OCR
Oxford Cambridge and RSA

GCE

Statistics (MEI)

Unit **G243**: Statistics 3 (Z3)

Advanced Subsidiary GCE

Mark Scheme for June 2016

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1.   **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2.   **Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand**

   a   Annotations should be used whenever appropriate during your marking.

   The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

   b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

   c   The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
1 (i)	Because both samples are large, the central limit theorem implies that the distributions of the sample means will be approximately Normal.	E1 E1	For samples large Condone 'sample large' For distribution of sample means Must mention 'sample means'
		E1	For mention of central limit theorem
	Because both samples are large, it is reasonable to approximate the population variances by the sample variances	E1 **[4]**	Do not need to mention 'samples are large' again, if already mentioned earlier.
1 (ii)	H_0: $\mu_A = \mu_B$ H_1: $\mu_A \neq \mu_B$ Where μ_A, μ_B denote the population mean marks for exams A and B respectively	B1 B1 B1	Condone absence of "population" if correct notation "μ" has been used, but do NOT accept $\overline{X}$ and $\overline{Y}$ or similar unless explicitly stated to be population means. Accept hypothesis explained in words, provided "population" appears.
	Use of 2–sample test based on N(0,1) soi	E1	
	Test statistic is $$\frac{57.2 - 55.7}{\sqrt{\dfrac{141.5}{60} + \dfrac{124.3}{60}}} = \frac{1.5}{2.105} = 0.713$$	M1 M1	Numerator Denominator NB do not allow a 'pooled estimate' of $\dfrac{124.3 + 141.5}{2}$
		A1	$\times \left(\dfrac{1}{60} + \dfrac{1}{60} \right)$ even though this gives 2.105. CAO Allow 0.71 or better with working
	2–tailed 10% point of N(0,1) is 1.645	B1	No further marks if CV is wrong
	$0.713 < 1.645$ Not significant	M1 A1	FT their test statistic provided both method marks scored for test statistic
	There is insufficient evidence to suggest that **the** exams are of different difficulties.	E1 **[11]**	

Question			Answer	Marks	Guidance
2	**(i)**		The pairing will eliminate any differences in abilities of different athletes and so will compare the times when listening to the two types of music.	E1 E1 **[2]**	Give 1 mark for any valid comment. For 2 marks must include either pairing or comparing
	(ii)		Wilcoxon signed rank test: H_0: Population median of differences is zero H_1: Population median of differences is not zero Differences are	B1 B1	Give B0B1 if no mention of population Do not allow population medians are different
			5.1 3.1 -6.9 6.6 -1.4 8.2 -5.0 -0.8 5.5 -1.5 11.5 6.0	B1	No marks if differences not used Allow one error
			Ranks of |d| are 6 4 10 9 2 11 5 1 7 3 12 8	M1 A1	CAO
			Test statistic is $1+2+3+5+10 = 21$ (or $4+6+7+8+9+11+12 = 57$)	M1 A1	FT if ranks wrong
			Refer to paired Wilcoxon table with $n=12$ Lower 2.5% 2-tailed value is 13 (or if 57 used, upper if 2.5% 2-tailed value is 65)	M1 A1	
			Not significant	E1	No further marks if CV or test statistic is wrong
			Insufficient evidence to suggest that the times when listening to the two types of music are different.	E1 **[11]**	
	(iii)		Because after completing one run the athletes might have been tired and not performed as well in the second run.	E1 **[1]**	Allow any sensible response
	(iv)		The researcher could have got half of the athletes to listen to each type of music on each evening. This would then have controlled for other factors such as weather conditions	E1 E1 **[2]**	Allow any sensible response

Question			Answer	Marks	Guidance
3	(i)		$S_{xy} = \Sigma xy - \dfrac{1}{n}\Sigma x\Sigma y = 326.5 - \frac{1}{15} \times 114.7 \times 42.45$	M1	For method for S_{xy}
			$= 1.899$		NB x-bar = 7.6467, y-bar = 2.83
			$S_{xx} = \Sigma x^2 - \dfrac{1}{n}(\Sigma x)^2 = 903.2 - \frac{1}{15} \times 114.7^2 = 26.127$	M1	For method for at least one of S_{xx} or S_{yy}
			$S_{yy} = \Sigma y^2 - \dfrac{1}{n}(\Sigma y)^2 = 120.8 - \frac{1}{15} \times 42.45^2 = 0.6665$	M1	For structure of r Dep on first two M1's Allow missing $\sqrt{}$
			$r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{1.899}{\sqrt{26.127 \times 0.6665}} = 0.455$	M1 A1 **[5]**	For $\sqrt{}$ Dep on first two M1's CAO (0.45 to 0.46)
3	(ii)		The population should have a bivariate Normal distribution. The scatter diagram is very roughly elliptical and so the assumption may be satisfied.	E1 E1 E1 **[3]**	For bivariate For Normal For roughly elliptical Condone 'cigar shaped'
3	(iii)		H_0: $\rho = 0$ H_1: $\rho > 0$ (one-tailed test)	B1 B1	Hypotheses in words must include 'population' and can get B1B1B0
			where ρ is the correlation coefficient for the underlying bivariate population.	B1	If correct symbol ρ used then no need for 'population'.
			For $n = 15$, one tailed 5% critical value = 0.4409	B1	No further marks if CV is wrong, but FT their test statistic
			$0.455 > 0.4409$ so significant There is evidence to suggest that there is positive correlation between the gain in weight of the mother during pregnancy and the weight of a new-born baby.	E1 E1 **[6]**	Condone '…between x and y'
3	(iv)		It could be that the fact that the baby is heavier causes the mother to gain more weight. It could be that there is a third underlying factor causing the relationship.	E1 E1 **[2]**	Allow valid alternatives (including 'there may be no correlation in the whole population')

Question			Answer	Marks	Guidance
4	(i)		Because different cars (of this model) may have different fuel consumption	E1	The sample size (of car) is too small Allow 'no replication'
			Because the fuel consumption may vary for the same car	E1 **[2]**	The distance of 10km is not enough Allow 'no randomisation' Do not allow 'should be tested over 100km'
4	(ii)		Allocate numbers 1 to 124 to the people. Use random numbers to choose 10 random numbers. If any repeats appear, choose further random numbers to replace them.	E1 E1 E1 **[3]**	Max SC2 for putting numbers in a hat etc
4	(iii)		Sample mean = 4.692	E1	Allow 4.69
			Sample standard deviation = 0.2858	E1 **[2]**	Allow 0.2858 to 0.286
4	(iv)		The two populations must be Normally distributed. The two populations must have equal variances.	E1 E1 **[2]**	Do not allow 'The data must be Normally distributed' nor 'The population is Normally distributed'
4	(v)		μ_{New} denotes the population mean fuel consumption for the new engines H$_1$: $\mu_{\text{N}} > \mu_{\text{O}} - 0.5$ H$_1$ takes this form since the correspondent thinks the reduction will be less so the new mean will be more than the old mean -0.5	B1 B1 B1	Must include 'population' oe If lower tail H$_1$ used allow max B1B0B0M1A1M1M1M1A1M1A1E0E0
			Pooled $s^2 = \dfrac{(6 \times 0.23^2) + (9 \times 0.2858^2)}{15} = 0.07017$	M1 A1	For attempt at pooling Allow 0.07015 to 0.07025
			Test statistic $= \dfrac{4.692 - (5.07 - 0.5)}{\sqrt{0.07017} \times \sqrt{\dfrac{1}{7} + \dfrac{1}{10}}} = \dfrac{0.122}{0.1305} = 0.935$	M1 M1 M1	for numerator for $\sqrt{0.07017}$ (with their variance) for $\sqrt{\dfrac{1}{7} + \dfrac{1}{10}}$
				A1	

Question		Answer	Marks	Guidance
		Refer to t_{15}	M1	
		1–tail 5% point is 1.753	A1	No further marks if CV is wrong
				If two tailed H_1 then A1 for 2.131 then E0E0
		0.935 < 1.753 so not significant	E1	FT their test statistic if all M marks earned for test
		There is insufficient evidence to suggest that the correspondent's belief that the reduction will be less than 0.5 litres per 100 km is correct.	E1	statistic
			[13]	
	(vi)	Wilcoxon rank sum test	B1	For Wilcoxon
		or Wilcoxon two sample test	B1	For rank sum or two sample
		The distribution of both populations have the same shape.	B1	Condone 'populations symmetrically distributed (about
			[3]	the median)'

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance
programme your call may be recorded or monitored