

Monday 23 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (54 marks)

- 1 (a) Given that $f(x) = \arccos x$,
- (i) sketch the graph of $y = f(x)$, [2]
- (ii) show that $f'(x) = -\frac{1}{\sqrt{1-x^2}}$, [3]
- (iii) obtain the Maclaurin series for $f(x)$ as far as the term in x^3 . [7]
- (b) A curve has polar equation $r = \theta + \sin \theta, \theta \geq 0$.
- (i) By considering $\frac{dr}{d\theta}$ show that r increases as θ increases.
Sketch the curve for $0 \leq \theta \leq 4\pi$. [4]
- (ii) You are given that $\sin \theta \approx \theta$ for small θ . Find in terms of α the approximate area bounded by the curve and the lines $\theta = 0$ and $\theta = \alpha$, where α is small. [3]

- 2 (a) The infinite series C and S are defined as follows.

$$C = a \cos \theta + a^2 \cos 2\theta + a^3 \cos 3\theta + \dots,$$

$$S = a \sin \theta + a^2 \sin 2\theta + a^3 \sin 3\theta + \dots,$$

where a is a real number and $|a| < 1$.

By considering $C + jS$, show that

$$S = \frac{a \sin \theta}{1 - 2a \cos \theta + a^2}.$$

Find a corresponding expression for C . [8]

- (b) P is one vertex of a regular hexagon in an Argand diagram. The centre of the hexagon is at the origin. P corresponds to the complex number $\sqrt{3} + j$.
- (i) Find, in the form $x + jy$, the complex numbers corresponding to the other vertices of the hexagon. [5]
- (ii) The six complex numbers corresponding to the vertices of the hexagon are squared to form the vertices of a new figure. Find, in the form $x + jy$, the vertices of the new figure. Find the area of the new figure. [4]

- 3 (a) (i) Find the eigenvalues and corresponding eigenvectors for the matrix \mathbf{A} , where

$$\mathbf{A} = \begin{pmatrix} 6 & -3 \\ 4 & -1 \end{pmatrix}. \quad [5]$$

- (ii) Write down a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{A} = \mathbf{PDP}^{-1}$. [2]

- (b) (i) The 3×3 matrix \mathbf{B} has characteristic equation

$$\lambda^3 - 4\lambda^2 - 3\lambda - 10 = 0.$$

Show that 5 is an eigenvalue of \mathbf{B} . Show that \mathbf{B} has no other real eigenvalues. [4]

- (ii) An eigenvector corresponding to the eigenvalue 5 is $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$.

Evaluate $\mathbf{B} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{B}^2 \begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix}$.

Solve the equation $\mathbf{B} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \\ 40 \end{pmatrix}$ for x, y, z . [4]

- (iii) Show that $\mathbf{B}^4 = 19\mathbf{B}^2 + 22\mathbf{B} + 40\mathbf{I}$. [3]

Section B (18 marks)

- 4 (i) Given that $\sinh y = x$, show that

$$y = \ln(x + \sqrt{1+x^2}). \quad (*)$$

Differentiate (*) to show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}. \quad [8]$$

- (ii) Find $\int \frac{1}{\sqrt{25+4x^2}} dx$, expressing your answer in logarithmic form. [3]

- (iii) Use integration by substitution with $2x = 5 \sinh u$ to show that

$$\int \sqrt{25+4x^2} dx = \frac{25}{4} \left(\ln \left(\frac{2x}{5} + \sqrt{1 + \frac{4x^2}{25}} \right) + \frac{2x}{5} \sqrt{1 + \frac{4x^2}{25}} \right) + c,$$

where c is an arbitrary constant. [7]

END OF QUESTION PAPER

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