Section 5: Sequences and recurrence relations

Solutions to Exercise

1. (i) $a_1 = 3\times1-1=2$
$a_2 = 3\times2-1=5$
$a_3 = 3\times3-1=8$
$a_4 = 3\times4-1=11$

(ii) $a_1 = 2\times3^1=6$
$a_2 = 2\times3^2=18$
$a_3 = 2\times3^3=54$
$a_4 = 2\times3^4=162$

(iii) $a_1 = 1^2=1$
$a_2 = 2^2=4$
$a_3 = 3^2=9$
$a_4 = 4^2=16$

(iv) $a_1 = (-1)^1 2^1=-2$
$a_2 = (-1)^2 2^2=4$
$a_3 = (-1)^3 2^3=-8$
$a_4 = (-1)^4 2^4=16$

(v) $a_1 = 2$
$a_2 = 2a_1+1=2\times2+1=5$
$a_3 = 2a_2+1=2\times5+1=11$
$a_4 = 2a_3+1=2\times11+1=23$

(vi) $a_1 = 3$
$a_2 = 1-a_1=1-3=-2$
$a_3 = 1-a_2=1-(-2)=3$
$a_4 = 1-a_3=1-3=-2$

2. (i) $u_5 = 0$, and then next terms are 1, 2, 3

(ii) $u_5 = \frac{1}{25}$, and then next terms are $\frac{1}{36}$, $\frac{1}{49}$, $\frac{1}{64}$

(iii) $u_5 = -\frac{1}{32}$, and then next terms are $+\frac{1}{64}$, $-\frac{1}{128}$, $+\frac{1}{256}$

(iv) $u_3 = -1$, $u_4 = -3$, and so $u_5 = -4$ and then next terms are -7, -11, -18
(this is an example of a Fibonacci sequence)

3. Since the value of the car reduces by 10% each year. The value each year is 0.9 times the value the previous year. Therefore $a_{k+1} = 0.9a_k$.

4. Since the value of the train set increases by 2% each year, if the original value is £150, the value one year later, a_1, is 150 x 1.02.
The value the next year a_2, is 150 x 1.02 x 1.02 = 150 x $(1.02)^2$.
This pattern continues and so the value at the end of the next year a_3 is 150 x $(1.02)^3$.
In general $a_k = 150 \times (1.02)^k$.