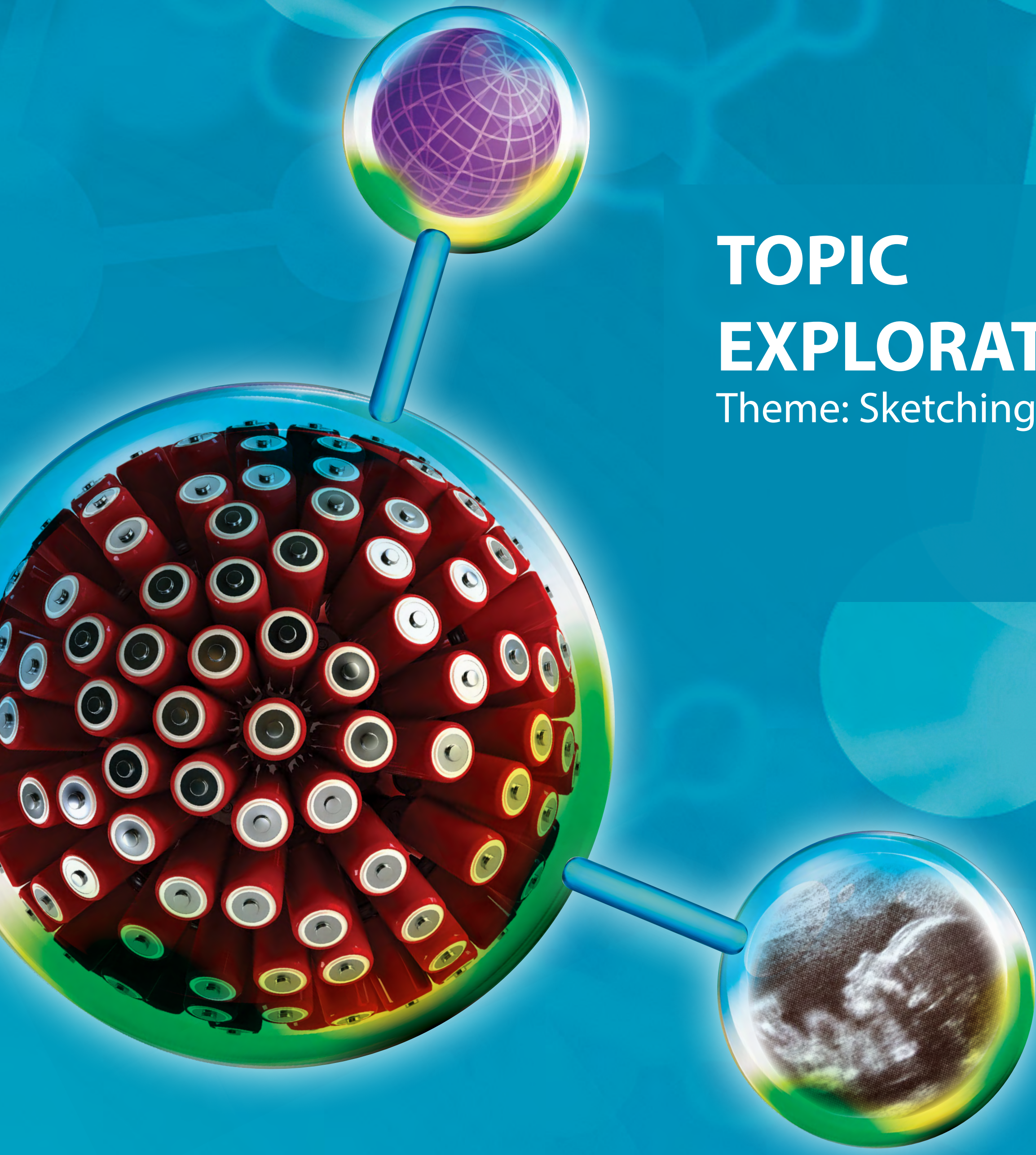


TOPIC EXPLORATION PACK

Theme: Sketching Graphs

A LEVEL PHYSICS A AND B

ocr.org.uk/science

Oxford Cambridge and RSA

Contents

This Topic Exploration Pack should accompany the OCR resource ‘Sketching Graphs’ learner activities, which you can download from the OCR website.

This activity offers an opportunity for maths skills development.

Introduction

KS4 Prior Learning

- Plot a linear function using a table of values
- Sketch a linear function with knowledge of $y = mx + c$
- Plot a quadratic function using a table of values
- Know the shape and features of the sine, cosine and tangent graphs
- Know the shape and features of the reciprocal ($1/x$) graph

KS5 Knowledge

- Sketch linear functions
- Sketch quadratic functions
- Sketch reciprocal graphs including $y = \frac{1}{x^2}$
- Sketch trigonometrical graphs
- Sketch exponential graphs
- Use logarithmic plots to test for exponential/power law relationships between variables.

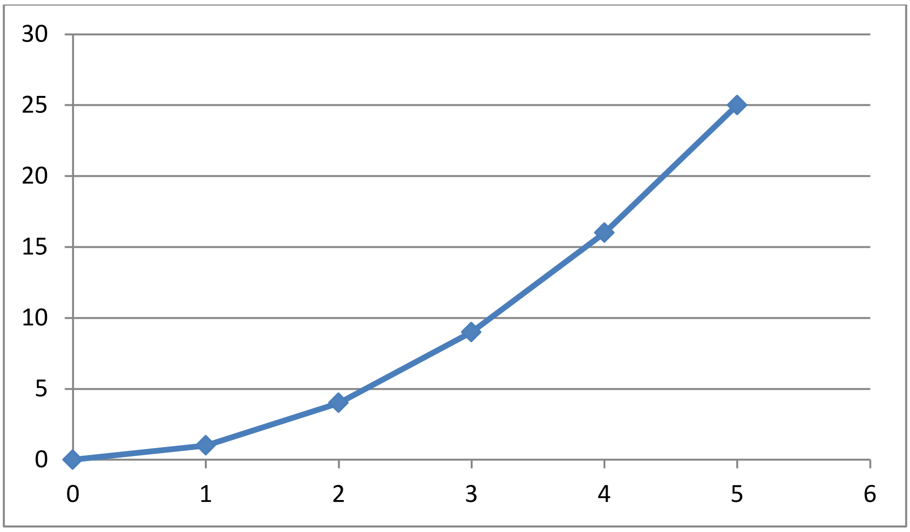
Delivery

There are a number of tricks for being able to sketch these graphs without using a table of values. Students *always* have a table of values method as a back up if they get stuck, however these should be used with caution. For instance take the simple function of $y = x^2$.

A student may decide to construct a table of values like so:

x	0	1	2	3	4	5
y	0	1	4	9	16	25

And get the graph:



There are clearly two errors with this:

- The graph ignores negative values of x. How can the student know in advance for what values of x does the graph exhibit it's 'interesting' features. In this example leaving out the negative x values misses out on the fundamental interesting feature of the graph in that at (0,0) there is a *turning point*.
- The graph has been joined up by straight lines. One cannot assume that between individual plotted points the graph is linear – a very common mistake.

With these points in mind we pay attention to correct ways to sketch these common functions which although loses accuracy in the sense of plotting, increases accuracy in fundamental behaviour of the graph. Alternatively a graph plotter could be used. Geogebra is a free software available to download and use as a web app that is suitable for this. Please find this at www.geogebra.org.

Sketching Linear Graphs

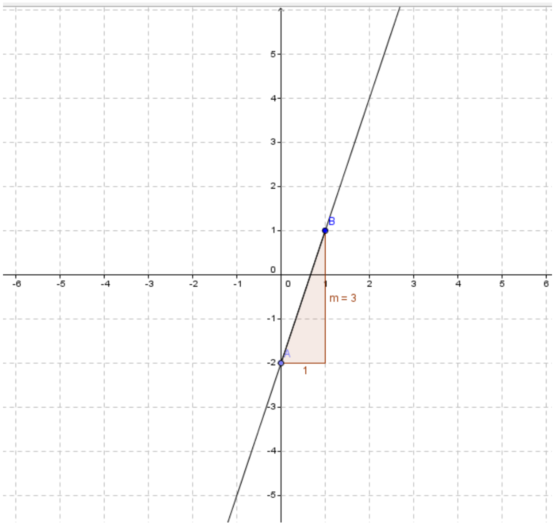
We will use the example of $y = 3x - 2$.

Students will be aware that the coefficient of x is the gradient and the -2 at the end is the y-intercept. This is fine in the context of a maths lesson but students will often fail to realise that the graph $v = 3t - 2$ is essentially the same as this but with different variable symbols.

This is the challenge of translating their maths knowledge to the physics laboratory. An easy way to get students to sketch this is to start at the vertical axis intercept -2. Once they are there they know that the gradient of 3 means that for every 1 across they go three up.

So plotting a point at (1,1) will give another point. Now the line can be drawn between these two points and the function is sketched.

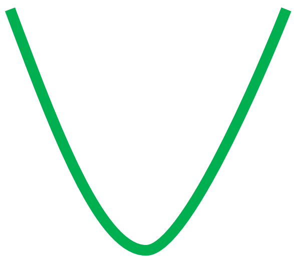
See the figure below for a diagram on how to do this.



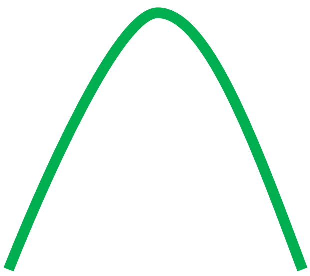
Sketching Quadratic Graphs

Sketching a quadratic graph requires a lot more work. There is a quick method that will get the graph sketched but not rely on any mathematical understanding (hopefully this will be achieved in the maths lessons!). Take for example the graph $y = x^2 - 5x + 3$.

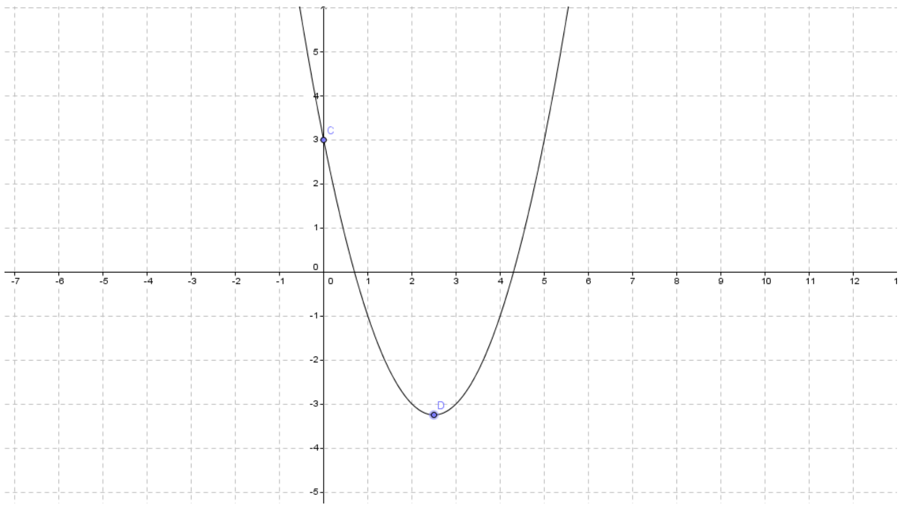
Again we start at the y intercept which in this case is +3. There are two types of quadratic – a positive curve and a negative curve. Because there is a positive in front of the x^2 then this will be a positive quadratic and will look like this:



A $-x^2$ curve will look like:



Now that the shape has been established the turning point has to be found. This is found by halving the coefficient in front of x and reversing the sign. For example, here we have -5. So the turning point will be at +2.5. Substituting this into the graph will give the *y* coordinate as -3.25. Hence we have the two vital points as (0,3) and (2.5,-3.25) as the turning point. The graph can now be sketched:



Note that the *x*-intercepts have been 'guessed'. All the student should remember is that the graph is symmetrical about the turning point. To find the *x*-intercepts the quadratic formula should be used:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Which in this case will yield:

$$x = \frac{+5 \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$

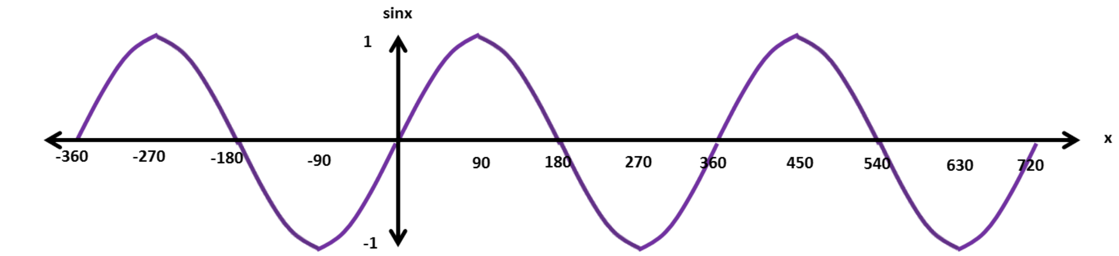
$$x = 4.3, 0.7$$

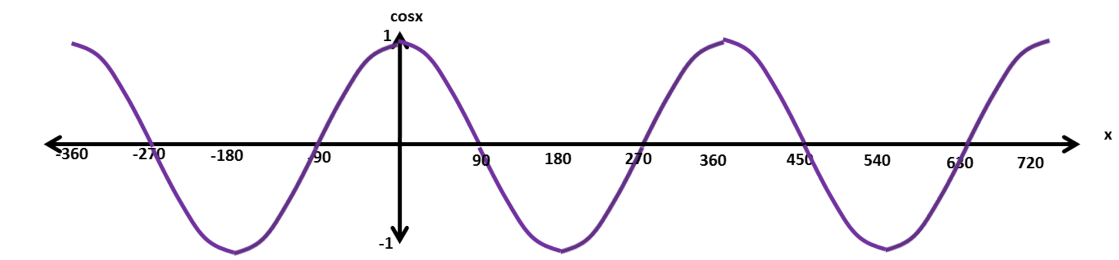
Alternatively a student with a calculator that has a quadratic solver may be able to get these solutions automatically.

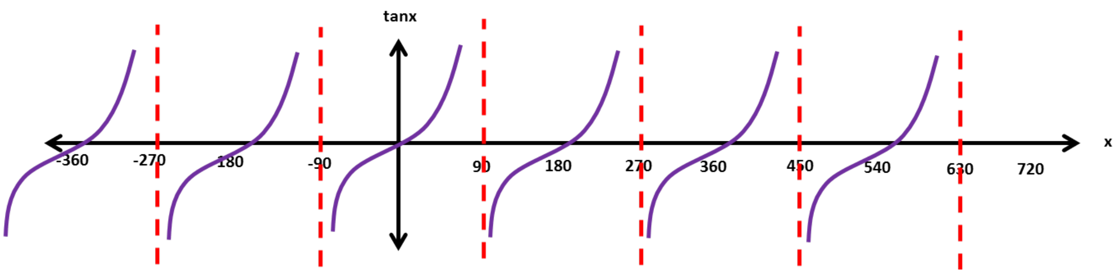
Sketching Trigonometrical Graphs

The best way to sketch the trig graphs is to remember the three basic shapes and then apply graph transformations in order to variations of the work. To understand how the basic shapes work please see **Activity 1** for an activity that creates the basic sine and cosine graph.

The basic graphs are shown below:

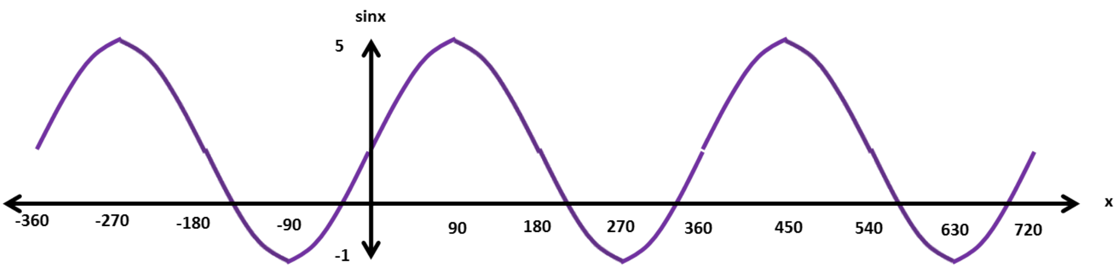




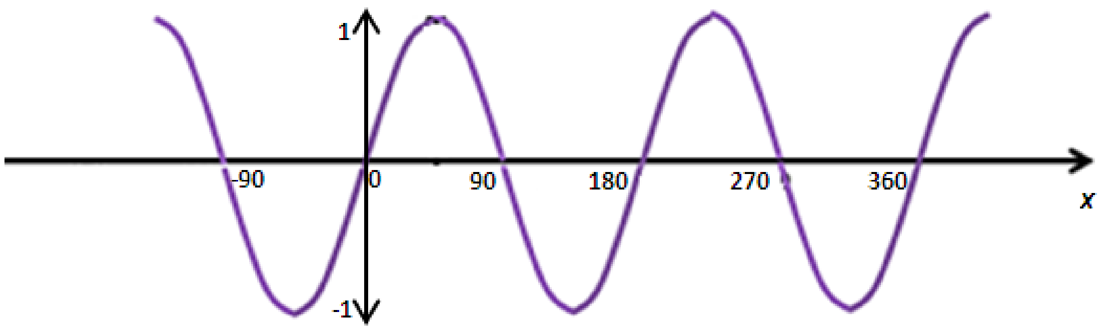


Students have to remember these shapes. An easy way to remember the sine and cosine is that between 0 and 360 the sine forms a *wave* starting at (0,0) whilst the cosine forms a *bucket* starting (0,1). Once these have been remembered, variations of these graphs can be sketched using *transformations*. For example $y = 2 + 3\sin x$.

Now if we start at the basic $\sin x$ we are changing this function in 2 ways.
First we are multiplying it by 3 – this represents a stretch in the *y*-direction of factor 3. So instead of going between -1 and 1, the graph now goes between -3 and 3. The +2 means that the whole graph moves up by 2 so now the graph will go between -1 and 5. This is shown below:



For other transformations consider $y = \cos(2x - 90)$. This is an altogether trickier transformation. Because all of the changes are 'inside' the brackets this means that changes occur in the x direction but are the *opposite* to what you would think. We can rewrite the equation in factored form as $y = \cos(2(x - 45)$. The 2 in front of the x represents a stretch in the x-direction of factor 1/2 and therefore results in a change of frequency. There is a phase shift in the positive *x*-direction moving the whole graph to the right by 45 degrees. See the graph below:



See **Activity 1** for a simple activity aimed at practising this skill.

Sketching Exponential Graphs

Sketching graphs of the form $y = ka^{\lambda x}$ is relatively straightforward. Please see **Activity 2** for a guided activity using a graphing software package that the students can go through to get an idea of the fundamental features of exponential graphs. It does require the use of Geogebra but it can easily be adapted for other software packages.

Using logarithms to test for exponential relationships

This is a difficult area to teach. Students may not have any knowledge of logarithms and therefore you will need to tread lightly. One can take the approach that ignorance is bliss and just explore this as a method with zero mathematical understanding. To understand the mathematics, the students will ideally be studying A level Mathematics (this comes up in the C2 module for most students which will usually not be studied until after Christmas) so it is important that initially it is treated as a method only. Understanding the mathematics will require a number of lessons on the theory of logarithms which while useful will take you away from what you want to achieve here. The idea is that given a set of data, can you find a mathematical relationship that links the two variables. The two models that are usually tested are:

$$y = ka^x$$
$$y = kx^n$$

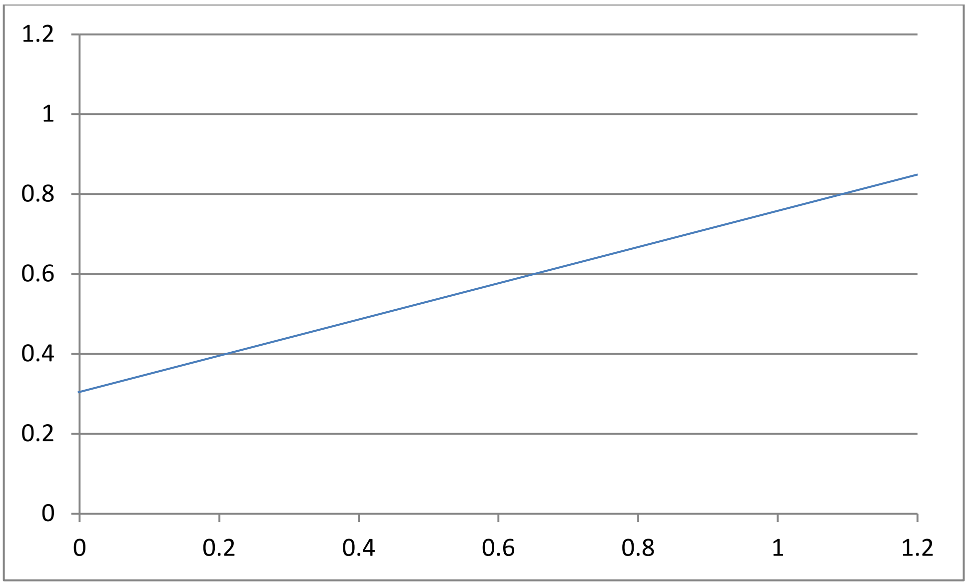
Notice in the first model the x variable is in the power and this is an example of an Exponential function. The second model has the x variable as the base and this is an example of a power function. Let's apply these models to the set of data:

Time, *t*	0	1	2	3	4	5	6	7
Population, *P*	2	7	27	98	359	1313	4807	17595

We will investigate the exponential model, ie $P = ka^t$. The aim is to find out the constants k and a. For the exponential model you should take the logarithm to base 10 of the DEPENDENT variable only; in this case P. Using a calculator and rounding to 2 decimal places gives:

Time	0	1	2	3	4	5	6	7
Population	2	7	27	98	359	1313	4807	17595
Log *P*	0.3	0.85	1.43	1.99	2.56	3.12	3.68	4.25

Now the graph of $\log P$ against t should be plotted; t as the x axis and $\log P$ as the y axis. If this model is suitable then the graph should be a straight line (plotted for $0 < x < 1.2$):



Students draw a line of best fit by 'eye' and they can see the data is roughly a straight-line. Now to find the constants we have to find the y-intercept and the gradient.

The gradient is $\frac{change\ in\ P}{change\ in\ t}$ whilst the y intercept can be found by 'eye'.

The y intercept is 0.3, whilst the gradient is $\frac{4.25-0.3}{7-0} = 0.56$.

For the exponential model the value of k is given by $k = 10^{y\ intercept} = 10^{0.3} = 2$ whilst a is found by $a = 10^{gradient} = 10^{0.56} = 3.63$. Hence the model is written as:

$$P = 2 \times 3.63^t$$

For a power relationship the method is roughly the same but for a few key differences. The differences are summed up in the table below:

Model	Logarithms	Graph	k	a
$P = ka^t$	Taking logarithms of P only	Plot log P against t	$k = 10^{y\ intercept}$	$a = 10^{gradient}$
$P = kt^a$	Take logarithms of P and t	Plot log P against log t	$k = 10^{y\ intercept}$	$a = gradient$

Notice that in the power model the gradient is simply the value of a - powers of 10 do not need to be taken. See **Activity 3** for a typical classroom activity for this.

Activity 1 – Sketching Trig Graphs

Resources: Activity Sheet 1, Activity Sheet 2

Instructions: Students have to sketch the following functions on the axis provided by using their knowledge of graph transformations. You may want to shrink the activity sheet 1 so you can fit more than one set of axis per page. Alternatively you may want them to sketch the graphs in different colours on the same set of axes. Activity sheet 2 provides the equations to be sketched and these should be handed out to the students as well.

Pedagogy: This will give students a firm grasp of sketching waves which will be particularly useful when looking at wave superposition and phase shifts etc.

Timing: This would look to take 10 minutes and can be used as an initial starter activity or as a pre-lesson homework task in order for the students to get used to sketching waves.

Activity 2 – Exploring Exponential Graphs

Resources: Activity Sheet 3, Laptop/Computer with Geogebra installed or equivalent graphing software.

Instructions: Book a set of laptops or a computer room. Give students Activity Sheet 3 with the instructions. The instructions are quite clear and let the students create their own dynamic graph on the Geogebra program. After they have created the dynamic graph they are then able to answer the questions in the back of the pack. If this particular program isn't installed they can access a free web app from the link http://www.geogebra.org/webstart/4.4/geogebra.html. Please check with your IT department that this can be accessed before doing this lesson. Alternatively learn how to create a dynamic curve on an alternative software and then just use the questions at the end to help them understand the effects of the parameters.

Pedagogy: This is an independent investigation where you can let the students independently discover the different effects the parameters of an exponential function have on the curve. The questions also help students relate these graphs to the real-life situations in Physics where they will encounter them.

Timing: This is a whole lesson task or alternatively can be given as an extended homework task.

Activity 3 – Modelling Rabbits

Resources: Activity Sheet 4, Graph paper

Instructions: Hand out activity sheet 4 and some graph paper. Given the data the students have to find the exact relationships between the population of rabbits and time. Two of the populations are exponential relationships and the other two are power relationships. Models of the form:

$$P = k \times t^a$$

and

$$P = k \times a^t$$

are assumed. The first job is for students to make a guess as to which data exhibits exponential or power behaviour. 2 of them are exponential relationships, the other two are power relationships. The exact answers are:

$$P_{Newton} = 2 \times 3.66^t$$
$$P_{Galileo} = 2 \times t^5$$
$$P_{Faraday} = 2 \times e^t$$
$$P_{Boyle} = 2 \times t^{4.5}$$

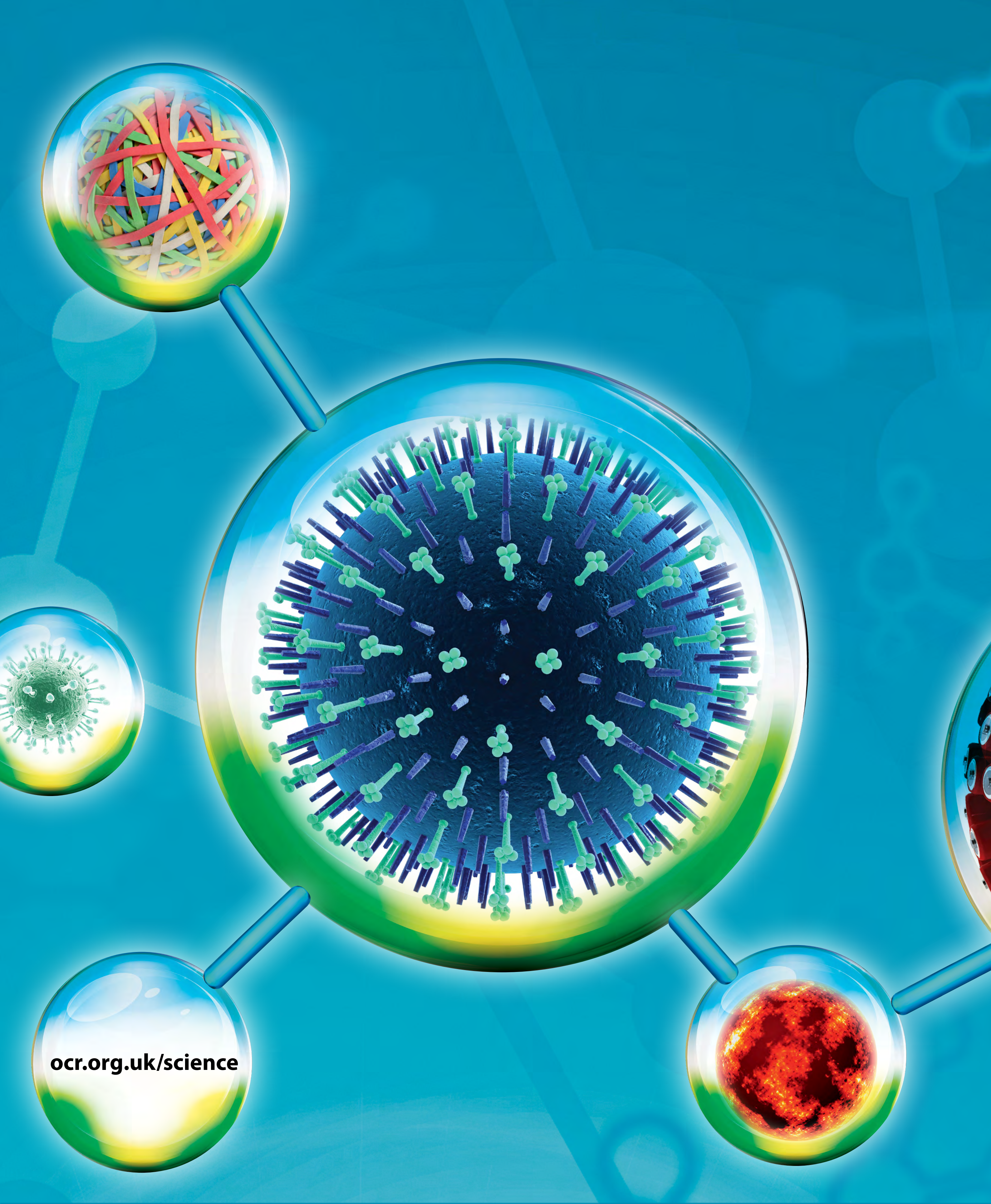
Students will need to sketch some axes, take logarithms appropriately and also plot and sketch lines of best fit in order to find the constants.

Pedagogy: This activity helps practise the main concepts of modelling data using logarithms.

Timing: This is a whole lesson task or alternatively can be given as an extended homework task.

Extension: These questions are really for those who have a good mathematical knowledge of the theory of logarithms:

- How can we test the relationship $P = a^{kt}$ where a and k are constants?
- How can we test the relationship $P = k + a^t$ where a and k are constants?



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