Section 5: Sequences and recurrence relations

Notes and Examples

In this section you will learn definitions and notation involving sequences and series, and some different ways in which sequences and series can be generated.

These notes contain subsections on

What is a sequence?

A sequence is a set of numbers in a given order.
Examples of sequences are

1, 2, 4, 8, 16, 32,…

0, 1, 0, 1, 0, 1, 0, 1, 0

2, 3, 5, 7, 11, 13, 17,..

The numbers in a sequence may form an algebraic pattern but they don't have to.

Notation for sequences involves using subscripts. A typical sequence may have notation as follows

$a_1, a_2, a_3, ...$

so that a_1 is the first term, a_2 is the second term, a_3 is the third term and so on.

Of course letters other than a might be used and the first term may have subscript 0 (this is particularly common when using sequences in computing) as in this example

$b_0, b_1, b_2, ...$

Sequences can be defined in several different ways.

Additional Mathematics (OCR): Algebra

Sequences defined in terms of their position

Here there is a direct formula for the kth term of the sequence in terms of k. The terms of the sequence can be found by substituting the numbers 1, 2, 3… for k.

Example 1

A sequence is defined

$$a_k = k^2 - 3$$

(i) Write down the first five terms of the sequence.

(ii) Find the 20th term of the sequence.

Solution

(i) Substituting $k = 1$, $k = 2$, … $k = 5$ into the expression $a_k = k^2 - 3$ gives the sequence

–2, 1, 6, 13, 22

(ii) Substituting $k = 20$

$$\begin{aligned} a_{20} &= 20^2 - 3 \\ &= 400 - 3 \\ &= 397 \end{aligned}$$

Sequences defined recursively

A recursive definition tells you how to find a term in a sequence from the previous term. The definition must also include the value of the first term of the sequence. You can then find the second term from the first term, the third term from the second term, and so on.

Example 2

A sequence is defined inductively as

$$a_{k+1} = 2a_k + 1, \ a_1 = 0$$

(i) Write down the first six terms of the sequence.

Solution

(i) Each term is found by doubling the previous term and adding 1. The first term is 0.

$$a_1 = 0$$

$$a_2 = 2a_1 + 1 = 2 \times 0 + 1 = 1$$

$$a_3 = 2a_2 + 1 = 2 \times 1 + 1 = 3$$

$$a_4 = 2a_3 + 1 = 2 \times 3 + 1 = 7$$

$$a_5 = 2a_4 + 1 = 2 \times 7 + 1 = 15$$

$$a_6 = 2a_5 + 1 = 2 \times 15 + 1 = 31$$

The first six terms are 0, 1, 3, 7, 15, 31

(ii) $\sum_{1}^{6} a_k = 0+1+3+7+15+31$

$= 57$

Sometimes the new term is defined in terms of more than one of the known, previous terms. Here is an example.

Example 3
A sequence is defined inductively as

$$a_{k+2} = 2a_{k+1} + a_k\,,\ a_1 = 0\,,\ a_2 = 1$$

Write down the first six terms of the sequence.

Solution
Each term is found by doubling the previous term and adding the term before that.
The first term is 0 and the second term is 1.

$a_1 = 0$

$a_2 = 1$

$a_3 = 2a_2 + a_1 = 2\times 1 + 0 = 2$

$a_4 = 2a_3 + a_2 = 2\times 2 + 1 = 5$

$a_5 = 2a_4 + a_3 = 2\times 5 + 2 = 12$

$a_6 = 2a_5 + a_4 = 2\times 12 + 5 = 29$

The first six terms are 0, 1, 2, 5, 12, 29