OCR
Oxford Cambridge and RSA

Friday 20 May 2016 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER
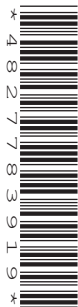
Candidates answer on the Printed Answer Book.

OCR supplied materials:
* Printed Answer Book 4725/01
* List of Formulae (MF1)

Other materials required:
* Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

* The Question Paper will be found inside the Printed Answer Book.
* Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
* **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
* Use black ink. HB pencil may be used for graphs and diagrams only.
* Answer **all** the questions.
* Read each question carefully. Make sure you know what you have to do before starting your answer.
* Do **not** write in the bar codes.
* You are permitted to use a scientific or graphical calculator in this paper.
* Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

* The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
* **You are reminded of the need for clear presentation in your answers.**
* The total number of marks for this paper is **72**.
* The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

* Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Find $\sum_{r=1}^{n}(3r+1)(r-1)$, giving your answer in a fully factorised form. **[5]**

2 The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{1}{3}\pi$. Giving your answers in the form $x+iy$, where x and y are exact real numbers, and showing clearly how you obtain them, find

   (i) z, **[2]**

   (ii) $\dfrac{1}{(z^*-5i)^2}$. **[5]**

3 The quadratic equation $kx^2+x+k=0$ has roots α and β.

   (i) Write down the values of $\alpha+\beta$ and $\alpha\beta$. **[1]**

   (ii) Find the value of $\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)$ in terms of k. **[5]**

4 The matrices **A**, **B** and **C** are given by $\mathbf{A}=(a\ \ 2\ \ 3)$, $\mathbf{B}=(b\ \ 0\ \ 5)$ and $\mathbf{C}=\begin{pmatrix}6\\4\\-1\end{pmatrix}$. Find

   (i) $5\mathbf{A}-3\mathbf{B}$, **[2]**

   (ii) $\mathbf{BC}$, **[2]**

   (iii) $\mathbf{CA}$. **[2]**

5 The sequence $u_1, u_2, u_3, \ldots$ is defined by
$$u_1 = 5 \text{ and } u_{n+1} = 3u_n+2 \text{ for } n \geqslant 1.$$

   Prove by induction that $u_n = 2\times 3^n - 1$. **[4]**

6 In an Argand diagram the points A and B represent the complex numbers $5+4i$ and $1+2i$ respectively.

   (i) Given that A and B are the ends of a diameter of a circle C, find the equation of C in complex number form. **[4]**

   The perpendicular bisector of AB is denoted by l.

   (ii) Sketch C and l on a single Argand diagram. **[2]**

   (iii) Find the complex numbers represented by the points of intersection of C and l. **[3]**

7 The matrix $\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}$ represents a transformation P.

   (i) Describe fully the transformation P. **[2]**

   The matrix **M** is given by $\mathbf{M}=\begin{pmatrix}-3 & -1\\-1 & 0\end{pmatrix}$.

   (ii) Given that **M** represents transformation Q followed by transformation P, find the matrix that represents the transformation Q and describe fully the transformation Q. **[6]**

8 **(i)** Show that $\dfrac{1}{2r+1} - \dfrac{1}{2r+3} \equiv \dfrac{2}{(2r+1)(2r+3)}$. **[1]**

(ii) Hence find $\displaystyle\sum_{r=1}^{n} \dfrac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[6]**

(iii) Find $\displaystyle\sum_{r=n}^{\infty} \dfrac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[3]**

9 **(i)** The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} a & 3 & -2 \\ 0 & a & 5 \\ 1 & 2 & 1 \end{pmatrix}$. Show that the determinant of $\mathbf{X}$ is $a^2 - 8a + 15$. **[3]**

(ii) Explain briefly why the equations

$$3x + 3y - 2z = 1$$
$$3y + 5z = 5$$
$$x + 2y + z = 2$$

do not have a unique solution and determine whether these equations are consistent or inconsistent. **[3]**

10 **(i)** Use an algebraic method to find the square roots of the complex number $9 + 40i$. **[6]**

(ii) Show that $9 + 40i$ is a root of the quadratic equation $z^2 - 18z + 1681 = 0$. **[1]**

(iii) By using the substitution $z = \dfrac{1}{u^2}$, find the roots of the equation $1681u^4 - 18u^2 + 1 = 0$. Give your answers in the form $x + iy$, where x and y are real. **[4]**

END OF QUESTION PAPER