

**GCE**

**Mathematics (MEI)**

Unit **4756**: Further Methods for Advanced Mathematics

Advanced GCE

**Mark Scheme for June 2014**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.


All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation	Meaning of annotation
	Blank Page – this annotation <b>must</b> be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

## Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

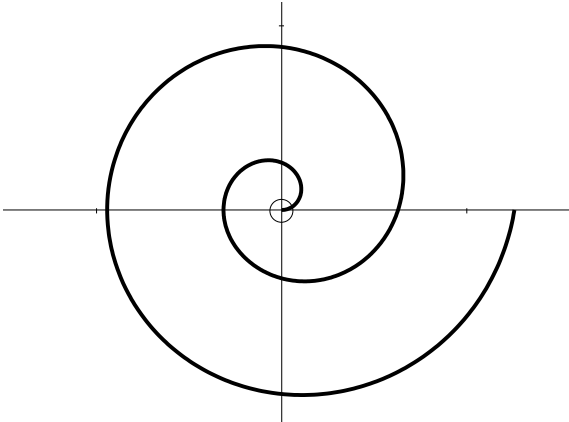
- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(a)	(i)		<p>B1</p> <p>B1</p> <p>[2]</p>	<p>Correct general shape (not multiple-valued, not straight, negative gradient throughout) relative to axes</p> <p>Dependent on first B1.</p> <p>Reasonably vertical at ends.</p> <p>Correct domain (labelled at <math>-1</math> and <math>1</math>)</p> <p>Correct range (labelled at <math>\pi</math>)</p> <p>Correct <math>y</math>-intercept (labelled at <math>\pi/2</math>)</p> <p>SC B1B0 for a fully correct curve in <math>[-1,1] \times [0,\pi]</math> but multiple-valued</p>	
1	(a)	(ii)	$\cos y = x \Rightarrow -\sin y \frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx} = \frac{-1}{\sin y}$ $\sin^2 y + \cos^2 y = 1 \Rightarrow \sin y = (\pm)\sqrt{1-x^2}$ $\Rightarrow \frac{dy}{dx} = \pm \frac{1}{\sqrt{1-x^2}} \text{ or } -\frac{1}{\sqrt{1-x^2}}$ <p>Taking <math>-</math> sign because gradient is negative</p>	<p>M1</p> <p>A1(ag)</p> <p>B1</p> <p>[3]</p>	<p>Differentiating w.r.t. <math>x</math> or <math>y</math></p> <p>Completion www with intermediate step</p> <p>Independent of B1 below</p> <p>Validly rejecting <math>+</math> sign.</p> <p>Dependent on A1 above</p>	$\frac{dx}{dy} = -\sin y$ $\frac{dy}{dx} = \pm \frac{1}{\sqrt{1-x^2}} \text{ or } \pm \text{ not considered}$ <p>scores max. 2</p> <p>Or <math>0 \leq y \leq \pi \Rightarrow \sin y \geq 0 \Rightarrow \frac{dy}{dx} \leq 0</math></p> <p>Or <math>f(x)</math> is decreasing</p>

Question			Answer	Marks	Guidance	
1	(a)	(iii)	$f(x) = \arccos x$ $\Rightarrow f'(x) = -(1-x^2)^{-\frac{1}{2}}$ $\Rightarrow f''(x) = \frac{1}{2}(1-x^2)^{-\frac{3}{2}} \times -2x = -x(1-x^2)^{-\frac{3}{2}}$ $\Rightarrow f'''(x) = -(1-x^2)^{-\frac{3}{2}} - x \times -\frac{3}{2}(1-x^2)^{-\frac{5}{2}} \times -2x$ $= -(1-x^2)^{-\frac{3}{2}} - 3x^2(1-x^2)^{-\frac{5}{2}}$ $\Rightarrow f(0) = \frac{\pi}{2}$ $f'(0) = -1, f''(0) = 0, f'''(0) = -1$ $\Rightarrow f(x) = \frac{\pi}{2} - x - \frac{x^3}{6} + \dots$	 M1 A1 M1 A1  B1  B1B1	Derivative in the form $kx(1-x^2)^{-\frac{3}{2}}$ o.e. Any correct form www Differentiating $f''(x)$ using product or quotient and chain rules. Dep. on 1st M1 Any correct form www  As first term of expansion  $-x$ www, $-\frac{x^3}{6}$ www	For second derivative  Allow a clear explanation that only the first term contributes to McLaurin expansion for 7/7  Independent of all other marks  Incorrect simplification above loses the last B1
		OR	$f'(x) = -(1-x^2)^{-\frac{1}{2}} \Rightarrow f'(x) = -1 - \frac{1}{2}x^2 \dots$ $\Rightarrow f(x) = \int \left(-1 - \frac{1}{2}x^2 \dots\right) dx = -x - \frac{1}{6}x^3 \dots + c$ $c = \arccos 0 = \frac{\pi}{2}$	 M2 Using binomial expansion A1A1 $-1, -\frac{1}{2}x^2$  B1B1 www  B1 Correct $c$ as term of expansion	With $x^2$	
				[7]		



Question			Answer	Marks	Guidance
1	(b)	(i)	$r = \theta + \sin \theta$ $\Rightarrow \frac{dr}{d\theta} = 1 + \cos \theta$ $\cos \theta \geq -1 \Rightarrow \frac{dr}{d\theta} \geq 0, \text{ so } r \text{ increases as } \theta$ <p>increases</p> 	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[4]</p>	<p><math>\frac{dr}{d\theta} \geq 0</math> stated. Dependent on first B1</p> <p>Do not condone &gt; No wrong statements</p> <p>One complete revolution with <math>r(0) = 0</math> and <math>r(2\pi) \geq r(3\pi/2) \geq r(\pi) \geq r(\pi/2) &gt; 0</math></p> <p>Correct general shape with two complete revolutions</p> <p>Independent. Condone <math>r(0) &gt; 0</math> for B0B1</p>
1	(b)	(ii)	$\text{Area} = \frac{1}{2} \int_0^\alpha r^2 d\theta = \frac{1}{2} \int_0^\alpha (\theta + \sin \theta)^2 d\theta$ <p>For small <math>\theta</math>, <math>\sin \theta \approx \theta \Rightarrow r \approx 2\theta</math></p> $\text{Area} \approx \frac{1}{2} \int_0^\alpha (2\theta)^2 d\theta = \frac{1}{2} \left[ \frac{4}{3} \theta^3 \right]_0^\alpha$ $= \frac{2}{3} \alpha^3$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Forming an integral expression in <math>\theta</math> for the required area</p> <p>Using <math>\sin \theta \approx \theta</math> and a complete method for integrating their expression</p> <p>Condone only omitted limits or <math>\frac{1}{2}</math></p> <p>Dependent on first M1</p>

Question		Answer	Marks	Guidance	
2	(a)	$C + jS = ae^{j\theta} + a^2e^{2j\theta} + \dots$ <p>This is a geometric series with <math>r = ae^{j\theta}</math></p> $\text{Sum to infinity} = \frac{ae^{j\theta}}{1 - ae^{j\theta}}$ $= \frac{ae^{j\theta}}{1 - ae^{j\theta}} \times \frac{1 - ae^{-j\theta}}{1 - ae^{-j\theta}}$ $= \frac{ae^{j\theta} - a^2}{1 - ae^{j\theta} - ae^{-j\theta} + a^2}$ $= \frac{a \cos \theta + aj \sin \theta - a^2}{1 - 2a \cos \theta + a^2}$ $= \frac{a \cos \theta - a^2}{1 - 2a \cos \theta + a^2} + \frac{aj \sin \theta}{1 - 2a \cos \theta + a^2}$ $\Rightarrow S = \frac{a \sin \theta}{1 - 2a \cos \theta + a^2}$ <p>and <math>C = \frac{a \cos \theta - a^2}{1 - 2a \cos \theta + a^2}</math></p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1*</p> <p>M1</p> <p>M1</p> <p>A1(ag)</p> <p>A1</p> <p>[8]</p>	<p>Forming <math>C + jS</math> as a series of powers</p> <p>Identifying G.P. and attempting sum. Dependent on first M1</p> <p>Multiplying numerator and denominator by <math>1 - ae^{-j\theta}</math> o.e.</p> <p>Multiplying out denominator. Dependent on M1*</p> <p>Introducing trig functions. Dependent on M1*</p> <p><b>NB answer space continued (BP)</b></p>	<p>...<math>a^2</math> (cos 2 sin 2<math>\theta</math>) insufficient. Powers must be correct</p> <p>Allow M1 for sum to <math>n</math> terms</p> <p>Correct sum to infinity implies M1M1</p> <p>Strictly this, or trig equivalent</p> <p>Use of FOIL with powers combined correctly (allow one slip)</p> <p>Condone e.g. <math>e^{-j\theta} = \cos \theta + j \sin \theta</math></p> <p>If trig used throughout award last M1 for using <math>\cos^2 \theta + \sin^2 \theta = 1</math></p> <p>Answer given. www which leads to <math>S</math>, e.g. condone sign error in num.</p>
2	(b)	(i)	$\sqrt{3} + j = 2e^{j\frac{\pi}{6}}; \text{ need to rotate by } \frac{\pi}{3} \text{ so vertices are}$ <p><math>2j</math></p> <p><math>-\sqrt{3} + j</math></p> <p><math>-\sqrt{3} - j</math></p> <p><math>-2j</math></p> <p><math>\sqrt{3} - j</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[5]</p>	<p>If vertices not given in form <math>x + jy</math>:</p> <p>B1 for <math>2e^{j\frac{7\pi}{6}}</math></p> <p>B1 for <math>2e^{j\frac{\pi}{2}}</math> and <math>2e^{j\frac{3\pi}{2}}</math></p> <p>B1 for <math>2e^{j\frac{5\pi}{6}}</math> and <math>2e^{j\frac{11\pi}{6}}</math></p> <p>i.e. maximum of 3/5.</p> <p>If B0 scored give SC B2 for five vertices in form <math>x + jy</math> obtained by repeatedly rotating their P by <math>\frac{\pi}{3}</math></p>

Question			Answer	Marks	Guidance
2	(b)	(ii)	Vertices are $4e^{j\frac{\pi}{3}} = 2 + 2\sqrt{3}j$ $4e^{j\pi} = -4$ and $4e^{j\frac{5\pi}{3}} = 2 - 2\sqrt{3}j$ Area = $\frac{1}{2} \times 4\sqrt{3} \times 6$ $= 12\sqrt{3}$	M1 A2 B1 <b>[4]</b>	Attempt to square at least one of their vertices in (i) Three correct in form $x + jy$ (and simplified) and no more awrt 20.8 Give A1 for any two of these, or all three and no extras in polar form Dependent on A2 above
3	(a)	(i)	Characteristic equation is $(6 - \lambda)(-1 - \lambda) + 12 = 0$ $\Rightarrow \lambda^2 - 5\lambda + 6 = 0$ $\Rightarrow \lambda = 2, 3$ When $\lambda = 2$ , $\begin{pmatrix} 4 & -3 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow 4x - 3y = 0$ $\Rightarrow$ eigenvector is $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ o.e. When $\lambda = 3$ , $\begin{pmatrix} 3 & -3 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow x - y = 0$ $\Rightarrow$ eigenvector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ o.e.	M1 A1 M1 A1 A1 <b>[5]</b>	Forming characteristic polynomial At least one equation relating $x$ and $y$ For either $\lambda = 2$ or $\lambda = 3$ ( $\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = (\lambda)\mathbf{x}$ M0 below
3	(a)	(ii)	$\mathbf{P} = \begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$	B1ft B1ft <b>[2]</b>	Do not ft $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ as eigenvector Columns must correspond Both fts must be of numerical values If one matrix diagonal, condone matrices not identified as $\mathbf{P}$ and $\mathbf{D}$

Question			Answer	Marks	Guidance	
3	(b)	(i)	$5^3 - 4 \times 5^2 - 3 \times 5 - 10 = 0 \Rightarrow \lambda = 5$ eigenvalue $\lambda^3 - 4\lambda^2 - 3\lambda - 10 = (\lambda - 5)(\lambda^2 + \lambda + 2)$ $\lambda^2 + \lambda + 2 = 0 \Rightarrow (\lambda + \frac{1}{2})^2 + \frac{7}{4} = 0 \Rightarrow$ no real roots	B1 M1 A1 A1(ag) [4]	Or showing that $(\lambda - 5)$ is a factor Obtaining quadratic factor Correct quadratic factor Correctly showing a correct quadratic equation has no real roots Two of three terms of quadratic correct e.g. $b^2 - 4ac = 1 - 8$ or correct use of quadratic formula	
3	(b)	(ii)	$\mathbf{B} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = 5 \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -10 \\ 5 \\ 20 \end{pmatrix}$ $\mathbf{B}^2 \begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix} = 5^2 \begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix} = \begin{pmatrix} 100 \\ -50 \\ -200 \end{pmatrix}$ $\mathbf{B} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \\ 40 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix}$ $\Rightarrow x = -4, y = 2, z = 8$	B1 B1 B2 [4]	Allow $5 \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ isw Allow $25 \begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix}$ or $5^2 \begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix}$ o.e. Accept vector form	Give B1 for two correct unknowns
3	(b)	(iii)	$\mathbf{C-H} \Rightarrow \mathbf{B}^3 - 4\mathbf{B}^2 - 3\mathbf{B} - 10\mathbf{I} = \mathbf{0}$ $\Rightarrow \mathbf{B}^3 = 4\mathbf{B}^2 + 3\mathbf{B} + 10\mathbf{I}$ and $\mathbf{B}^4 = 4\mathbf{B}^3 + 3\mathbf{B}^2 + 10\mathbf{B}$ $= 4(4\mathbf{B}^2 + 3\mathbf{B} + 10\mathbf{I}) + 3\mathbf{B}^2 + 10\mathbf{B}$ $\Rightarrow \mathbf{B}^4 = 19\mathbf{B}^2 + 22\mathbf{B} + 40\mathbf{I}$	M1 M1 A1(ag) [3]	Idea of $\lambda \leftrightarrow \mathbf{B}$ . Condone omitted $\mathbf{I}$ Multiplying by $\mathbf{B}$ and substituting for $\mathbf{B}^3$ Completion	Condone use of $\mathbf{M}$ throughout

Question		Answer	Marks	Guidance	
4	(i)	$x = \sinh y \Rightarrow x = \frac{e^y - e^{-y}}{2}$ $\Rightarrow e^y - e^{-y} = 2x$ $\Rightarrow e^{2y} - 2xe^y - 1 = 0$ $\Rightarrow (e^y - x)^2 = 1 + x^2$ $\Rightarrow e^y = x \pm \sqrt{1 + x^2}$ $\Rightarrow y = \ln(x(\pm)\sqrt{1 + x^2})$ $x - \sqrt{1 + x^2} < 0 \text{ so take + sign}$	<p>B1</p> <p>M1</p> <p>A1(ag)</p> <p>B1</p>	<p>x in exponential form</p> <p>Solving to reach <math>e^y</math></p> <p>Completion www</p> <p>Validly rejecting negative root. Dependent on A1 above</p>	<p>Allow one slip. Ignore variables. Allow unsimplified <math>y = \ln x \pm \sqrt{x^2 + 1} </math> A0 e.g. <math>e^y &gt; 0</math>; <math>e^y \geq 0</math> B0</p>
		<p><b>OR</b></p> $\ln(x + \sqrt{1 + x^2}) = \ln(\sinh y + \sqrt{1 + \sinh^2 y})$ $= \ln(\sinh y + \cosh y)$ $= \ln(e^y)$ $= y$	<p>M1</p> <p>B1</p> <p>B1</p> <p>A1</p>	<p>Explanation why + is taken</p> <p>Completion www</p>	<p>e.g. <math>\sinh y - \cosh y &lt; 0</math></p>
		$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{1 + x^2}} \times \frac{d}{dx}(x + \sqrt{1 + x^2})$ $= \frac{1}{x + \sqrt{1 + x^2}} \times \left(1 + \frac{x}{\sqrt{1 + x^2}}\right)$ $= \frac{1}{x + \sqrt{1 + x^2}} \times \left(\frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}}\right)$ $= \frac{1}{\sqrt{1 + x^2}}$	<p>M1</p> <p>B1</p> <p>A1</p> <p>A1(ag)</p> <p>[8]</p>	<p>Attempting <math>\frac{1}{u} \times \frac{du}{dx}</math></p> <p><math>\frac{d}{dx}(x + \sqrt{1 + x^2}) = 1 + \frac{x}{\sqrt{1 + x^2}}</math> o.e.</p> <p>Any correct form of <math>\frac{dy}{dx}</math> in terms of x</p> <p>Obtained www with valid intermediate step, e.g. (*)</p>	<p>Or implicit differentiation of <math>e^y = x + \sqrt{1 + x^2}</math> as far as <math>\frac{dy}{dx} =</math></p> <p><b>NB answer space continued (BP)</b></p>

Question	Answer	Marks	Guidance
4 (ii)	$\int \frac{1}{\sqrt{25+4x^2}} dx = \frac{1}{2} \int \frac{1}{\sqrt{\frac{25}{4} + x^2}} dx$ $= \frac{1}{2} \ln \left( x + \sqrt{x^2 + \frac{25}{4}} \right) + c$	M1 A1 A1 <b>[3]</b>	$\operatorname{arsinh} kx$ or $\ln(kx + \sqrt{k^2x^2 + \dots})$ $\frac{1}{2} \operatorname{arsinh} \frac{2x}{5}$ or $\ln \left( \frac{2x}{5} + \sqrt{1 + \frac{4x^2}{25}} \right)$ o.e. Fully correct in logarithmic form Condone omitted $c$
4 (iii)	$2x = 5 \sinh u \Rightarrow \frac{dx}{du} = \frac{5}{2} \cosh u$ $\int \sqrt{25+4x^2} dx = \int \sqrt{25+25 \sinh^2 u} \times \frac{5}{2} \cosh u du$ $= \int \frac{25}{2} \cosh^2 u du$ $= \int \left( \frac{25}{4} \cosh 2u + \frac{25}{4} \right) du$ $= \frac{25}{8} \sinh 2u + \frac{25}{4} u + c$ $= \frac{25}{4} \sinh u \cosh u + \frac{25}{4} u + c$ $= \frac{25}{4} \times \frac{2x}{5} \times \sqrt{1 + \frac{4x^2}{25}} + \frac{25}{4} \operatorname{arsinh} \frac{2x}{5} + c$ $= \frac{25}{4} \left( \ln \left( \frac{2x}{5} + \sqrt{1 + \frac{4x^2}{25}} \right) + \frac{2x}{5} \sqrt{1 + \frac{4x^2}{25}} \right) + c$	M1 A1 M1* A2 M1 A1(ag) <b>[7]</b>	Finding $\frac{dx}{du}$ and complete substitution Substituting for all elements correctly Simplifying an expression of the form $k \cosh^2 u$ to an integrable form Any correct form. Condone omitted $c$ Give A2ft for $\frac{k}{4} \sinh 2u + \frac{ku}{2}$ Give A1ft A0 for one error Using double “angle” formula Dependent on M1* Completion www with convincing intermediate step Condone “upside-down” substitution for dx e.g. $\frac{25}{8} e^{2u} + \frac{25}{4} + \frac{25}{8} e^{-2u}$ e.g. $\frac{25}{16} e^{2u} + \frac{25}{4} u - \frac{25}{16} e^{-2u} + c$ Using exponential definition of $\sinh 2u$ and substituting for $u$ scores this M1 if an expression with a constant denominator is found validly e.g. reversing terms <b>NB answer space continued (BP)</b>

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