OCR
Oxford Cambridge and RSA

AS Level Further Mathematics A

Advanced Subsidiary GCE (H235)

Further Mathematics A

Formulae Booklet

The information in this booklet is for the use of candidates following the Advanced Subsidiary course.

The formulae booklet will be printed for distribution with the examination papers.

Copies of this booklet may be used for teaching.

This document consists of **8** pages.

Instructions to Exams Officer/Invigilator

- **Do not send this Formulae Booklet for marking; it should be retained in the centre or destroyed.**

Pure Mathematics

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1} b + {}^n\mathrm{C}_2 a^{n-2} b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r} b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

Matrix transformations

Reflection in the line $y = \pm x$: $\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Rotations through θ about the coordinate axes. The direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation.

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\mathbf{R}_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Complex numbers

Circles: $|z-a| = k$

Half lines: $\arg(z-a) = \alpha$

Lines: $|z-a| = |z-b|$

Vectors and 3-D coordinate geometry

Cartesian equation of the line through the point A with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ in direction $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ is $\frac{x-a_1}{u_1} = \frac{y-a_2}{u_2} = \frac{z-a_3}{u_3} (= \lambda)$

Vector product: $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

Statistics

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \textbf{ or } \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$$

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $\mathrm{P}(X = x_i) = p_i$

Expectation: $\mu = \mathrm{E}(X) = \sum x_i p_i$

Variance: $\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	$P(X=x)$	$\mathrm{E}(X)$	$\mathrm{Var}(X)$
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Uniform distribution over 1, 2, …, n $\mathrm{U}(n)$	$\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2-1)$
Geometric distribution $\mathrm{Geo}(p)$	$(1-p)^{x-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson $\mathrm{Po}(\lambda)$	$e^{-\lambda}\frac{\lambda^x}{x!}$	λ	λ

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_\nu^2$

Correlation and regression

For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum(x_i - \overline{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n},\ S_{yy} = \sum(y_i - \overline{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n},$$

$$S_{xy} = \sum(x_i - \overline{x})(y_i - \overline{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

Product moment correlation coefficient: $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{\sum x_i y_i - \dfrac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \dfrac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \dfrac{\left(\sum y_i\right)^2}{n}\right)\right]}}$

The regression coefficient of y on x is $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sum(x_i - \overline{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \overline{y} - b\overline{x}$

Spearman's rank correlation coefficient: $r_s = 1 - \dfrac{6\sum d_i^2}{n(n^2 - 1)}$

Critical values for the product moment correlation coefficient, r

1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					*n*				
1	-	-	-	-	31	0.3009	0.3550	0.4158	0.4556
2	-	-	-	-	32	0.2960	0.3494	0.4093	0.4487
3	0.9877	0.9969	0.9995	0.9999	33	0.2913	0.3440	0.4032	0.4421
4	0.9000	0.9500	0.9800	0.9900	34	0.2869	0.3388	0.3972	0.4357
5	0.8054	0.8783	0.9343	0.9587	35	0.2826	0.3338	0.3916	0.4296
6	0.7293	0.8114	0.8822	0.9172	36	0.2785	0.3291	0.3862	0.4238
7	0.6694	0.7545	0.8329	0.8745	37	0.2746	0.3246	0.3810	0.4182
8	0.6215	0.7067	0.7887	0.8343	38	0.2709	0.3202	0.3760	0.4128
9	0.5822	0.6664	0.7498	0.7977	39	0.2673	0.3160	0.3712	0.4076
10	0.5494	0.6319	0.7155	0.7646	40	0.2638	0.3120	0.3665	0.4026
11	0.5214	0.6021	0.6851	0.7348	41	0.2605	0.3081	0.3621	0.3978
12	0.4973	0.5760	0.6581	0.7079	42	0.2573	0.3044	0.3578	0.3932
13	0.4762	0.5529	0.6339	0.6835	43	0.2542	0.3008	0.3536	0.3887
14	0.4575	0.5324	0.6120	0.6614	44	0.2512	0.2973	0.3496	0.3843
15	0.4409	0.5140	0.5923	0.6411	45	0.2483	0.2940	0.3457	0.3801
16	0.4259	0.4973	0.5742	0.6226	46	0.2455	0.2907	0.3420	0.3761
17	0.4124	0.4821	0.5577	0.6055	47	0.2429	0.2876	0.3384	0.3721
18	0.4000	0.4683	0.5425	0.5897	48	0.2403	0.2845	0.3348	0.3683
19	0.3887	0.4555	0.5285	0.5751	49	0.2377	0.2816	0.3314	0.3646
20	0.3783	0.4438	0.5155	0.5614	50	0.2353	0.2787	0.3281	0.3610
21	0.3687	0.4329	0.5034	0.5487	51	0.2329	0.2759	0.3249	0.3575
22	0.3598	0.4227	0.4921	0.5368	52	0.2306	0.2732	0.3218	0.3542
23	0.3515	0.4132	0.4815	0.5256	53	0.2284	0.2706	0.3188	0.3509
24	0.3438	0.4044	0.4716	0.5151	54	0.2262	0.2681	0.3158	0.3477
25	0.3365	0.3961	0.4622	0.5052	55	0.2241	0.2656	0.3129	0.3445
26	0.3297	0.3882	0.4534	0.4958	56	0.2221	0.2632	0.3102	0.3415
27	0.3233	0.3809	0.4451	0.4869	57	0.2201	0.2609	0.3074	0.3385
28	0.3172	0.3739	0.4372	0.4785	58	0.2181	0.2586	0.3048	0.3357
29	0.3115	0.3673	0.4297	0.4705	59	0.2162	0.2564	0.3022	0.3328
30	0.3061	0.3610	0.4226	0.4629	60	0.2144	0.2542	0.2997	0.3301

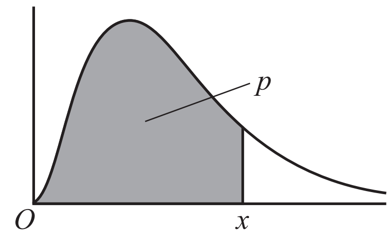
Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					*n*				
1	-	-	-	-	31	0.3012	0.3560	0.4185	0.4593
2	-	-	-	-	32	0.2962	0.3504	0.4117	0.4523
3	-	-	-	-	33	0.2914	0.3449	0.4054	0.4455
4	1.0000	-	-	-	34	0.2871	0.3396	0.3995	0.4390
5	0.9000	1.0000	1.0000	-	35	0.2829	0.3347	0.3936	0.4328
6	0.8286	0.8857	0.9429	1.0000	36	0.2788	0.3300	0.3882	0.4268
7	0.7143	0.7857	0.8929	0.9286	37	0.2748	0.3253	0.3829	0.4211
8	0.6429	0.7381	0.8333	0.8810	38	0.2710	0.3209	0.3778	0.4155
9	0.6000	0.7000	0.7833	0.8333	39	0.2674	0.3168	0.3729	0.4103
10	0.5636	0.6485	0.7455	0.7939	40	0.2640	0.3128	0.3681	0.4051
11	0.5364	0.6182	0.7091	0.7545	41	0.2606	0.3087	0.3636	0.4002
12	0.5035	0.5874	0.6783	0.7273	42	0.2574	0.3051	0.3594	0.3955
13	0.4835	0.5604	0.6484	0.7033	43	0.2543	0.3014	0.3550	0.3908
14	0.4637	0.5385	0.6264	0.6791	44	0.2513	0.2978	0.3511	0.3865
15	0.4464	0.5214	0.6036	0.6536	45	0.2484	0.2945	0.3470	0.3822
16	0.4294	0.5029	0.5824	0.6353	46	0.2456	0.2913	0.3433	0.3781
17	0.4142	0.4877	0.5662	0.6176	47	0.2429	0.2880	0.3396	0.3741
18	0.4014	0.4716	0.5501	0.5996	48	0.2403	0.2850	0.3361	0.3702
19	0.3912	0.4596	0.5351	0.5842	49	0.2378	0.2820	0.3326	0.3664
20	0.3805	0.4466	0.5218	0.5699	50	0.2353	0.2791	0.3293	0.3628
21	0.3701	0.4364	0.5091	0.5558	51	0.2329	0.2764	0.3260	0.3592
22	0.3608	0.4252	0.4975	0.5438	52	0.2307	0.2736	0.3228	0.3558
23	0.3528	0.4160	0.4862	0.5316	53	0.2284	0.2710	0.3198	0.3524
24	0.3443	0.4070	0.4757	0.5209	54	0.2262	0.2685	0.3168	0.3492
25	0.3369	0.3977	0.4662	0.5108	55	0.2242	0.2659	0.3139	0.3460
26	0.3306	0.3901	0.4571	0.5009	56	0.2221	0.2636	0.3111	0.3429
27	0.3242	0.3828	0.4487	0.4915	57	0.2201	0.2612	0.3083	0.3400
28	0.3180	0.3755	0.4401	0.4828	58	0.2181	0.2589	0.3057	0.3370
29	0.3118	0.3685	0.4325	0.4749	59	0.2162	0.2567	0.3030	0.3342
30	0.3063	0.3624	0.4251	0.4670	60	0.2144	0.2545	0.3005	0.3314

Critical values for the χ^2 distribution

If X has a χ^2 distribution with v degrees of freedom then, for each pair of values of p and v, the table gives the value of x such that

$$P(X \le x) = p.$$



p	0.01	0.025	0.05	0.90	0.95	0.975	0.99	0.995	0.999
$v = 1$	0.0^31571	0.0^39821	0.0^23932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

Mechanics

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Newton's experimental law

Between two smooth spheres $v_1 - v_2 = -e(u_1 - u_2)$

Between a smooth sphere with a fixed plane surface $v = -eu$

Motion in a circle

Tangential velocity is $v = r\dot{\theta}$

Radial acceleration is $\frac{v^2}{r}$ or $r\dot{\theta}^2$ towards the centre

Tangential acceleration is $\dot{v} = r\ddot{\theta}$

Discrete Mathematics

Sorting algorithms

Bubble sort:

> Start at the left hand end of the list unless specified otherwise.
>
> Compare the first and second values and swap if necessary. Then compare the (new) second value with the third value and swap if necessary. Continue in this way until all values have been considered.
>
> Fix the last value then repeat with the reduced list until either there is a pass in which no swaps occur or the list is reduced to length 1, then stop.

Shuttle sort:

> Start at the left hand end of the list unless specified otherwise.
>
> Compare the second value with the first and swap if necessary, this completes the first pass. Next compare the third value with the second and swap if necessary, if a swap happened shuttle back to compare the (new) second with the first as in the first pass, this completes the second pass.
>
> Next compare the fourth value with the third and swap if necessary, if a swap happened shuttle back to compare the (new) third value with the second as in the second pass (so if a swap happens shuttle back again). Continue in this way for $n-1$ passes, where n is the length of the list.

Network algorithms

Dijkstra's algorithm

START with a graph G. At each vertex draw a box, the lower area for temporary labels, the upper left hand area for the order of becoming permanent and the upper right hand area for the permanent label.

STEP 1 Make the given start vertex permanent by giving it permanent label 0 and order label 1.

STEP 2 For each vertex that is not permanent and is connected by an arc to the vertex that has just been made permanent (with permanent label = P), add the arc weight to P. If this is smaller than the best temporary label at the vertex, write this value as the new best temporary label.

STEP 3 Choose the vertex that is not yet permanent which has the smallest best temporary label. If there is more than one such vertex, choose any one of them. Make this vertex permanent and assign it the next order label.

STEP 4 If every vertex is now permanent, or if the target vertex is permanent, use 'trace back' to find the routes or route, then STOP; otherwise return to STEP 2.

Prim's algorithm (graphical version)

START with an arbitrary vertex of G.

STEP 1 Add an edge of minimum weight joining a vertex already included to a vertex not already included.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Prim's algorithm (tabular version)

START with a table (or matrix) of weights for a connected weighted graph.

STEP 1 Cross through the entries in an arbitrary row, and mark the corresponding column.

STEP 2 Choose a minimum entry from the uncircled entries in the marked column(s).

STEP 3 If no such entry exists STOP; otherwise go to STEP 4.

STEP 4 Circle the weight w_{ij} found in STEP 2; mark column i; cross through row i.

STEP 5 Return to STEP 2.

Kruskal's algorithm

START with all the vertices of G, but no edges; list the edges in increasing order of weight.

STEP 1 Add an edge of G of minimum weight in such a way that no cycles are created.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Additional Pure Mathematics

Vector product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$, where $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$, in that order, form a right-handed triple.