

## Section 5: Sequences and recurrence relations

## Solutions to Exercise

1. (i)  $a_1 = 3 \times 1 - 1 = 2$   
 $a_2 = 3 \times 2 - 1 = 5$   
 $a_3 = 3 \times 3 - 1 = 8$   
 $a_4 = 3 \times 4 - 1 = 11$

(ii)  $a_1 = 2 \times 3^1 = 6$   
 $a_2 = 2 \times 3^2 = 18$   
 $a_3 = 2 \times 3^3 = 54$   
 $a_4 = 2 \times 3^4 = 162$

(iii)  $a_1 = 1^2 = 1$   
 $a_2 = 2^2 = 4$   
 $a_3 = 3^2 = 9$   
 $a_4 = 4^2 = 16$

(iv)  $a_1 = (-1)^1 2^1 = -2$   
 $a_2 = (-1)^2 2^2 = 4$   
 $a_3 = (-1)^3 2^3 = -8$   
 $a_4 = (-1)^4 2^4 = 16$

(v)  $a_1 = 2$   
 $a_2 = 2a_1 + 1 = 2 \times 2 + 1 = 5$   
 $a_3 = 2a_2 + 1 = 2 \times 5 + 1 = 11$   
 $a_4 = 2a_3 + 1 = 2 \times 11 + 1 = 23$

(vi)  $a_1 = 3$   
 $a_2 = 1 - a_1 = 1 - 3 = -2$   
 $a_3 = 1 - a_2 = 1 - (-2) = 3$   
 $a_4 = 1 - a_3 = 1 - 3 = -2$

2. (i)  $u_5 = 0$ , and then next terms are 1, 2, 3

(ii)  $u_5 = \frac{1}{25}$ , and then next terms are  $\frac{1}{36}$ ,  $\frac{1}{49}$ ,  $\frac{1}{64}$

## Additional Mathematics (OCR): Algebra

(iii)  $u_5 = -\frac{1}{32}$ , and then next terms are  $+\frac{1}{64}$ ,  $-\frac{1}{128}$ ,  $+\frac{1}{256}$

(iv)  $u_3 = -1$ ,  $u_4 = -3$ , and so  $u_5 = -4$  and then next terms are  $-7$ ,  $-11$ ,  $-18$   
(this is an example of a Fibonacci sequence)

3. Since the value of the car reduces by 10% each year. The value each year is 0.9 times the value the previous year. Therefore  $a_{k+1} = 0.9a_k$ .

4. Since the value of the train set increases by 2% each year, if the original value is £150, the value one year later,  $a_1$ , is  $150 \times 1.02$ .

The value the next year  $a_2$  is  $150 \times 1.02 \times 1.02 = 150 \times (1.02)^2$ .

This pattern continues and so the value at the end of the next year  $a_3$  is  $150 \times (1.02)^3$ .

In general  $a_k = 150 \times (1.02)^k$ .

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