

OCR

Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

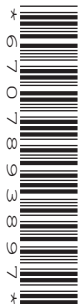
OCR supplied materials:

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

- 1 (a) (i) By differentiating the equation $a \tan y = x$ show that

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c . \quad [3]$$

The cartesian equation of an ellipse is $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

- (ii) Show that the polar equation of the ellipse may be written in the form

$$r^2 = \frac{36 \sec^2 \theta}{9 + 4 \tan^2 \theta} . \quad [3]$$

- (iii) By using the substitution $3u = 2 \tan \theta$ show that the area enclosed by the ellipse and the lines $\theta = 0$ and $\theta = \frac{\pi}{4}$ is $3 \arctan\left(\frac{2}{3}\right)$. [7]

- (b) Obtain the first three terms of the Maclaurin series for $f(x)$, where $f(x) = \arctan(1+x)$. [5]

- 2 (a) The infinite series C and S are defined as follows.

$$C = -\frac{1}{2}\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{8}\cos 3\theta + \dots$$

$$S = -\frac{1}{2}\sin\theta + \frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 3\theta + \dots$$

By considering $C + jS$, show that

$$S = \frac{-2\sin\theta}{5 + 4\cos\theta}.$$

Find a corresponding expression for C .

[9]

- (b) In an Argand diagram, O is the origin and points A and B are represented by the complex conjugate pair z_1 and z_2 respectively, where $0 < \arg z_1 < \frac{\pi}{2}$. The triangle OAB has side OA of length a .

(i) Show the above information on an Argand diagram.

[1]

(ii) Show that $z_1 z_2$ is real, giving its value in terms of a .

[2]

Triangle OAB is rotated anti-clockwise about the origin through γ radians, where $0 < \gamma < 2\pi$, and then enlarged through the origin with scale factor 3. The resulting new positions of A and B are represented by the complex numbers z_3 and z_4 respectively, where z_3 and z_4 form another complex conjugate pair.

(iii) State the value of γ .

[1]

(iv) Find, in polar form (modulus-argument form), the complex number $\frac{z_3}{z_1}$.

[2]

(v) Given that, in the original triangle OAB , AB also has length a , find the complex number $\frac{z_1}{z_4}$, giving your answer in the form $x + jy$, where x and y are exact real numbers.

[3]

- 3 (a) You are given the matrix $\mathbf{M} = \begin{pmatrix} k & 2 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & -2 \end{pmatrix}$.

(i) Find the value of k for which \mathbf{M} does not have an inverse.

[3]

(ii) Find \mathbf{M}^{-1} in terms of k .

[4]

- (b) The matrix \mathbf{Q} is given by $\mathbf{Q} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$.

(i) Find the eigenvalues and corresponding eigenvectors of \mathbf{Q} .

[5]

(ii) State a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{Q} = \mathbf{PDP}^{-1}$.

[2]

(iii) Show that, for $n \geq 1$, $\mathbf{Q}^n = \frac{1}{8} \begin{pmatrix} 6 + 2\varphi & 3\varphi - 3 \\ 4\varphi - 4 & 6\varphi + 2 \end{pmatrix}$, where $\varphi = 9^n$.

[4]

Section B (18 marks)

4 (i) Prove, from definitions involving exponentials, that $\operatorname{sech}^2 x + \tanh^2 x = 1$. [4]

(ii) Prove that

$$\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right).$$

State the set of values of x for which this is valid. [5]

(iii) Solve the equation

$$3(\tanh^2 x - \operatorname{sech}^2 x) = \tanh x - 2,$$

giving your answers in an exact logarithmic form. [5]

(iv) Find the exact value of

$$\int_{\operatorname{arsinh} 2}^{\operatorname{arsinh} 3} \frac{1}{\tanh x - \operatorname{sech} x} dx. [4]$$

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.