# Section Check In – 1.01 Proof

## Questions

 1.\*  and . Show that .

 2.\* Show that the sum of all even numbers less than or equal to  is divisible by .

 [You may find the arithmetic series results helpful.]

 3. Prove by exhaustion that, in the set of natural numbers less than 50, there are fewer square numbers than prime numbers.

 4. Let *p* be a prime number such that . Prove, by exhaustion, that for all such *p*,  is divisible by .

 5. The following result is known as the “difference of two squares”

 .

 Find and prove a similar formula for the difference of two cubes .

 6. Prove that  is even if and only if  is odd.

 7.\* (a) Find a counter example to disprove the conjecture that curves of the form 
 do not cross the *x*-axis.

 (b) Find a counter example to disprove the conjecture that an asymptote cannot be touched or crossed by a curve by considering rational functions of the form

  where .

 To find this counter example you will need to choose constants ,  and , and are
 recommended to solve  and to explore the behaviour of this curve as .

 8.\* Prove by contradiction that the curves  and  do not cross.

 9. A new hotel is to be built and will have a cylindrical swimming pool.
Given that the pool’s tiled surface area will be m2 show that its maximum volume is approximatelym3.

10.\* (a) Prove by contradiction that for any integer ,  and  do not have a prime factor
 in common.

 (b) Explain why this implies that  must have at least two distinct prime factors.

 (c) What can you conclude about the number of distinct prime factors which  has?

 *[This was the basis of Filip Saidak’s proof of the infinity of primes in 2005*.]

**Extension**

 If the circle  and the line  do not meet, prove that

 .

## Worked solutions

1. 

2. 

 

 

 

 Therefore the sum of all even numbers less than or equal to  is divisible by .

3. Proof by exhaustion:

All square numbers less than  are 

 All prime numbers less than  are 

 Therefore there are fewer squares than primes.

4. Proof by exhaustion:

 

Therefore the product of the two numbers adjacent, at either side, of the low primes  is divisible by .

5. 

 Proof:

 RHS 

 

 

  LHS

 Hence proven.

6. Need  and  for if and only if proofs.

 Therefore, initially need to prove that  is even  is odd.

 Now  is even so  for .

 So .

  is divisible by 2 and is therefore even.

 Hence  is therefore odd.

 Now need to prove that  odd  is even.

 If  odd then  for  even.

 So .

 Now  is even  is divisible by  and  is also divisible by.

 So  is divisible by  and is therefore even.

 Therefore  is even if and only if  is odd.

7. (a) You could use a graphing tool such as www.desmos.com or a graphical calculator to
 search for your counter example and/or to check that your example is a true counter
 example with a curve which crosses the *x*-axis.

 For instance,

 

*x*

f(*x*)

 (b) You could use a graphing tool such as www.desmos.com or a graphical calculator to search for your counter example and/or to check that your example is a true counter example with an asymptote which is also crossed by the curve itself.

 For instance, 

*x*

f(*x*)

8. Suppose the curves cross.

 At the crossing points, .

 

 

 or  but there are no real values of *x* for either of these. So the original assumption that the curves cross must be untrue. The curves do not cross.

9. Volume is 

 Surface area is 

 So  and 

 For the maximum volume we need , i.e. 

 So 

 Substituting this into our equations for  and  gives m3.

10. (a) Suppose  and  have a prime factor, *p*, in common.

 

  where *s* and *t* are integers.

 Subtracting,  where each of  and  are integers but 1 only has itself
 as a factor so this is a contraction.

 Hence,  and  cannot have a prime factor, *p*, in common.

 (b) Each of  and  have at least one prime factor and these are not the same.

 (c)  is 1 more than  and so has no prime factor in common with .

  has at least two prime factors so  has at least three prime factors.

**Extension**

 and 







If circle and line do not meet then 

Which leads to 

Hence to . Which is proven.

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