GCE

Mathematics (MEI)

Unit **4753**: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2015

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a   Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance	
1	$y = e^{2x}\cos x$	M1	product rule used	consistent with their derivs
	$\Rightarrow$ $dy/dx = 2e^{2x}\cos x - e^{2x}\sin x$	A1	cao – mark final ans	e.g. $2e^{2x} - e^{2x}\tan x$ is A0
	$dy/dx = 0 \Rightarrow e^{2x}(2\cos x - \sin x) = 0$	M1	their derivative = 0	
	$\Rightarrow$ $2\cos x = \sin x$			
	$\Rightarrow$ $2 = \sin x/\cos x = \tan x$	M1	$\sin x / \cos x = \tan x$ used	or $\sin^2 x + \cos^2 x = 1$ used
	$\Rightarrow$ $x = 1.11$	A1	1.1 or 0.35π or better, or arctan 2, not 63.4° but condone ans given in both degrees and radians here	1.1071487 …, 0.352416…π, penalise incorrect rounding
	$\Rightarrow$ $y = 4.09$	A1cao	art 4.1	no choice
		[6]		
2	let $u = 2x - 1$, $du = 2\,dx$			
	$\int \sqrt[3]{2x-1}\;dx = \int \frac{1}{2}u^{\frac{1}{3}}\,du$	M1 M1	substituting $u = 2x - 1$ in integral $\times\,½$ o.e.	i.e. $u^{1/3}$ or $\sqrt[3]{u}$ seen in integral condone no du, or dx instead of du
	$= \frac{3}{8}u^{\frac{4}{3}} + c$	M1	integral of $u^{1/3} = u^{4/3}/(4/3)$ (oe) soi	not $x^{1/3}$
	$= \frac{3}{8}(2x-1)^{\frac{4}{3}} + c$	A1cao	o.e., but must have $+ c$ and single fraction mark final answer	so $\frac{3}{4}(2x-1)^{\frac{4}{3}} + c$ is M1M0M1A0
	or			
	$\int \sqrt[3]{2x-1}\;dx = ½ \times (2x-1)^{4/3} \div 4/3$	M1 M1 M1	$(2x-1)^{4/3}$ seen $\div\,4/3$ (oe) soi $\times\,½$	e.g. correct power of $(2x-1)$ e.g. ¾ $(2x-1)^{4/3}$ seen
	$= \frac{3}{8}(2x-1)^{\frac{4}{3}} + c$	A1cao	o.e., but must have $+ c$ and single fraction mark final ans	so $\frac{3}{8}(2x-1)^{\frac{4}{3}}$ is M1M1M1A0
		[4]		

Question	Answer	Marks	Guidance	
3	let $u = \ln x$, $\mathrm{d}v/\mathrm{d}x = x^3$, $\mathrm{d}u/\mathrm{d}x = 1/x$, $v = \frac{1}{4} x^4$	M1	u, u', v', v all correct	
	$\int_1^2 x^3 \ln x \, \mathrm{d}x = \left[\frac{1}{4} x^4 \ln x\right]_1^2 - \int_1^2 \frac{1}{4} x^4 . \frac{1}{x} \mathrm{d}x$	A1	$\frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 . \frac{1}{x}[\mathrm{d}x]$	ignore limits
	$= \left[\frac{1}{4} x^4 \ln x\right]_1^2 - \int_1^2 \frac{1}{4} x^3 \mathrm{d}x$	M1dep	simplifying $x^4 / x = x^3$ in second term (soi)	dep 1st M1
	$= \left[\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4\right]_1^2$	A1cao	$\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4$ o.e.	
	$= 4\ln 2 - 15/16$	A1cao	o.e. must be exact, but can isw	must evaluate $\ln 1 = 0$ and combine $-1 + 1/16$
		[5]		
4	$h = r$ so $V = \pi h^3/3$	B1	o.e. e.g $\pi h^3 \tan 45° /3$	
	$\mathrm{d}V/\mathrm{d}t = 5$	B1	soi (can be implied from $V = 5t$)	e.g. from a correct chain rule
	$\mathrm{d}V/\mathrm{d}h = \pi h^2$	B1ft	must be $\mathrm{d}V/\mathrm{d}h$ soi, ft their $\pi h^3/3$	but must have substituted for r
	$\mathrm{d}V/\mathrm{d}t = (\mathrm{d}V/\mathrm{d}h) . \mathrm{d}h/\mathrm{d}t$	M1	any correct chain rule in V, h and t (soi)	e.g. $\mathrm{d}h/\mathrm{d}t = \mathrm{d}h/\mathrm{d}V \times \mathrm{d}V/\mathrm{d}t$,
	$\Rightarrow \quad 5 = 100\pi \, \mathrm{d}h/\mathrm{d}t$			
	$\Rightarrow \quad \mathrm{d}h/\mathrm{d}t = 5/100\pi = 0.016 \text{ cm s}^{-1}$	A1	0.016 or better; accept $1/(20\pi)$ o.e., but mark final answer	0.01591549... penalise incorrect rounding
	or $V = 5t$ so $\pi h^3/3 = 5t$	B1		
	$\Rightarrow \pi h^2 \mathrm{d}h/\mathrm{d}t = 5$	M1	or $5 \, \mathrm{d}t/\mathrm{d}h = \pi h^2$ o.e.	
	$\Rightarrow \mathrm{d}h/\mathrm{d}t = 5/\pi h^2 = 5/100\pi = 0.016 \text{ cm s}^{-1}$	A1	0.016 or better; accept $1/(20\pi)$ o.e., but mark final answer	penalise incorrect rounding
		[5]		

	Question		Answer	Marks	Guidance	
5			$y^2 + 2x \ln y = x^2$ $1^2 + 2\times1\times\ln1 = 1^2$ so (1, 1) lies on the curve.	B1	clear evidence of verification needed	at least "1 + 0 = 1"
				M1	d/dx (y^2) = 2ydy/dx	must be correct
				M1	d/dx (2x ln y) = 2ln y + 2x/y dy/dx	must be correct
			$2y\dfrac{dy}{dx} + 2\ln y + 2x.\dfrac{1}{y}.\dfrac{dy}{dx} = 2x$	A1 cao		condone dy/dx = … unless pursued
			$[\Rightarrow \dfrac{dy}{dx} = \dfrac{2x - 2\ln y}{2y + 2x/y}$]			
			when $x = 1$, $y = 1$, $\dfrac{dy}{dx} = \dfrac{2 - 2\ln 1}{2 + 2}$	M1	substituting both $x = 1$ and $y = 1$ into their dy/dx or their equation in x, y and dy/dx	$2\dfrac{dy}{dx} + 2\ln1 + 2\dfrac{dy}{dx} = 2$
			$= \frac{1}{2}$	A1cao	not from wrong working	
				[6]		
6	**(i)**		arcsin $x = \pi/6 \Rightarrow x = \sin \pi/6$	M1		
			$= \frac{1}{2}$	A1	allow unsupported answers	
				[2]		
6	**(ii)**		sin $\pi/4 = \cos \pi/4 = 1/\sqrt2$ $\Rightarrow$ arcsin $(1/\sqrt2) =$ arccos $(1/\sqrt2)$ $\Rightarrow$ $x = 1/\sqrt2$	B2	o.e. e.g. $\sqrt2/2$, must be exact; SCB1 0.707…	
				[2]		
7	**(i)**		$\mathrm{ff}(x) = \mathrm{f}(\dfrac{1-x}{1+x}) = \dfrac{1 - \dfrac{1-x}{1+x}}{1 + \dfrac{1-x}{1+x}}$	M1	substituting $(1-x)/(1+x)$ for x in f(x)	
			$= \dfrac{1 + x - 1 + x}{1 + x + 1 - x} = \dfrac{2x}{2} = x$*	A1	correctly simplified to x **NB AG**	
			$\mathrm{f}^{-1}(x) = \mathrm{f}(x) = (1-x)/(1+x)$	B1	or just f$^{-1}(x) = $ f(x)	

Question			Answer	Marks	Guidance	
				[3]		
7	(ii)		$g(-x) = \dfrac{1-(-x)^2}{1+(-x)^2}$	M1	substituting $-x$ for x in g(x) condone use of 'f' for g	if brackets are omitted or misplaced allow M1A0
			$= \dfrac{1-x^2}{1+x^2} = g(x)$	A1	must indicate that $g(-x) = g(x)$ somewhere	condone use of 'f' for g
			Graph is symmetrical about the y-axis.	B1	allow 'reflected', 'reflection' for symmetrical	must state axis (y-axis or $x = 0$)
				[3]		
8	(i)		$f'(x) = \dfrac{x.2(x-2)-(x-2)^2}{x^2}$	M1	quotient (or product) rule, condone sign errors only	e.g. $\dfrac{\pm x.2(x-2)\pm(x-2)^2}{x^2}$
				A1	correct exp, condone missing brackets here	PR: $(x-2)^2.(-x^{-2}) + (1/x).2(x-2)$
			$= \dfrac{2x^2-4x-x^2+4x-4}{x^2}$			
			$= (x^2 - 4)/x^2 = 1 - 4/x^2$ *	A1	simplified correctly **NB AG**	with correct use of brackets
			or f(x) = $(x^2 - 4x + 4)/x$			
			$= x - 4 + 4/x$	M1	expanding bracket and dividing each term by x	must be 3 terms: $(x^2 - 4)/x$ is M0
				A1	correctly	e.g. $x - 4 + 2/x$ is M1A0
			$\Rightarrow$ f'(x) = 1 $-4/x^2$ *	A1	not from wrong working **NB AG**	
			$f''(x) = 8/x^3$	B1	o.e. e.g. $8x^{-3}$ or $8x/x^4$	
			f'(x) = 0 when $x^2 = 4$, $x = \pm2$	M1	$x = \pm2$ found from $1 - 4/x^2 = 0$	allow for $x = -2$ unsupported
			so at Q, $x = -2$, $y = -8$.	A1	$(-2, -8)$	
			f''(-2) [$= -1$] < 0 so maximum	B1dep	dep first B1. Can omit -1, but if shown must be correct. Must state < 0 or negative.	must use 2nd derivative test
				[7]		

Question			Answer	Marks	Guidance	
8	(ii)		$f(1) = (-1)^2/1 = 1$ $f(4) = (2)^2/4 = 1$	B1	verifying f(1) = 1 and f(4) = 1	or $(x-2)^2 = x \Rightarrow x^2 - 5x + 4 = 0$ $\Rightarrow (x-1)(x-4) = 0$, $x = 1, 4$
			$\int_1^4 \frac{(x-2)^2}{x}\,dx = \int_1^4 (x - 4 + 4/x)\,dx$	M1	expanding bracket and dividing each term by x 3 terms: $x - 4/x$ is M0	if $u = x - 2$ $\int \frac{u^2}{u+2}\,du = \int (u - 2 + \frac{4}{u+2})\,du$
			$= \left[x^2/2 - 4x + 4\ln x \right]_1^4$	A1	$x^2/2 - 4x + 4\ln x$	$u^2/2 - 2u + 4\ln(u+2)$
			$= (8 - 16 + 4\ln 4) - (\tfrac{1}{2} - 4 + 4\ln 1)$ $= 4\ln 4 - 4\tfrac{1}{2}$	A1cao		
			Area enclosed = rectangle − curve	M1	soi	
			$= 3 \times 1 - (4\ln 4 - 4\tfrac{1}{2}) = 7\tfrac{1}{2} - 4\ln 4$	A1cao	o.e. but must combine numerical terms and evaluate ln 1 – mark final ans	
			or Area $= \int_1^4 [1 - \frac{(x-2)^2}{x}]\,dx$	M1	no need to have limits	
			$= \int_1^4 (5 - x - 4/x)\,dx$	M1 A1	expanding bracket and dividing each term by x $5 - x - 4/x$	must be 3 terms in $(x-2)^2$ expansion
			$= \left[5x - x^2/2 - 4\ln x \right]_1^4$	A1	$5x - x^2/2 - 4\ln x$	
			$= 20 - 8 - 4\ln 4 - (5 - \tfrac{1}{2} - 4\ln 1)$ $= 7\tfrac{1}{2} - 4\ln 4$	A1cao **[6]**	o.e. but must combine numerical terms and evaluate ln 1 – mark final ans	
8	(iii)		$[g(x) =]\ f(x+1) - 1$	M1	soi [may not be stated]	
			$= \frac{(x+1-2)^2}{x+1} - 1$	A1		
			$= \frac{x^2 - 2x + 1 - x - 1}{x+1} = \frac{x^2 - 3x}{x+1}$ *	A1	correctly simplified – not from wrong working **NB AG**	
				[3]		

Question			Answer	Marks	Guidance	
8	**(iv)**		Area is the same as that found in part (ii)	M1	award M1 for $\pm$ ans to 8(ii) (unless zero)	
			$4\ln 4 - 7\frac{1}{2}$	A1cao	need not justify the change of sign	
				[2]		
9	**(i)**		At P, $(e^x - 2)^2 - 1 = 0$ $\Rightarrow$ $e^x - 2 = [\pm]1$, $e^x = [1 \text{ or }] 3$	M1	square rooting – condone no $\pm$	
			or $(e^x)^2 - 4e^x + 3 = 0$ $\Rightarrow (e^x - 1)(e^x - 3) = 0$, $e^x = 1$ or 3	M1	expanding to correct quadratic and solve by factorising or using quadratic formula	condone e^x^2
			$\Rightarrow x = [0 \text{ or }] \ln 3$	A1	x-coordinate of P is ln 3; must be exact	condone P = ln 3, but not $y = \ln 3$
				[2]		
9	**(ii)**		$f'(x) = 2(e^x - 2)e^x$	M1	chain rule	e.g. $2u \times$ their deriv of e^x
				A1	correct derivative	$2(e^x - 2)x$ is M0
			$= 0$ when $e^x = 2$, $x = \ln 2$ *	A1	not from wrong working **NB AG**	or verified by substitution
			or $f(x) = e^{2x} - 4e^x + 3$	M1	expanding to 3 term quadratic with $(e^x)^2$ or e^{2x}	condone e^x^2
			$\Rightarrow f'(x) = 2e^{2x} - 4e^x$	A1	correct derivative, not from wrong working	
			$= 0$ when $2e^{2x} = 4e^x$, $e^x = 2$, $x = \ln 2$ *	A1	or $2e^x(e^x - 2) = 0 \Rightarrow e^x = 2$, $x = \ln 2$ not from wrong working **NB AG**	or verified by substitution
			$y = f(\ln(2)) = -1$	B1		
				[4]		

| Question	Answer	Marks	Guidance
9 (iii)	$\int_0^{\ln 3}[(e^x - 2)^2 - 1]dx = \int_0^{\ln 3}[(e^x)^2 - 4e^x + 4 - 1]dx$	M1	expanding brackets must have 3 terms: $(e^x)^2 - 4$ is M0, condone e^x^2	or if $u = e^x$, $\int_1^3 [u^2 - 4u + 4 - 1]/u\,du$
	$= \int_0^{\ln 3}[e^{2x} - 4e^x + 3]dx$	A1	$\int e^{2x} - 4e^x + 3$ [dx] (condone no dx)	$= \int u - 4 + 3/u\,du$
	$= \left[\dfrac{1}{2}e^{2x} - 4e^x + 3x\right]_0^{\ln 3}$	B1 A1ft	$\int e^{2x} = \frac{1}{2}\,e^{2x}$ $[\frac{1}{2}\,e^{2x} - 4e^x + 3x]$	$= [\ \frac{1}{2}\,u^2 - 4u + 3\ln u]$
	$= (4.5 - 12 + 3\ln 3) - (0.5 - 4)$			
	$= 3\ln 3 - 4$ [so area $= 4 - 3\ln 3$]	A1	condone $3\ln 3 - 4$ as final ans; mark final ans	
		[5]		
9 (iv)	$y = (e^x - 2)^2 - 1$ $x \leftrightarrow y$ $x = (e^y - 2)^2 - 1$ $\Rightarrow$ $x + 1 = (e^y - 2)^2$	M1	attempt to solve for y (might be indicated by expanding and then taking lns)	or x if x and y not interchanged yet or adding (or subtracting) 1
	$\Rightarrow$ $\pm\sqrt{(x + 1)} = e^y - 2$ (+ for $y \geq \ln 2$) $\Rightarrow$ $2 + \sqrt{(x + 1)} = e^y$	A1	condone no $\pm$	
	$\Rightarrow$ $y = \ln(2 + \sqrt{(x + 1)}) = f^{-1}(x)$ Domain is $x \geq -1$ Range is $y \geq \ln 2$	A1 B1 B1	must have interchanged x and y in final ans must be $\geq$ and x (not y) or $f^{-1}(x) \geq \ln 2$, must be $\geq$ (not x or $f(x)$) if $x > -1$ and $y > \ln 2$ SCB1	if not specified, assume first ans is domain and second range
		M1 A1	recognisable attempt to reflect curve, or any part of curve, in $y = x$ good shape, cross on $y = x$ (if shown), correct domain and range indicated. [see extra sheet for examples]	$y = x$ shown indicative but not essential e.g. -1 and $\ln 2$ marked on axes
		[7]		

Appendix 1 Annotation notes

1. All questions on practice and standardisation scripts should be fully annotated, unless they are all correct or worth no credit.
2. For all other scripts, each part question should be annotated according to the following guidelines:

    - if part marks awarded, annotate fully or:
    - If fully correct, one tick; if worth no credit, one cross, or a yellow line
    - if one mark only is lost, you can indicate this by A0 or B0 in the appropriate place
    - if 1 mark only is earned, you can indicate this by M1 or B1 in the appropriate place
    - if M0, then you need not annotate dependent A marks

3. The following questions can be divided into sub-parts, which can be treated as above:

    - 8(i) First 3 marks, last 4 marks
    - 8(ii) First 4 marks, last 2 marks
    - 9(iv) first 3 marks, next 2 marks, last 2 marks

NB Please annotate all blank pages with a 'BP' mark (especially AOs), and blank answer spaces with a mark to show they have been seen (e.g. with a tick, a carat or yellow line) . Don't forget to scroll down to the bottom of each page, and annotate this if necessary to show this. Also, please indicate you have seen the spare copy of the graph in 9(iv)

Please put the annotations close to where they apply, rather than using the margins.

Appendix 2: Marked examples of 9(iv)
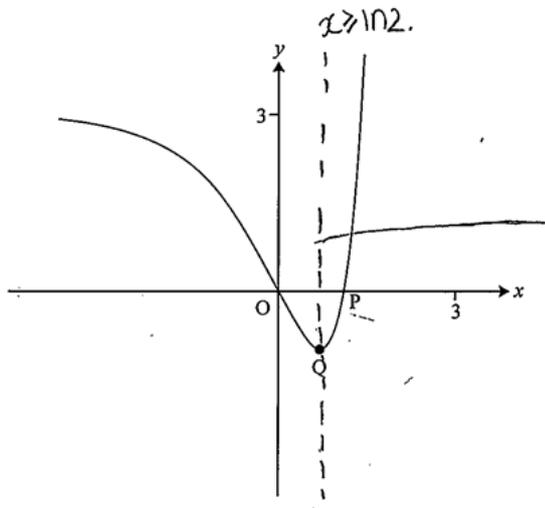

M1A0 (incorrect domain)
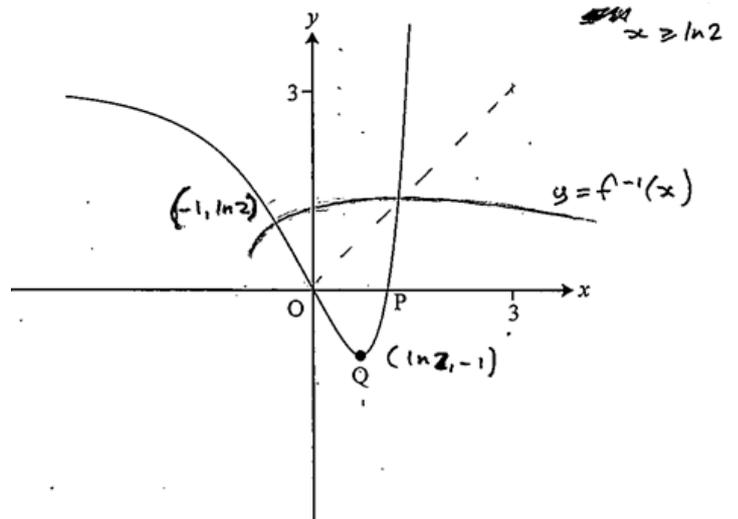

M1A1 (condone –ve gradient for x > 3)
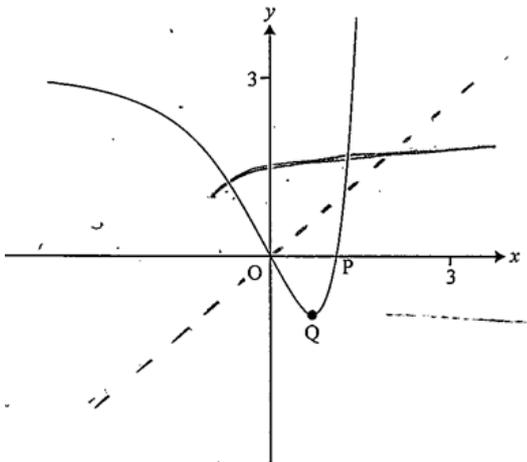

M1 A0 (domain not indicated)
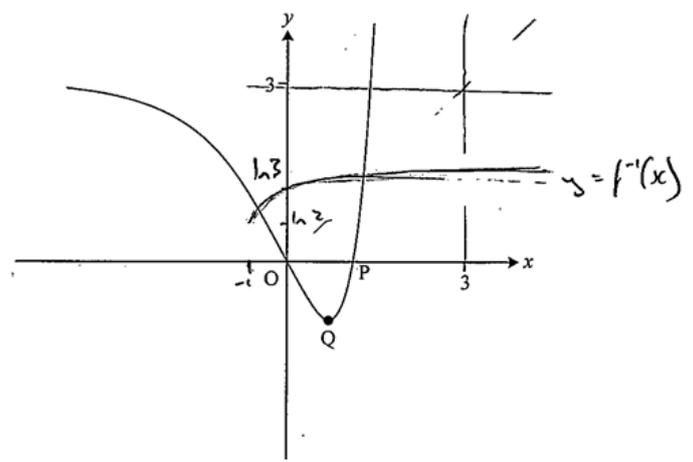
M1 A0
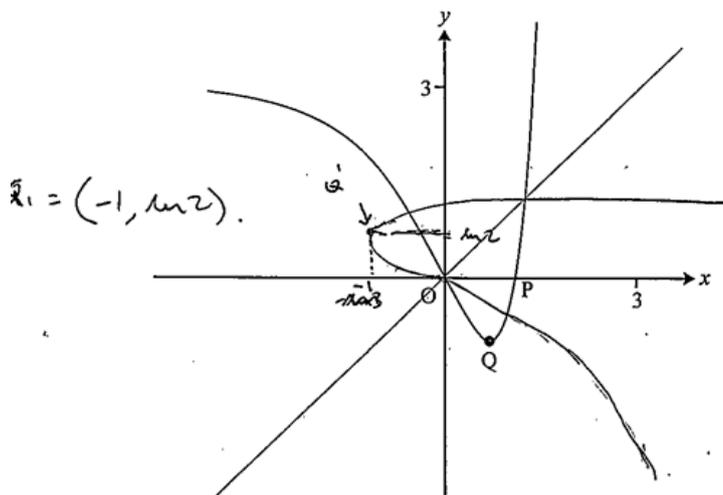
M1 A1
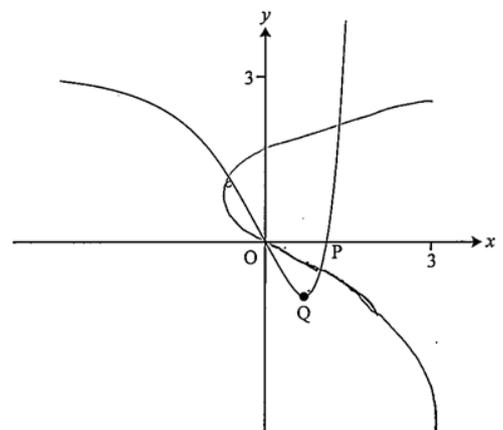
M1 A0
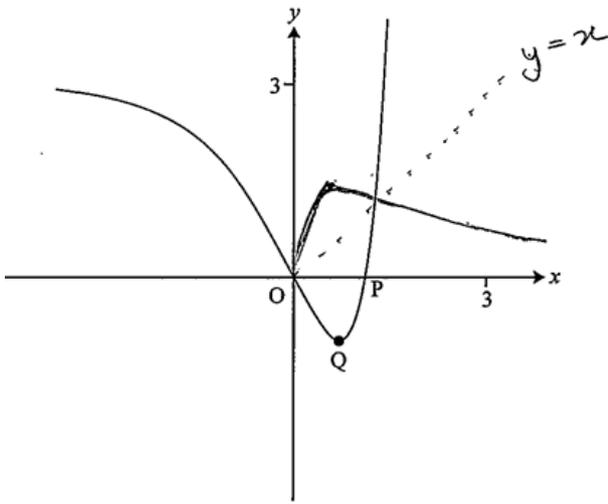
M1A0
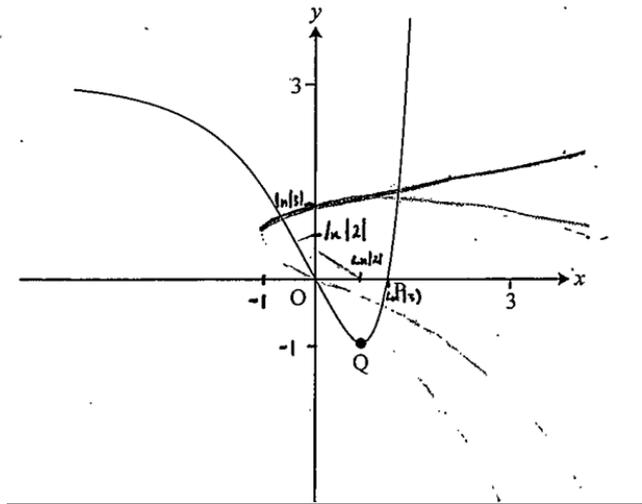
M1bodA1 (condone gradient at (-1, ln2)
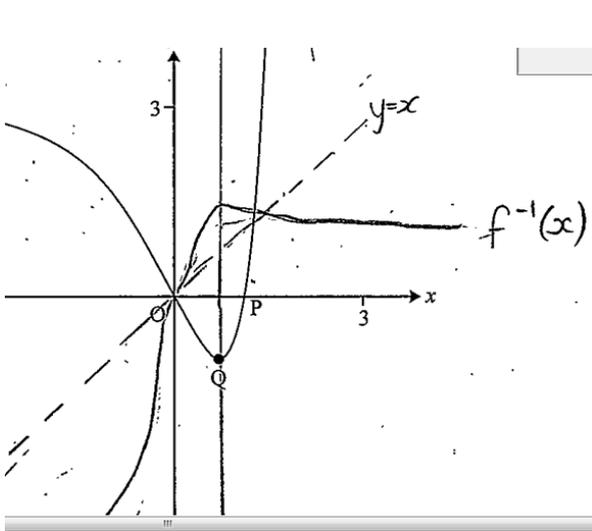
M1 A0
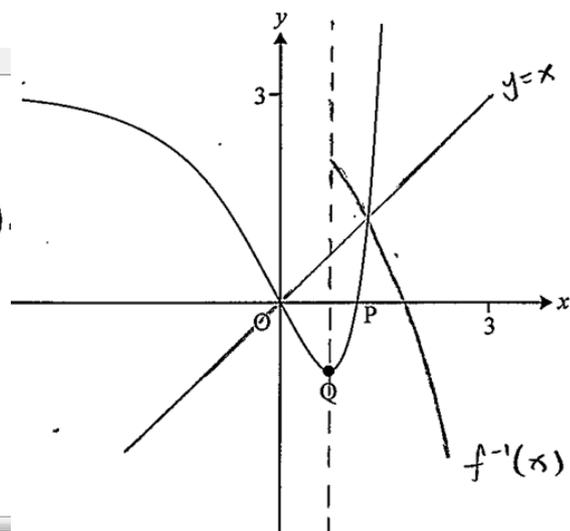
M1 A0
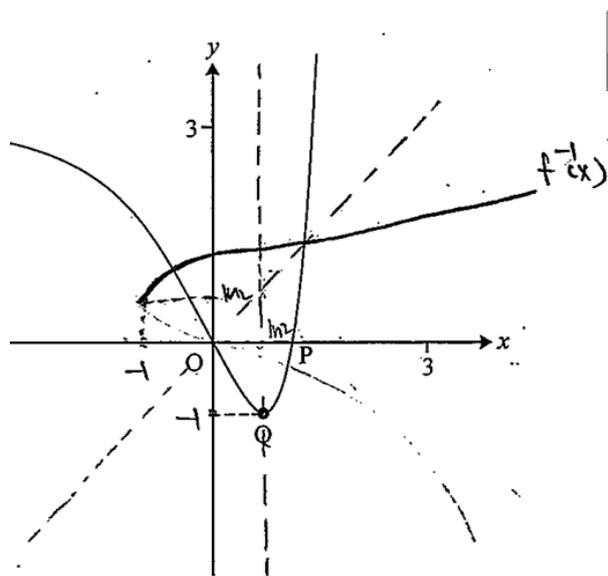
M1 A1
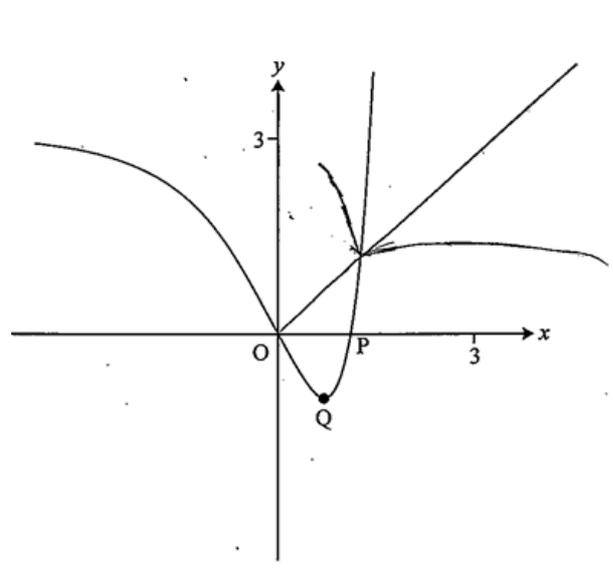
bod M1 A0 (evidence of reflection of part of curve here)

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