OCR
Oxford Cambridge and RSA

GCE

Statistics (MEI)

Unit **G243**: Statistics 3 (Z3)

Advanced Subsidiary GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a   Annotations should be used whenever appropriate during your marking.

   The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

   E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures

are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao"

even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.
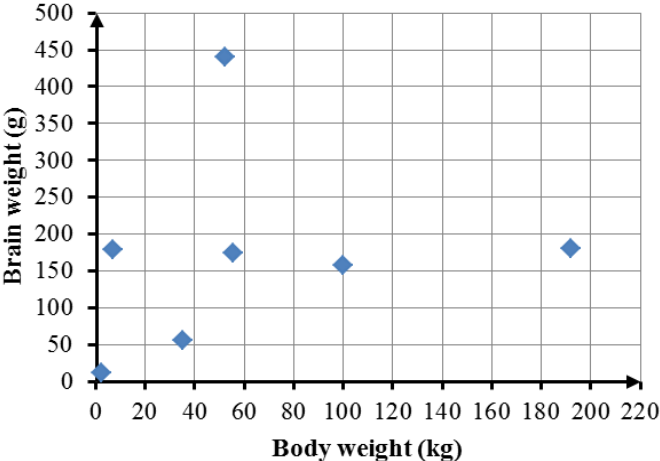
A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
PLEASE NOTE THAT I HAVE PUT THE GRADE FOR EACH MARK NEXT TO THE MARK. I HAVE ALSO INCLUDED THE AOS, WHICH I WILL DELETE LATER						
1	(i)		They must both be random.	E1e **[1]**	Ignore any other statements	AO1 1
	(ii)		H_0: the medians of the two populations are the same H_1: the medians of the two populations are different	B1e B1c	For medians Need population for second mark Note: Explicit statement re shapes of distributions (eg that they are the same shape) is not required. Do not allow 'Population median difference is zero'	AO1 2 AO3 5 AO4 2 AO5 1
			Wilcoxon rank sum test (or Mann-Whitney form thereof)	M1e	soi	
			Ranks are National news rank 17 16 10 15 7 12 14 18 6 19 4 Local news rank 5 11 1 3 2 8 9 13	M1e	Combined ranking	
			Smaller rank sum (for local news) is 52	A1e	FT (M–W stat = 0+0+0+2+2+3+4+5 = 16)	
			Refer to (8,11) table	B1e	soi	
			2–tail 5% critical value is 55 [or 19 for M–W]	M1e A1e		
			51 < 55 so significant There is sufficient evidence to suggest that, (on average), latency of national news programmes is different to that of local television news programmes	E1c E1a **[10]**		
1	(iii)	A	Wilcoxon signed rank test	E1e	For Wilcoxon	AO2

Question			Answer	Marks	Guidance	
				E1c **[2]**	For signed rank Mark final answer	**2**
	(iii)	B	The paired sample t test would be more likely to detect the difference, since the ranking involved in the calculation in the Wilcoxon signed rank test loses information.	E1a E1a **[2]**		**AO4** **2**
2	(i)		The pairing will eliminate any differences in individual subjects' blood pressures and so will compare the blood pressures standing and seated.	E1a E1a **[2]**		**AO4** **2**
	(ii)		The population of differences of blood pressures must be Normally distributed	E1a E1a **[2]**	For population of differences For Normally distributed E0 for data is Normally distributed Ignore any other comments	**AO2** **2**
	(iii)		H_0: $\mu_D = 0$ H_1: $\mu_D \neq 0$ Where μ_D denotes the population mean for differences. No further marks unless paired comparison t test Differences (seated $-$ standing) are -1 4 12 10 3 -9 5 -4 8 $\overline{d} = 3.1111$ $s_{n-1} = 6.7905$ Test statistic is $\dfrac{3.1111 - 0}{6.7905 / \sqrt{9}} = 1.374$ Refer to t_8 Two tailed 5% critical value is 2.306	B1e B1e B1a M1e A1e M1c A1c M1e A1e	Allow μ in place of μ_D Condone absence of "population" if correct notation "μ" has been used, but do NOT accept $\overline{D}$ or similar unless explicitly stated to be population means. Hypotheses explained in words only must include "population" For differences For both FT their $\overline{d}$ and s_{n-1} CAO but FT from here if M1 awarded For t_8 No FT if wrong	**AO1** **2** **AO2** **1** **AO3** **5** **AO4** **2** **AO5** **1**

Question			Answer	Marks	Guidance	
			So not significant There is insufficient evidence to suggest that the blood pressures when people are standing or seated are different	E1c E1a **[11]**		
3	**(i)**			G1e G1e G1e **[3]**	Linear axes, including labels Correct zero All points correct (allow 1 error)	**AO3** **3**
	(ii)		Because the points do not appear to lie in an elliptical pattern.	E1a **[1]**	For not elliptical Ignore other comments	**AO1** **1**
	(iii)		<table><tr><td>A</td><td>7</td><td>179</td><td>2</td><td>5</td><td>3</td><td>9</td></tr><tr><td>B</td><td>35</td><td>56</td><td>3</td><td>2</td><td>-1</td><td>1</td></tr><tr><td>C</td><td>3</td><td>12</td><td>1</td><td>1</td><td>0</td><td>0</td></tr><tr><td>D</td><td>55</td><td>175</td><td>5</td><td>4</td><td>-1</td><td>1</td></tr><tr><td>E</td><td>100</td><td>157</td><td>6</td><td>3</td><td>-3</td><td>9</td></tr><tr><td>F</td><td>52</td><td>440</td><td>4</td><td>7</td><td>3</td><td>9</td></tr><tr><td>G</td><td>192</td><td>180</td><td>7</td><td>6</td><td>-1</td><td>1</td></tr></table>	M1e M1e	For ranking (allow all ranks reversed) Allow slight errors For d^2	**AO1** **4** **AO5** **1**

Question			Answer	Marks	Guidance	
			$\Sigma d^2 = 30$	A1e	For Σd^2 CAO	
			$r_s = 1 - \dfrac{6 \times 30}{7 \times 48} = 1 - 0.5357 = 0.4643$	M1e A1c **[5]**	For method for r_s FT their ranks provided $\lvert r_s \rvert < 1$ NB No ranking scores zero	
	(iv)		H_0: no association between average body weight and average brain weight in the population H_1: positive association between average body weight and average brain weight in the population	B1e B1e B1c	NB $H_0 \, H_1$ <u>not</u> ito ρ For mention of population, either in hypotheses or in concluding statement	AO3 2 AO4 4 AO5 1
			One tail test critical value at 5% level is 0.7143 Since $0.4643 < 0.7143$, there is insufficient evidence to reject H_0.	B1e M1e	For 0.7143 For comparison with c.v., provided $\lvert r_s \rvert < 1$	
				A1c	For 'Do not reject H_0' oe	
			Thus there is not sufficient evidence to suggest that average body weights and average brain weights are positively associated.	E1a **[7]**	For conclusion in words in context – FT their r_s and sensible cv	
4	(i)		The sample size is too small to provide sufficient detail of variability. The heights of the loaves may vary between different batches.	E1e E1c **[2]**	Allow other sensible comments	AO2 2
	(ii)		Select 10 loaves of bread from each batch as follows: Allocate numbers 1 to 60 to the loaves. Choose 10 random numbers and select the loaf corresponding to each number. If a repeat occurs, choose a further random number to replace it.	E1e E1e E1e E1c **[4]**		AO3 4

Question			Answer	Marks	Guidance	
	(iii)		$\overline{x} = \dfrac{709.5}{50} = 14.19$	B1e	14.2 or better	**AO5 3**
			$s^2 = \dfrac{10092 - \dfrac{709.5^2}{50}}{49} = \dfrac{24.195}{49} = 0.49377...$	M1c	For S_{xx}	
			$s = \sqrt{0.49377...} = 0.70269...$	A1c **[3]**	0.703 or better	
	(iv)		H_0: $\mu_A = \mu_B$ H_1: $\mu_A \neq \mu_b$ Where μ_A, μ_B denote the population mean heights of loaves of varieties A and B respectively	B1e B1c B1a	Condone absence of "population" if correct notation "μ" has been used, but do NOT accept $\overline{X}$ and $\overline{Y}$ or similar unless explicitly stated to be population means. Accept hypothesis explained in words, provided "population" appears.	**AO1 2 AO2 1 AO3 5 AO4 2 AO5 1**
			2–sample test based on N(0,1)	E1e		
			Test statistic is $\dfrac{14.51 - 14.19}{\sqrt{\dfrac{0.49377..}{50} + \dfrac{0.718^2}{50}}} = \dfrac{0.32}{0.1420...} = 2.252$ Allow ±2.252	M1e M1c A1a	Numerator Denominator CAO	
			2–tailed 5% point of N(0,1) is 1.96 Allow ±1.96	B1e		
			2.252 > 1.96 Significant	M1e A1c	FT their test statistic provided at least one M1 earned FT their critical value provided from a Normal distribution	
			There is sufficient evidence to suggest that on average the heights of loaves of varieties A and B are different.	E1a **[11]**		

Question			Answer	Marks	Guidance	
	(v)		Because the central limit theorem implies that (for large samples such as these) the distributions of the sample means are Normally distributed.	E2,1,0a,a **[2]**	Do not allow 'The <u>data</u> will be approximately normally distributed' Condone 'Sample' rather than 'Samples'	**AO2** **2**
	(vi)		Unpaired *t* test. Both populations must be Normally distributed. The two population variances must be equal.	E1c E1c E1a E1a **[4]**	For unpaired For *t* test	**AO4** **4**

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance
programme your call may be recorded or monitored