OCR
Oxford Cambridge and RSA

Friday 17 June 2016 – Afternoon

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Answer **all** the questions.

1 In this question, give all non-real numbers in the form $re^{i\theta}$ where $r > 0$ and $0 < \theta < 2\pi$.

(i) Solve $z^5 = 1$. **[2]**

(ii) Hence, or otherwise, solve $z^5 + 32 = 0$. Sketch an Argand diagram showing the roots. **[4]**

2 Find the shortest distance between the lines $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$. **[4]**

3 The differential equation

$$\frac{2}{y} - \frac{x}{y^2}\frac{dy}{dx} = \frac{1}{x^2}$$

is to be solved subject to the condition $y = 1$ when $x = 1$.

(i) Show that $y = \frac{1}{u}$ transforms the differential equation into

$$x\frac{du}{dx} + 2u = \frac{1}{x^2}.$$ **[3]**

(ii) Find y in terms of x. **[7]**

4 Let A be the set of non-zero integers.

(i) Show that A does not form a group under multiplication. **[2]**

(ii) State the largest subset of A which forms a group under multiplication. Show that this is a group. **[3]**

5 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 85\cos x.$$ **[8]**

6 The planes Π_1 and Π_2 have equations

$$\mathbf{r}.\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 3 \quad \text{and} \quad \mathbf{r}.\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = 5$$

respectively. They intersect in the line l.

(i) Find cartesian equations of l. **[4]**

The plane Π_3 has equation $\mathbf{r}.\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} = 1$.

(ii) Show that Π_3 is parallel to l but does not contain it. **[3]**

(iii) Verify that $(2, 0, 1)$ lies on planes Π_1 and Π_3. Hence write down a vector equation of the line of intersection of these planes. **[3]**

7 **(i)** Use de Moivre's theorem to show that

$$\sin 6\theta \equiv \cos\theta\left(6\sin\theta - 32\sin^3\theta + 32\sin^5\theta\right).$$ **[5]**

(ii) Hence show that, for $\sin 2\theta \neq 0$,

$$-1 \leqslant \frac{\sin 6\theta}{\sin 2\theta} < 3.$$ **[7]**

8 A non-commutative multiplicative group G of order eight has the elements

$$\{e, a, a^2, a^3, b, ab, a^2b, a^3b\},$$

where e is the identity and $a^4 = b^2 = e$.

(i) Show that $ba \neq a^n$ for any integer n. **[2]**

(ii) Prove, by contradiction, that $ba \neq a^2b$ and also that $ba \neq ab$. Deduce that $ba = a^3b$. **[6]**

(iii) Prove that $ba^2 = a^2b$. **[2]**

(iv) Construct group tables for the three subgroups of G of order four. **[7]**

END OF QUESTION PAPER