OCR
Oxford Cambridge and RSA

Friday 16 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = 6\cos t\,.$$

(i) Find the general solution. **[8]**

The particle is initially at rest, and its displacement remains bounded as $t \to \infty$.

(ii) Find the particular solution for x. **[4]**

(iii) Show that for large values of t the motion of the particle is oscillatory. Find the approximate amplitude of the oscillations. **[2]**

On another occasion, the displacement of the particle satisfies the differential equation

$$\frac{d^3x}{dt^3} + 2\frac{d^2x}{dt^2} - 3\frac{dx}{dt} = 0.$$

In this case, the particle is initially at the origin with acceleration $6\,\mathrm{m\,s^{-2}}$ and velocity $k\,\mathrm{m\,s^{-1}}$, where k is a positive constant.

(iv) Find the particular solution for x in terms of t and k. **[8]**

(v) Show that, whatever the value of k, the displacement of the particle cannot remain bounded for large values of t. **[2]**

2 **(a)** A small particle moving in a fluid satisfies the differential equation

$$\frac{dv}{dt} = -0.25\left(v^2 + 2v\right),$$

where $v\,\mathrm{m\,s^{-1}}$ is its velocity at time t s.

Given that $v = 20$ when $t = 0$, find the particular solution for v in terms of t. **[8]**

(b) The differential equation

$$x\frac{dy}{dx} - 4y = x^3\sqrt{x}$$

is to be solved for $x > 0$, subject to the initial condition $y = 0$ when $x = 1$.

(i) Find the particular solution for y in terms of x. **[10]**

(ii) Find the x-coordinate of the stationary point on this solution curve. **[2]**

(iii) State the value of $\frac{dy}{dx}$ when $x = 1$. State also the limiting value of y as $x \to 0$. Hence, given that the stationary point is a minimum, sketch a graph of your solution to part **(i)**. **[4]**

3 **(a)** The differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{x^2 + y^2}$ is to be investigated, firstly by means of a tangent field and then numerically.

(i) Describe fully the isocline $\frac{\mathrm{d}y}{\mathrm{d}x} = m$ where m is a positive constant. **[1]**

(ii) In the Answer Book, sketch on the given axes the isoclines $\frac{\mathrm{d}y}{\mathrm{d}x} = m$ for $m = \frac{1}{2}$, 1, 2 and 3 and hence draw a sketch of the tangent field. **[4]**

(iii) Sketch on your tangent field the solution curve through the point (0.5, 0.5) and the solution curve through the point (2, 0). **[3]**

The differential equation is now to be solved numerically using Euler's method. The algorithm is given by

$$x_{r+1} = x_r + h, \qquad y_{r+1} = y_r + hy'_r,$$

with $(x_0, y_0) = (0.5, 0.5)$.

(iv) Use a step length of 0.05 to estimate y when $x = 0.65$. **[4]**

(v) How might the accuracy of your estimate for y be improved? **[1]**

(b) Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y = x^2 - 1.$$

(i) Find the complementary function and the particular integral and hence state the general solution for this differential equation. **[6]**

(ii) Find the particular solution for which $y = 3$ when $x = 0$. **[2]**

(iii) Show that y is always positive and sketch the solution curve for y. You do not need to find the coordinates of any stationary points. **[3]**

Question 4 begins on page 4

4 Two species of insects, X and Y, compete for survival on an island. The populations of the species are x and y respectively at time t, where t is measured in tens of years. The situation is modelled by the simultaneous differential equations

$$\frac{dx}{dt} = 2x + 2y,$$

$$\frac{dy}{dt} = 6y - 4x.$$

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[7]**

(ii) Find the corresponding general solution for y. **[4]**

When $t = 0$, $\frac{dx}{dt} = 10$ and the population of species Y is k times the population of species X, where k is a positive constant.

(iii) Find the particular solutions for x and y, in terms of t and k. **[5]**

Consider the case $k = 6$.

(iv) Determine whether the model predicts that species X or species Y dies out first. State the value of t at which this first species dies out. **[7]**

(v) Comment on why the time predicted by the model for the second species to die out is unreliable. **[1]**

END OF QUESTION PAPER