Oxford Cambridge and RSA

GCE

Mathematics

Unit **4723**: Core Mathematics 3

Advanced GCE

Mark Scheme for June 2017

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			State volume is $\pi\int(4+4e^{\frac{1}{2}x}+e^x)\,dx$	B1	Condone absence of dx; no need for limits here; π may be implied here by its appearance later in solution; integrand must be expanded
			Obtain integral of form $px+qe^{\frac{1}{2}x}+re^x$	*M1	With non-zero constants p, q, r; with or without π here
			Obtain correct $4x+8e^{\frac{1}{2}x}+e^x$ or $\pi(4x+8e^{\frac{1}{2}x}+e^x)$	A1	Or unsimplified equiv; condone presence of + c
			Apply limits 0 and 4 correctly to their integral	M1	Dep *M; with at least one non-zero term obtained from use of limit 0; limits used the wrong way round is M0
			Obtain $\pi(e^4+8e^2+7)$	A1 **[5]**	Or simplified equiv; + c now is A0; ignore subsequent working if necessary
2	i		Attempt calculation of form $k(y_0+4y_1+2y_2+4y_3+y_4)$	M1	Any non-zero constant k with attempts at y values (in terms of ln or decimals); M0 if attempt does not involve exactly four strips; M0 if each y value initially 'amended', to $\ln(2x+4)$ for example
			Obtain $k(\ln 1\ln 5+4\ln 3\ln 7+2\ln 5\ln 9+4\ln 7\ln 11+\ln 9\ln 13)$	A1	Or equiv involving decimals indicating use of correct values
			Use $k=\frac{2}{3}$	A1	
			Obtain 26.62	A1 **[4]**	Allow greater accuracy 26.6159…; any value rounding to 26.62 with no errors seen
	ii		State or imply that integrand now involves $-\ln x$ or $2\ln(x+4)$ or both	M1	
			Obtain –53.23 or –53.24 as final answer	A1ft **[2]**	Following their Simpson rule answer from **(i)**, ie -2 times their answer; allow greater accuracy; correct answer with no working earns B2; second use of Simpson's rule leading to correct answer earns B2, but B0 if incorrect; concluding with 53.23 or 53.24 (perhaps with some reference to area below axis) is A0

Question			Answer	Marks	Guidance
3	i		Draw V-shaped graph with vertex on positive x-axis	B1	And graph extending at least a little into second quadrant; condone minimal smoothing at the vertex; allow graph which is asymmetrical about vertical line through vertex unless it is an extreme case
			State $(\frac{7}{2}a, 0)$ and $(0, 7a)$	B1 **[2]**	Can be earned if first B1 not awarded; allow for $\frac{7}{2}a$ and $7a$ marked on axes of graph or cases where zero coordinates are not given but are clearly implied
	ii		Attempt to find two critical values	M1	By squaring both sides (giving 3 terms on left) and solving quadratic equation <u>or</u> by solving two linear equations (one with signs of $2x$ and $4a$ the same and one with the signs different) <u>or</u> using graph with horizontal line representing $y = 4a$
			Obtain $\frac{3}{2}a$ and $\frac{11}{2}a$	A1	
			Conclude with $\frac{3}{2}a < x < \frac{11}{2}a$	A1 **[3]**	Allow the logically correct '$x > \frac{3}{2}a$ and $x < \frac{11}{2}a$' but not conclusions such as '$x > \frac{3}{2}a$, $x < \frac{11}{2}a$'; giving a a particular value means only M1 is available; use of $\leq$ signs is final A0
	iii		Relate $\ln N$ to their upper limit of **(ii)** with $a = 1.5$ <u>or</u> proceed directly from inequality in **(iii)** to $2\ln N < 16.5$	M1	
			State the single value 3827	A1 **[2]**	A0 for $N \leq 3827$; A0 for $N < 3827.6$
4	i		Use identity $\sec^2\theta = 1 + \tan^2\theta$	B1	Identity must be used not merely quoted
			Attempt solution of 3-term quadratic equation in $\tan\theta$	M1	If using factorisation, M1 earned if their factors correct; if using formula, M1 earned if substitution of their values into correct formula correct; for incorrect equation and two values produced with no working, check that values are correct given their equation so that M1 can be awarded
			Obtain at least $\tan\theta = -4$ from the correct equation	A1 **[3]**	Ignore second value given provided no error at this stage is involved; so $\frac{2}{3}$ and –4 is A1, –4 only is A1, $\frac{2}{3}$ only is A0, $\frac{3}{2}$ and –4 is A0 ; allow solution such as $y = -4$ when clear that y is $\tan\theta$; ignore subsequent work with angles

Question			Answer	Marks	Guidance
	ii	a	Attempt substitution into $\frac{2\tan\theta}{1-\tan^2\theta}$	M1	Using any value from **(i)**
			Use –4 to obtain $\frac{8}{15}$ and no other value	A1 **[2]**	Or exact equiv; full details to be shown; indication of use of calculator is M0; finding $\tan 2\theta$ for both angles is M1A0; answer $\frac{8}{15}$ with no working is M0A0; final answer $\frac{-8}{-15}$ is A0
		b	State or imply $\cot(2\theta+135°)$ is $1 \div \tan(2\theta+135°)$	B1	Either at beginning of solution or towards the end
			Attempt substitution of their value from **(a)** into $\frac{1-\tan 2\theta \tan 135°}{\tan 2\theta + \tan 135°}$ or into $\frac{\tan 2\theta + \tan 135°}{1-\tan 2\theta \tan 135°}$	M1	Allow with $\tan 135°$ still present
			Obtain $-\frac{23}{7}$ and no other value	A1 **[3]**	Or exact equiv; full details to be shown; allow $\frac{23}{-7}$
5			Differentiate to obtain $k(4x-3)^{-\frac{1}{2}}$	M1	For any non-zero constant k
			Obtain correct $2(4x-3)^{-\frac{1}{2}}$	A1	Or unsimplified equiv
			Use negative reciprocal of gradient to find intersection of normal with x-axis	M1	Using their attempt at first derivative; <u>either</u> using equation of normal ($y=-\frac{5}{2}x+\frac{45}{2}$) <u>or</u> relevant right-angled triangle
			Obtain $-\frac{5}{2}$ for gradient of normal and hence $x=9$ or equiv such as base of triangle is 2	A1	
			Integrate to obtain $p(4x-3)^{\frac{3}{2}}$	M1	For any non-zero constant p
			Obtain correct $\frac{1}{6}(4x-3)^{\frac{3}{2}}$	A1	Or unsimplified equiv
			Use limits $\frac{3}{4}$ and 7 to obtain $\frac{125}{6}$ for area under curve	A1	Allow calculation apparently using only upper limit
			Use triangle area to obtain $\frac{155}{6}$ for shaded area	A1 **[8]**	

Question			Answer	Marks	Guidance
6	i		Translation parallel to x-axis by – 1	B1	Must use term 'translate' or 'translation', not 'move', not 'shift', etc.; translate by $\begin{pmatrix}-1\\0\end{pmatrix}$ is acceptable; 'in x direction' is acceptable; 'translate in negative x direction by –1' is B0
			Stretch parallel to y-axis, factor 3	B1 **[2]**	Must use term 'stretch'; 'in y direction' is acceptable; condone 'in y axis'; the two transformations can be given in either order
	ii		State <u>either</u> <3 <u>or</u> >0 or both	M1	Allow any letter; accept $<$ or $\le$, $>$ or $\ge$ here for method mark
			State correct $0<\mathrm{f}(x)\le 3$ or $0<y\le 3$ or $0<\mathrm{f}\le 3$	A1 **[2]**	
	iii		Obtain expression of form $\frac{a}{x}+b$ or $\frac{a+bx}{x}$	M1	For non-zero constants a and b; or equiv in terms of y
			Obtain correct $\dfrac{3}{x}-1$ or $\dfrac{3-x}{x}$	A1	In terms of x now
			Reflection in line $y=x$	B1 **[3]**	Or clear equiv such as one is the mirror image of the other
	iv		<u>Either</u> Attempt correct process to find $\mathrm{ff}(x)$	M1	
			Obtain $\dfrac{3}{\frac{3}{x+1}+1}$ or $\dfrac{3x+3}{x+4}$	A1	Or equiv
			Solve to obtain $x=5$	A1	
			<u>Or</u> Attempt $\mathrm{f}^{-1}\mathrm{f}^{-1}(2)$ with their f^{-1}	M1	
			Obtain $\frac{1}{2}$ as first value	A1	
			Obtain 5	A1 **[3]**	

Question			Answer	Marks	Guidance
7	i		State $x=\dfrac{\ln y}{\ln a}$	B1	Ignore any subsequent manipulation of right-hand side
			Differentiate to obtain $\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{1}{y\ln a}$	B1	$\dfrac{\mathrm{d}x}{\mathrm{d}y}$ must be used; quotient rule may be used but must be correct
			Rearrange to confirm $\dfrac{\mathrm{d}y}{\mathrm{d}x}=a^x\ln a$	B1 **[3]**	AG – at least one intermediate step needed
	ii	a	Obtain derivative $4x^3+4^x\ln 4$	B1	Or equiv
			Equate attempt at first derivative to –8 and rearrange to form $x=\sqrt[3]{\ldots}$	M1	Where expression under cube root involves two terms at least one of which involves *x*; allow M1 if there is one sign slip
			Confirm $x=\sqrt[3]{-2-4^{x-1}\ln 4}$	A1 **[3]**	AG – necessary detail needed
		b	Carry out iteration process	M1	Showing at least 3 values after –1
			Obtain –1.27 for *x*-coordinate	A1	Condone correct value eventually obtained after error in iteration process; answer required to precisely 2 dp; ($-1 \to -1.277858 \to -1.272179 \to -1.272275$); iterates must be present and showing at least 3 dp; answer only and no iterates shown earns 0/3; treat sequence starting at value other than –1 as mis-read
			Obtain 2.79 for *y*-coordinate	A1 **[3]**	Answer required to precisely 2 dp; using –1.27 to obtain 2.77 is A0; M1A0A1 is possible where iterates shown are not to at least 3 dp(but values are perhaps in calculator)
8	i		Use $\sin 2\theta=2\sin\theta\cos\theta$	B1	Must be used not merely stated
			Obtain $6\sin\theta+8\cos\theta$	B1	May be implied
			Obtain $R=10$	B1	From correct $6\sin\theta+8\cos\theta$
			Attempt appropriate trigonometry to find α	M1	Allow for $\tan\alpha=\frac{6}{8}$ or equiv
			Obtain 53.1°	A1 **[5]**	Or greater accuracy 53.13…; with no errors seen

Question			Answer	Marks	Guidance
	ii		State or imply equation is $10\sin(\beta+63.1°)=3$	B1ft	Following their R and α
			Carry out correct process to find one value of β	M1	Not available for finding negative angle; must involve use of 2nd quadrant angle
			Obtain $99.4°$ (or $314°$)	A1	Or greater accuracy $99.4122...°$
			Carry out correct process to find second value of β	M1	Must involve use of '5th' quadrant angle
			Obtain $314°$ (or $99.4°$)	A1	Accept value rounding to 314 providing no error; and no others between 0 and 360
					[Note: Solving $10\sin(\theta+53.1°)=3$ can earn M1 M1 if correct processes followed; if continue to find correct angles by subtracting $10°$, A1 A1 available; B1 can be retrospectively given even if answers are wrong]
				[5]	
9	a		Differentiate using quotient rule or equiv	M1	With negative sign in numerator, with $(x^2+3)^2$ in denominator and at least one of the two terms in the numerator correct
			Obtain $\dfrac{p(x^2+3)-2x(px+q)}{(x^2+3)^2}$ or equiv	A1	
			Equate derivative to zero and attempt discriminant	M1	Provided equation is a 3-term quadratic with p and q present
			Obtain $4q^2+12p^2$ and observe it is positive	A1	With at least one reference to squared value being positive
				[4]	
	b		Differentiate to obtain form $e^{x^2}(px^3+qx)$	M1	
			Obtain $\dfrac{dy}{dx}=2xe^{x^2}(ax^2+b)+2axe^{x^2}$	A1	Or equiv
			Obtain $\dfrac{d^2y}{dx^2}=e^{x^2}(4ax^4+10ax^2+4bx^2+2a+2b)$	A1	Or equiv
			Equate coefficient of $x^2e^{x^2}$ to zero	M1	Provided second derivative involves $e^{x^2}x^4$, $e^{x^2}x^2$ and e^{x^2} terms and no others
			Confirm $5a+2b=0$	A1	AG – necessary detail needed
				[	

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