

**Level 3 Certificate**

**Mathematical Techniques and Applications for  
Engineers**

Unit **H865/01** Component 1

OCR Level 3 Certificate

**Mark Schemes for June 2015**

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.


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Annotations in scoris

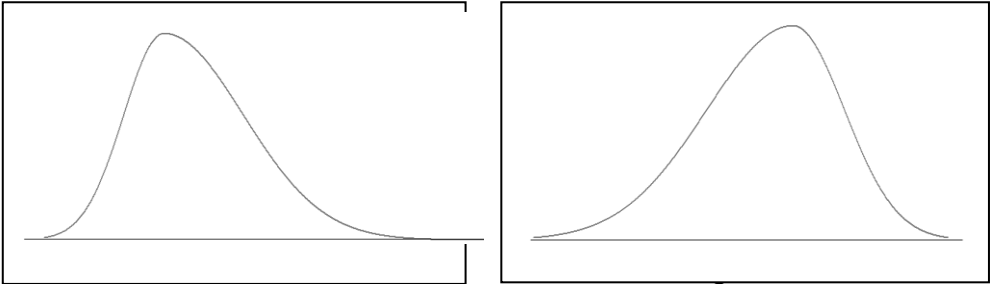
The following annotations are available:

	Blank Page – this annotation <b>must</b> be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓	correct response
✗	incorrect response
ecf	error carried forward
BOD	Benefit of doubt

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
1	$6x - (8x + 5) = 6x - 8x - 5 = -2x - 5$ $-2x$ $-5$	<p>[1]</p> <p>[1]</p>	
2	$x^2 + 5x - 14 = (x + 7)(x - 2)$ $(x + 7)$ $(x - 2)$	<p>[1]</p> <p>[1]</p>	
3	$[(x + 4)/8] - (x - 5)/2 = (x + 4 - 4x + 20)/8 \text{ or } ((2x + 8) - (8x - 40))/16$ $\text{or } (x + 4)/8 - (4x - 20)/8 \text{ or } (2x + 8)/16 - (8x - 40)/16$ $= (-3x + 24)/8 \text{ or } (-6x + 48)/16$ $(-3x + 24) \text{ or } (-6x + 48)$ $8 \quad \text{or } 16$	<p>[1]</p> <p>[1]</p>	<p>If denominator is subsequently removed from final answer i.e.</p> <p>final answer = <math>(-3x + 24)</math> or <math>(-6x + 48)</math></p> <p>award one mark only.</p> <p>Also accept <math>-3x/8 + 3</math> for 2 marks</p>
4	$6(3x - 5) = 2(4x - 7)$ $18x - 30 = 8x - 14$ $10x = 16$ $x = 1.6$	<p>[1]</p> <p>[1]</p>	<p>One mark for <math>18x - 30 = 8x - 14</math></p> <p>or <math>10x = 16</math></p> <p>or <math>-10x = -16</math></p> <p>CAO</p>

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
5	(a)		Award one mark for correct numerical results with or without the unit.
5	(b)	[1] [1]	Accept $\frac{2}{3}\pi$ or answers between 2.05 and 2.15 radians.
6	Sine wave.	[1]	Accept $y=\sin(x)$ or just $y=\sin$ or $\sin$
7	<p>In the triangle ABC angle A = <math>30^\circ</math> and length of side 'a' = 100 mm.</p> <p>Angle C = <math>90 - 30 = 60^\circ</math></p> <p>.Side c = <math>a/\tan A =</math></p> <p><math>100/\tan 30^\circ = 173.2</math> mm.</p> <p>-----</p> <p>Alternative solution</p> <p>Angle C = <math>90 - 30 = 60^\circ</math></p> <p><math>100/\sin 30 = c/\sin 60</math></p> <p><math>c = 100\sin 60/\sin 30 = 173.2</math> mm</p>	[1] [1] ----- or [1] [1]	Accept answers between 173 and 173.5 mm. Unit not required for full marks.
8	<p>Given cosine <math>x = 3/5</math>. </p> <p>By observation the right-angled triangle will have sides of 3, 4 and 5 units long.</p> <p>So sine <math>x = 4/5</math> or 0.8</p>	[1] [1]	One mark for triangle drawn with correct values.  CAO.

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
9	Given $y = 3 \cos x + 6x^3$ So $dy/dx = -3 \sin x + 18x^2$  $-3 \sin x$  $+ 18x^2$	  <b>[1]</b> <b>[1]</b>	Allow $-3 \sin x + 18x^2$  Allow $3(-) \sin x + 18x^2$
10	Given $y = \sin x + (1/x) = \sin x + x^{-1}$  Then $dy/dx = \cos x - x^{-2} = \cos x - (1/x^2)$  $\cos x$  $-(1/x^2)$ or $-x^{-2}$	  <b>[1]</b>  <b>[1]</b>	Allow $\cos(-)x - (1/x^2)$
11	$\int 2 \sin 5x \, dx = -0.4 \cos 5x + C$  $-0.4 \cos 5x$ or $-2/5 (\cos 5x)$  $+C$	  <b>[1]</b>  <b>[1]</b>	
12	$\int_2^5 4x^3 \, dx = \left[ \frac{4x^4}{4} \right]_2^5 = 5^4 - 2^4 = 625 - 16 = 609$  $\left[ \frac{x^4}{1} \right]_2^5$  $5^4 - 2^4$  609	  <b>[1]</b>  <b>[1]</b>  <b>[1]</b>	Do not accept error carried forward if incorrect integration is used.

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
13	<p>In a statistical distribution, the curve appears distorted either to the left or to the right.</p> 	<b>[2]</b>	<p>Diagram 1 mark</p> <p>Explanation 1 mark</p> <p>Award 2 marks for a clearly annotated diagram</p>
14	<p>Given 6 9 7 11 8 10 6</p> <p>Mode = 6</p> <p>Values arranged in order of magnitude:</p> <p>6 6 7 <u>8</u> 9 10 11</p> <p>Median point = 8</p>	<p><b>[1]</b></p> <p><b>[1]</b></p>	
15	<p><math>P(A)</math> = probability that event A happens.</p> <p>.</p> <p><math>P(B)</math> = probability that event B happens.</p> <p>.</p>	<p><b>[1]</b></p> <p><b>[1]</b></p>	<p>Accept similar explanations but these <b>MUST</b> include reference 'Probability' or 'Likelihood' or 'possibility' and must mention (event) A (and B).</p> <p>eg</p> <p>'likelihood of (event) A (happening)'</p> <p>Do not accept eg:</p> <p>'A will happen' or 'A happens' etc</p>
<b>Total</b>		<b>[30]</b>	

Question		Expected Answer	Mark	Rationale/Additional Guidance
<b>Section B</b>				
1	(a)	Given $W = (mv^2) / 2$ Substitute $m = 5$ and $v = 10$ .  Then $W = (5 \times 10^2) / 2$ $= 250$	[1]	
1	(b)	Given that $A = B(1 + 2CD)$ Open the brackets then $A = B + 2BCD$ Then $2BCD = A - B$ So $C = (A - B) / 2BD$  -----  Alternative solution $A/B = (1 + 2CD)$ $A/B - 1 = 2CD$ $C = (A/B - 1) / 2D$	[1] [1] [1]  ----- or [1] [1] [1]	Accept any other correct method.
1	(c)	Given $v^2 = u^2 + 2as$ Subtract both sides by $2as$ Then $u^2 = v^2 - 2as$ Square root both sides So $u = \sqrt{v^2 - 2as}$	[1]  [1]	Accept any other correct method.



Question		Expected Answer	Mark	Rationale/Additional Guidance
<b>Section B</b>				
1	(d)	Given $A = B/(B + 2)$ Multiply both sides by $(B + 2)$ then $AB + 2A = B$ Subtract both sides by $B$ then $AB - B = - 2A$ Then $B(A - 1) = - 2A$ So $B = - 2A/(A - 1)$ or $2A/(1 - A)$	[1] [1] [1] [1]	Accept any other correct method.
		<b>Total</b>	<b>[10]</b>	

Question	Expected Answer	Mark	Rationale/Additional Guidance
2 (a)	Area of triangle A = $\frac{1}{2}bh$ So perpendicular height $h = 2A/b$ $= (2 \times 1000)/40$ $= 50 \text{ mm.}$	[1]  [1]	Unit not required for full marks
2 (b)	Given $x = 8 \text{ m}$ , $y = 10 \text{ m}$ and $z = 12 \text{ m}$ .  Area of triangle A = $\sqrt{[s(s-x)(s-y)(s-z)]}$ where $s = (x + y + z)/2$ So $s = (8 + 10 + 12)/2 = 15$ Then Area A = $\sqrt{[15(15-8)(15-10)(15-12)]}$ $= \sqrt{1575}$ $= 39.69 \text{ m}^2$  -----  Alternative solutions  $x^2 = y^2 + z^2 - 2yz\cos X$ $\cos X = (x^2 - y^2 - z^2) / (-2yz)$ $\cos X = (8^2 - 10^2 - 12^2) / (-2 \times 10 \times 12) = 0.75$ $X = 41.41^\circ$ Area = $(10 \times 12 \times \sin 41.41) / 2$ $= 39.69 \text{ m}^2$  $y^2 = x^2 + z^2 - 2xz\cos Y$ $\cos Y = (y^2 - x^2 - z^2) / (-2xz)$ $\cos Y = (10^2 - 8^2 - 12^2) / (-2 \times 8 \times 12) = 0.56$ $Y = 55.77$ Area = $(8 \times 12 \times \sin 55.77) / 2$ $= 39.69 \text{ m}^2$	[1] [1] [1] [1] [1]  ----- or [1] [1] [1] [1] [1] or [1] [1] [1] [1]	BEWARE $\frac{1}{2}(8 \times 10) = 40$ (no marks for this)  Accept answers between $39 \text{ m}^2$ and $40 \text{ m}^2$ provided correct working is seen  Unit not required for full marks

Question	Expected Answer	Mark	Rationale/Additional Guidance
	$z^2 = x^2 + y^2 - 2xy\cos Z$ $\cos Z = (z^2 - x^2 - y^2) / (-2xy)$ $\cos Z = (12^2 - 8^2 - 10^2) / (-2 \times 8 \times 10) = 0.125$ $Z = 82.82^\circ$ $\text{Area} = (8 \times 10 \times \sin 82.82) / 2$ $= 39.69 \text{ m}^2$	or [1] [1] [1] [1] [1]	
2 (c)	Given angle B = $30^\circ$ , side a = 12 m and side c = 16 m.  Area of triangle A = $\frac{1}{2}ac \sin B$ $= \frac{1}{2} \times 12 \times 16 \sin 30^\circ$ $= 96 \times 0.5$ $= 48 \text{ m}^2$	[1] [1] [1]	Unit not required for full marks.
	<b>Total</b>	<b>[10]</b>	

Question	Expected Answer	Mark	Rationale/Additional Guidance
3 (a)	Given $s = 60(20e^{-t/20} + t - 20)$ . Velocity = $ds/dt = 60(-e^{-t/20} + 1)$ Acceleration = $dv/dt = d^2s/dt^2 = 60(e^{-t/20}/20)$ When $t = 10s$ Acceleration = $3(e^{-10/20})$ = $1.82 \text{ ms}^{-2}$	[1] [1] [1] [1]	Unit not required for full marks
3 (b)	Given $S = 2\pi r^2 + 400/r = 2\pi r^2 + 400r^{-1}$ $ds/dr = 4\pi r - 400r^{-2} = 4\pi r - 400/r^2$ Surface area is a minimum when $ds/dr = 0$ So $4\pi r - 400/r^2 = 0$ $4\pi r = 400/r^2$ $4\pi r^3 = 400$ $r^3 = 400/4\pi$ So $r = 3.17$	[1] [1] [1] [1] [1]	Allow solutions by trial and error (4 marks) for answers between 3.0 and 3.3 (2 marks)
	<b>Total</b>	<b>[10]</b>	

Question			Expected Answer			Mark	Rationale/Additional Guidance
4	(a)	(i)	Time (minutes)	Tally	Cumulative frequency (f)	[1]	
			$0 < t \leq 20$	8	8		
			$20 < t \leq 40$	18	26		
			$40 < t \leq 60$	15	41		
			$60 < t \leq 80$	5	46		
			$80 < t \leq 100$	3	49		
			$100 < t \leq 120$	1	50		



Question			Expected Answer	Mark	Rationale/Additional Guidance
5	(a)	(i)	<p>Given <math>v = u + at</math>.</p> <p><math>v = 60</math> when <math>t = 10</math> and <math>v = 30</math> when <math>t = 4</math>.</p> <p>So <math>60 = u + 10a</math> Equation 1</p> <p>And <math>30 = u + 4a</math> Equation 2</p> <p>Subtract eqn 2 from eqn 1</p> <p>Then <math>30 = 6a</math></p> <p>So <math>a = 30/6 = 5</math></p> <p>Substitute <math>a = 5</math> into eqn 1</p> <p>So <math>60 = u + (10 \times 5)</math></p> <p>Then <math>u = 60 - 50 = 10</math></p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>	
5	(a)	(ii)	<p>When <math>t = 6</math> use equation <math>v = 10 + 5t</math></p> <p><math>v = 10 + (5 \times 6)</math></p> <p><math>= 40</math></p>	[1]	
5	(b)		<p>Given <math>3x^2 + 14x + 8 = 0</math>.</p> <p>By inspection <math>3x^2 = (3x)(x)</math></p> <p>And <math>8 = (1)(8)</math> or <math>(2)(4)</math></p> <p>By factorisation <math>(3x + 2)(x + 4) = 0</math></p> <p>So <math>(3x + 2) = 0</math> or <math>(x + 4) = 0</math></p> <p>When <math>3x + 2 = 0</math></p> <p>Then <math>3x = -2</math> so <math>x = -2/3</math> or <math>0.666\dots</math> or <math>0.67</math></p> <p>Likewise when <math>(x + 4) = 0</math></p> <p>Then <math>x = -4</math></p>	<p>[1]</p> <p>[1]</p> <p>[1]</p>	

Question	Expected Answer	Mark	Rationale/Additional Guidance
	Alternative method $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-14 \pm \sqrt{14^2 - 4 \times 3 \times 8}}{2 \times 3}$ $x = \frac{-14 \pm \sqrt{14^2 - 4 \times 3 \times 8}}{2 \times 3}$ $x = \frac{-14 \pm \sqrt{100}}{6}$ $x = \frac{-14 \pm 10}{6}$ $x = -4/6 \text{ or } -2/3 \text{ or } 0.666\dots \text{ or } 0.67$ $x = -24/6 \text{ or } -4$	or  [1]       [1]  [1]	
	<b>Total</b>	<b>[10]</b>	



Question		Expected Answer	Mark	Rationale/Additional Guidance
6	(a)	<p>In a triangle ABC.            AB = 5 m, AC = 8 m and angle ABC = <math>42^\circ</math>.</p> <p>Let angle ABC be angle B.            Let angle ACB be angle C.</p> <p>Sine Rule: <math>AC/\sin B = AB/\sin C</math>            so <math>\sin C = (AB \sin B)/AC</math>            Then <math>\sin C = (5 \sin 42^\circ)/8 = 0.4182</math>            So <math>C = \sin^{-1} 0.4182 = 24.72^\circ</math>            So Angle ACB is <math>24.72^\circ</math></p>	<p>[1]            [1]            [1]            [1]</p>	Accept answers between $24$ and $25^\circ$ .
6	(b)	<p>In a triangle ABC.            Side a = 40 mm, side b = 70 mm and angle C = <math>50^\circ</math></p> <p>Cosine Rule: <math>\cos C = (a^2 + b^2 - c^2)/2ab</math>            So <math>c^2 = a^2 + b^2 - 2ab \cos C</math>  <math>= (40^2 + 70^2 - [2 \times 40 \times 70 \cos 50^\circ])</math>  <math>= 1600 + 4900 - [5600 \times 0.6428]</math>  <math>= 1600 + 4900 - 3600</math>  <math>= 2900</math>            So <math>c = \sqrt{2900} = 53.85</math> mm.</p>	<p>[1]            [1]            [1]            [1]</p>	Accept answers between 53 and 54mm.
6	(c)	<p>Given: <math>\sin 30^\circ + \cos 60^\circ + \tan 45^\circ = 2</math>            LHS = <math>0.5 + 0.5 + 1 = 2</math>            RHS = 2            Therefore <math>2 = 2</math> QED</p>	<p>[1]            [1]</p>	
<b>Total</b>			<b>[10]</b>	

Question		Expected Answer	Mark	Rationale/Additional Guidance
7	(a)	Integrate $\cos 2x + 1/x^2 + \sqrt{x}$ with respect to $x$ . So $\int \cos 2x + 1/x^2 + \sqrt{x} \, dx = \int \cos 2x + x^{-2} + x^{0.5} \, dx$ $= \frac{1}{2} \sin 2x + (x^{-1})/-1 + (x^{1.5})/1.5 + C$ $= \frac{1}{2} \sin 2x - 1/x + \frac{2}{3} \sqrt{x^3} + C$	[5]	Award one mark for $\int \cos 2x + x^{-2} + x^{0.5} \, dx$ . Award one mark for $\frac{1}{2} \sin 2x$ Award one mark for $-1/x$ or $-x^{-1}$ Award one mark for $+ \frac{2}{3}$ or $1/1.5$ Award one mark for $\sqrt{x^3}$ or $x^{3/2}$ or $x^{1.5}$ +C is not required for full marks
7	(b)	Given $v = 4 + 6t$ where $t_1 = 0$ and $t_2 = 5$ . $\text{Distance} = \int_0^5 (4 + 6t) \, dt$ $= [4t + 3t^2]_0^5$ $= (4 \times 5) + (3 \times 5^2) - 0$ $= 95 \text{ metres}$	[1]  [1]  [1]	$\text{Award one mark for Distance} = \int_0^5 (4 + 6t) \, dt.$ $\text{Award one mark for } [4t + 3t^2]_0^5$ Unit not required for full marks

Question		Expected Answer	Mark	Rationale/Additional Guidance
7	(c)	<p>Given <math>F = 8s - s^2</math> where <math>s_1 = 2</math> and <math>s_2 = 6</math></p> $\text{Work done} = \int_2^6 (8s - s^2) ds$ $= [4s^2 - \frac{1}{3}s^3]_2^6$ $= ((4 \times 6^2) - \{\frac{1}{3} \times 6^3\}) - ((4 \times 2^2) - \{\frac{1}{3} \times 2^3\})$ $= (144 - 72) - (16 - 2\frac{2}{3})$ $= 72 - 13\frac{1}{3}$ $= 58\frac{2}{3} \text{ or } 58.666\dots \text{ or } 58.67 \text{ newtons}$	<p>[1]</p> <p>[1]</p>	
		Total	[10]	

Question		Expected Answer	Mark	Rationale/Additional Guidance
8	(a)	<p>An event that is affected by previous events.</p> <p>Suitable example involving an experiment 'without replacement' or equivalent.</p>	<p>[1]</p> <p>[1]</p>	
8	(b)	<p>Total number of screws = <math>110 + 120 + 70 = 300</math>.            Assume that three separate draws take place.            Draw One:            Probability of selecting a steel screw is <math>120/300</math>.            Draw Two:            There are now 119 steel screws in a total of 299            Probability of selecting a steel screw is <math>119/299</math>.            Draw Three:            There are now 118 steel screws in a total of 298.            Probability of selecting a steel screw is <math>118/298</math></p> <p>The probability of choosing a steel screw on the first draw and the second and the third draw is:  <math>(120/300) \times (119/299) \times (118/298)</math> is 0.063</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>	

Question		Expected Answer	Mark	Rationale/Additional Guidance
8	(c)	Mutually exclusive Not Mutually exclusive Not Mutually exclusive Mutually exclusive	[4]	Award one mark for each correct response.
		<b>Total</b>	[1]	

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