

OCR

Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

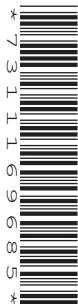
OCR supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

- 1 Express $\sin \theta - 2.4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.
Hence write down the maximum value of the function $f(\theta) = 1 - \sin \theta + 2.4 \cos \theta$, where $0 \leq \theta \leq 2\pi$. [5]
- 2 The finite region bounded by the curve $y = \ln x$, the x -axis, the y -axis and the line $y = 1$ is rotated through 360° about the y -axis. Find the exact volume of the solid of revolution generated. [4]
- 3 Find the first three terms of the binomial expansion of $\frac{1+2x}{(2-x)^3}$ in ascending powers of x .
State the set of values of x for which the expansion is valid. [7]
- 4 A curve has parametric equations $x = \sin 2\theta$, $y = 1 + 2 \cos \theta - \cos 2\theta$, where $0 < \theta < \pi$.
(i) Find $\frac{dy}{dx}$ in terms of θ . [3]
(ii) Find the exact coordinates of the point on the curve where the gradient is zero. [4]
- 5 Fig. 5 shows the curve with equation $y = \sqrt{1+x^3}$.

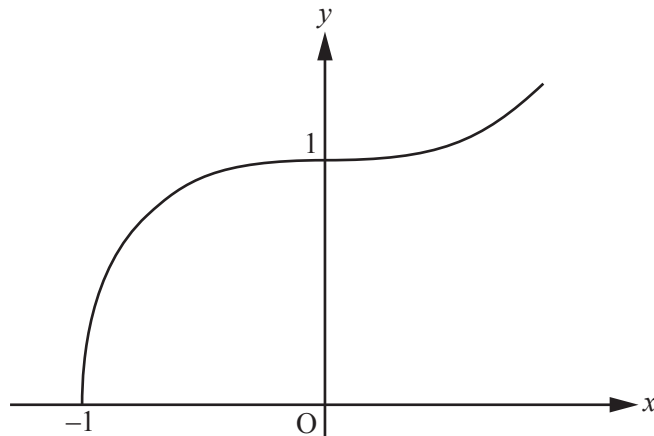


Fig. 5

- (i) Use the trapezium rule with 4 strips to estimate the finite area enclosed by the curve and the x - and y -axes, giving your answer correct to 3 significant figures. [3]
- (ii) Use a quarter circle of radius 1 to estimate this area, giving your answer correct to 3 significant figures. [1]
- (iii) State, with a reason, which of these estimates is closer to the true area. [1]

- 6 In Fig. 6, triangle ADC is right-angled at C, with $CD = h$. The point B on AC is such that $AB = x$, angle $ADB = \alpha$ and angle $BDC = \beta$.

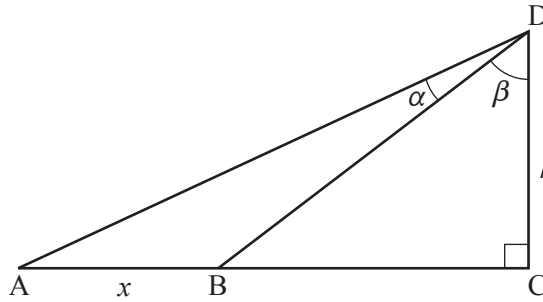


Fig. 6

- (i) Find BC and AC in terms of h , α and β .

Hence show that $x = \frac{h \tan \alpha \sec^2 \beta}{1 - \tan \alpha \tan \beta}$. [5]

- (ii) Given that $x = h$ and $\beta = 30^\circ$, find α , giving your answer correct to 1 decimal place. [3]

Section B (36 marks)

- 7 Three points A, B and C have coordinates A (2, 1, 1), B (1, -3, -1) and C (-4, -1, 0).

- (i) Find the lengths AB and AC, and use a scalar product to calculate the angle BAC.

Hence find the area of triangle ABC. [7]

The lines with vector equations

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}), \quad \mathbf{r} = \mathbf{i} - 3\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -4\mathbf{i} - \mathbf{j} + \nu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

pass through the points A, B and C respectively.

- (ii) Show that these three lines meet at a point D. [6]

You are given that the plane ABC has equation $\mathbf{r} \cdot (\mathbf{j} - 2\mathbf{k}) = -1$. The normal through D to the plane ABC meets the plane at E.

- (iii) Find the coordinates of E. [3]

The volume of a tetrahedron is $\frac{1}{3} \times \text{area of base} \times \text{height}$.

- (iv) Find the volume of the tetrahedron ABCD. [3]

- 8 The speed $v \text{ m s}^{-1}$ of an object at time t seconds is modelled by the differential equation

$$\frac{dv}{dt} = -kv(4 + v^2),$$

where k is a positive constant. Initially, $v = 4$.

(i) Find constants A , B and C such that $\frac{1}{v(4 + v^2)} = \frac{A}{v} + \frac{Bv + C}{4 + v^2}$. [5]

(ii) Hence show by integration that $v = \frac{4}{\sqrt{5e^{8kt} - 4}}$. [9]

(iii) After 1 second the speed of the object is 2 m s^{-1} . Find the value of k . [3]

END OF QUESTION PAPER

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