

OCR

Oxford Cambridge and RSA

Friday 16 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

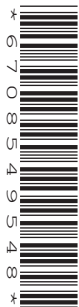
OCR supplied materials:

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = 6 \cos t.$$

- (i) Find the general solution. [8]

The particle is initially at rest, and its displacement remains bounded as $t \rightarrow \infty$.

- (ii) Find the particular solution for x . [4]
 (iii) Show that for large values of t the motion of the particle is oscillatory. Find the approximate amplitude of the oscillations. [2]

On another occasion, the displacement of the particle satisfies the differential equation

$$\frac{d^3x}{dt^3} + 2\frac{d^2x}{dt^2} - 3\frac{dx}{dt} = 0.$$

In this case, the particle is initially at the origin with acceleration 6 m s^{-2} and velocity $k \text{ m s}^{-1}$, where k is a positive constant.

- (iv) Find the particular solution for x in terms of t and k . [8]
 (v) Show that, whatever the value of k , the displacement of the particle cannot remain bounded for large values of t . [2]

- 2 (a) A small particle moving in a fluid satisfies the differential equation

$$\frac{dv}{dt} = -0.25(v^2 + 2v),$$

where $v \text{ m s}^{-1}$ is its velocity at time t s.

Given that $v = 20$ when $t = 0$, find the particular solution for v in terms of t . [8]

- (b) The differential equation

$$x \frac{dy}{dx} - 4y = x^3 \sqrt{x}$$

is to be solved for $x > 0$, subject to the initial condition $y = 0$ when $x = 1$.

- (i) Find the particular solution for y in terms of x . [10]
 (ii) Find the x -coordinate of the stationary point on this solution curve. [2]
 (iii) State the value of $\frac{dy}{dx}$ when $x = 1$. State also the limiting value of y as $x \rightarrow 0$. Hence, given that the stationary point is a minimum, sketch a graph of your solution to part (i). [4]

- 3 (a) The differential equation $\frac{dy}{dx} = \sqrt{x^2 + y^2}$ is to be investigated, firstly by means of a tangent field and then numerically.
- (i) Describe fully the isocline $\frac{dy}{dx} = m$ where m is a positive constant. [1]
- (ii) In the Answer Book, sketch on the given axes the isoclines $\frac{dy}{dx} = m$ for $m = \frac{1}{2}, 1, 2$ and 3 and hence draw a sketch of the tangent field. [4]
- (iii) Sketch on your tangent field the solution curve through the point $(0.5, 0.5)$ and the solution curve through the point $(2, 0)$. [3]

The differential equation is now to be solved numerically using Euler's method. The algorithm is given by

$$x_{r+1} = x_r + h, \quad y_{r+1} = y_r + hy'_r,$$

with $(x_0, y_0) = (0.5, 0.5)$.

- (iv) Use a step length of 0.05 to estimate y when $x = 0.65$. [4]
- (v) How might the accuracy of your estimate for y be improved? [1]
- (b) Consider the differential equation

$$\frac{dy}{dx} + y = x^2 - 1.$$

- (i) Find the complementary function and the particular integral and hence state the general solution for this differential equation. [6]
- (ii) Find the particular solution for which $y = 3$ when $x = 0$. [2]
- (iii) Show that y is always positive and sketch the solution curve for y . You do not need to find the coordinates of any stationary points. [3]

Question 4 begins on page 4

- 4 Two species of insects, X and Y, compete for survival on an island. The populations of the species are x and y respectively at time t , where t is measured in tens of years. The situation is modelled by the simultaneous differential equations

$$\frac{dx}{dt} = 2x + 2y,$$

$$\frac{dy}{dt} = 6y - 4x.$$

- (i) Eliminate y to obtain a second order differential equation for x in terms of t . Hence find the general solution for x . [7]

- (ii) Find the corresponding general solution for y . [4]

When $t = 0$, $\frac{dx}{dt} = 10$ and the population of species Y is k times the population of species X, where k is a positive constant.

- (iii) Find the particular solutions for x and y , in terms of t and k . [5]

Consider the case $k = 6$.

- (iv) Determine whether the model predicts that species X or species Y dies out first. State the value of t at which this first species dies out. [7]

- (v) Comment on why the time predicted by the model for the second species to die out is unreliable. [1]

END OF QUESTION PAPER

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