OCR
Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\ \mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 A car moves horizontally in a straight line with speed v at time t. The total resistance force on the car has magnitude kv where k is a positive constant. The car is powered by a rocket, which ejects burnt fuel backwards at a constant mass rate λ and at a constant speed u relative to the car. The initial mass of the car and the fuel is M and at time t, when some fuel still remains to be burnt, the mass of the car and the remaining fuel is m.

(i) Derive the differential equation $m\frac{\mathrm{d}v}{\mathrm{d}t}+u\frac{\mathrm{d}m}{\mathrm{d}t}=-kv$. **[3]**

(ii) Given that the initial speed of the car is zero, show that

$$v=\frac{\lambda u}{k}\left(1-\left(\frac{M-\lambda t}{M}\right)^{\frac{k}{\lambda}}\right),$$

and hence show that for small values of t the speed of the car is approximately $\frac{\lambda ut}{M}$. **[9]**

2 A particle of mass 3 kg moves along the x-axis by means of a driving force applied in the positive x-direction. There are no other forces acting on the particle. When the particle is x m from the origin O, its velocity is $v\,\mathrm{m\,s^{-1}}$. Initially $v = 3$ and the particle is at O. The magnitude of the driving force is F N, where $F=\mathrm{e}^{0.1x}\left(v^2-1\right)^{\frac{1}{3}}$.

(i) By solving a suitable equation of motion satisfied by the particle, show that F may be written as

$$F=\frac{2}{3}\mathrm{e}^{0.1x}\sqrt{10\mathrm{e}^{0.1x}-1}\,.$$ **[9]**

(ii) By using the work-energy principle, and without further integration, show that

$$\int_0^{10}\mathrm{e}^{0.1x}\sqrt{10\mathrm{e}^{0.1x}-1}\,\mathrm{d}x=k\left(\left(10\mathrm{e}-1\right)^{\frac{3}{2}}-27\right),$$

stating the exact value of the constant k. **[3]**

Section B (48 marks)

3 Fig. 3 shows a smooth wire in the form of a semi-circle with centre O and radius a. The wire is fixed in a vertical plane. The points C and D are at the ends of the wire at the same horizontal level as O. A small ring, P, of mass λm can move freely on the wire. One end of a light inextensible string of length $2a$ is attached to P. The string passes over a small smooth fixed pulley at C; a particle of mass μm hangs freely from its other end, vertically below C. One end of a second light inextensible string of length $2a$ is attached to P. This string passes over a small smooth fixed pulley at D; a particle of mass μm hangs freely from its other end, vertically below D. The radius OP makes an angle 2θ with the downward vertical, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, and λ and μ are positive constants.

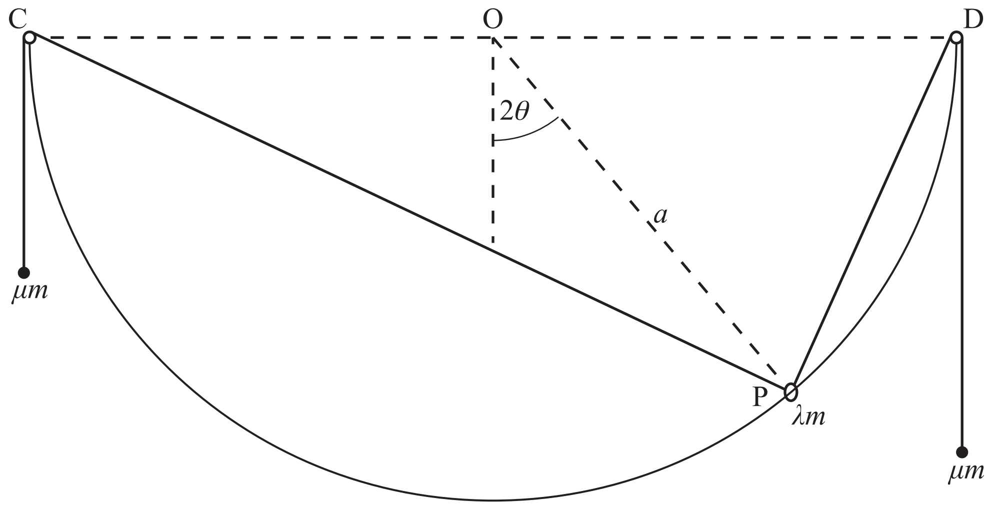

Fig. 3

(i) Find the potential energy, V, of the system relative to the level of CD, and hence show that

$$\frac{dV}{d\theta} = 2mga(\lambda \sin 2\theta - \sqrt{2}\mu \sin\theta).$$ **[8]**

(ii) Show that there are three values of θ for which the system is in equilibrium provided that $\lambda < \mu < \sqrt{2}\lambda$. **[5]**

(iii) Given that there are three positions of equilibrium, establish whether each of these positions is stable or unstable. **[8]**

You are now given that $\mu = 6$ and $\lambda = 3\sqrt{2}$.

(iv) Investigate the stability of the single equilibrium position of the system. **[3]**

4 A triangular lamina OAB of mass M kg has OA = OB and AB = $2a$ m. OX = $3a$ m, where X is the mid-point of AB. Fig. 4 shows this lamina in an x–y plane with origin O and OX horizontal. The mass per unit area $\rho\,\text{kg}\,\text{m}^{-2}$ of the lamina is given by $\rho = k\left(1+\dfrac{x}{a}\right)$ where x m is the horizontal distance from O and k is a positive constant.

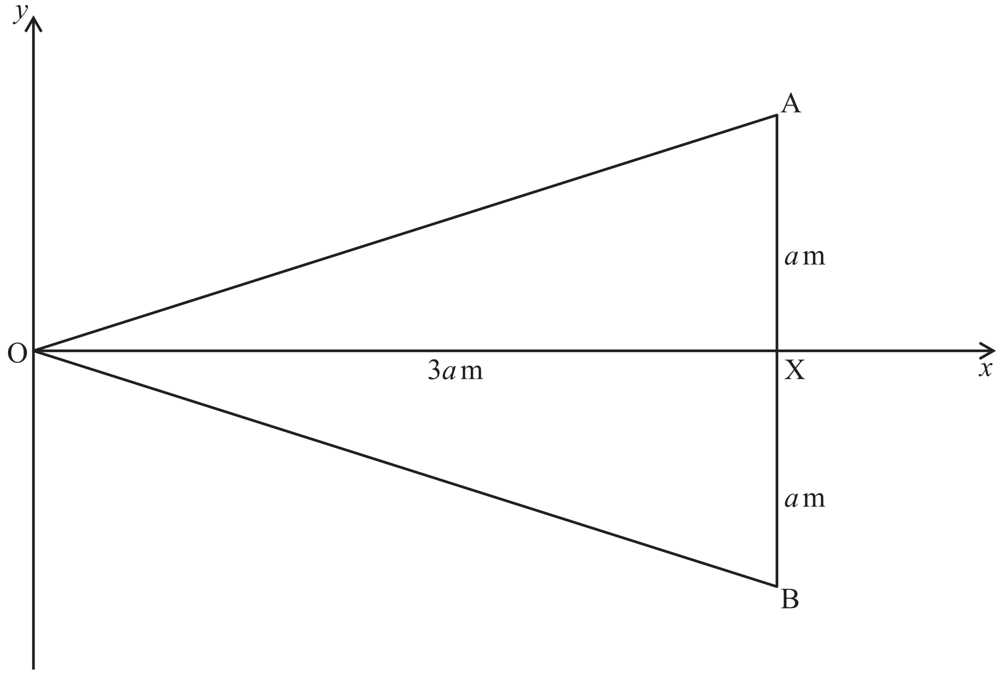

Fig. 4

(i) Show that $M = 9ka^2$. **[5]**

(ii) Show, using integration, that the moment of inertia of the lamina about an axis through O perpendicular to the plane of the lamina is $\dfrac{238}{45}Ma^2$. [You may assume the standard formula for the moment of inertia of a thin rod about an axis through its centre perpendicular to the rod.] **[7]**

The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis through O perpendicular to the lamina. The lamina is released from rest with OX making an angle ϕ with the downward vertical. At time t s after the lamina is released, OX makes an angle θ with the downward vertical.

(iii) Show that the angular velocity $\dot{\theta}$ of the lamina when it has turned through an angle θ satisfies

$$a\dot{\theta}^2 = \sigma g(\cos\theta - \cos\phi),$$

stating the exact value of the constant σ. **[8]**

You are now given that $a = 2.25$ and that ϕ is small.

(iv) Show that the motion is approximately simple harmonic, and find the approximate time when the lamina first comes instantaneously to rest. **[4]**

END OF QUESTION PAPER