

OCR

Oxford Cambridge and RSA

Tuesday 19 June 2018 – Afternoon

A2 GCE MATHEMATICS

4735/01 Probability & Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

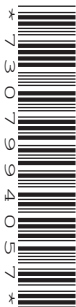
OCR supplied materials:

- Printed Answer Book 4735/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

- 1 A Wilcoxon signed-rank test is carried out at the 5% level of significance on a random sample of size 32. The hypotheses are $H_0: m = m_0$, $H_1: m < m_0$ where m is the population median and m_0 is a specific numerical value. The value obtained for the test statistic T is 162. Find the outcome of the test. [5]

- 2 The distances from home to work, in km, of 8 men and 5 women were recorded and are given below. The workers were chosen at random.

Men	4	7	10	13	16	17	20	21
Women	1	2	14	18	22			

Carry out a Wilcoxon rank-sum test at the 5% significance level to test whether there is a significant difference between the distances from home to work between men and women. [8]

- 3 Events A and B are such that $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cup B) = 0.8$.

(i) Find $P(A \cap B)$. [2]

(ii) Find $P(A \cap B')$. [1]

(iii) Find $P(A|B)$. [2]

Events A and B are as above and a third event C is such that $P(A \cup B \cup C) = 1$, $P(A \cap B \cap C) = 0.05$, $P(A \cap C) = P(B \cap C)$ and $P(A \cap B' \cap C') = 3P(A' \cap B \cap C')$.

(iv) Find $P(C)$. [5]

- 4 The random variable X has a χ^2 distribution with ν degrees of freedom. The moment generating function of X is

$$M_X(t) = (1 - 2t)^{-\frac{1}{2}\nu}.$$

(i) Show that $E(X) = \nu$. [3]

(ii) Find $\text{Var}(X)$. [3]

(iii) Obtain the moment generating function of the sum Y of two independent χ^2 random variables, one with 6 degrees of freedom and the other with 8 degrees of freedom. [2]

(iv) Identify the distribution of Y . [2]

- 5 The independent discrete random variables U and V can each take the values 1, 2 and 3, all with probability $\frac{1}{3}$. The random variables X and Y are defined as follows:

$$X = |U - V|, Y = U + V.$$

- (i) In the Printed Answer Book complete the table showing the joint probability distribution of X and Y . [3]
- (ii) Find $\text{Cov}(X, Y)$. [4]
- (iii) State with a reason whether X and Y are independent. [2]
- (iv) Find $P(Y = 3|X = 1)$. [2]
- 6 In each round of a quiz a contestant can answer up to three questions. Each correct answer scores 1 point and allows the contestant to go on to the next question. A wrong answer scores 0 points and the contestant is allowed no further question in that round. If all 3 questions are answered correctly 1 bonus point is scored, making a total score of 4 for the round. For a certain contestant, A , the probability of giving a correct answer is $\frac{3}{4}$, independently of any other question. The random variable X_r is the number of points scored by A during the r^{th} round.
- (i) Find the probability generating function of X_r . [4]
- (ii) Use the probability generating function found in part (i) to find the mean and variance of X_r . [6]
- (iii) Write down an expression for the probability generating function of $X_1 + X_2$ and find the probability that A has a total score of 4 at the end of two rounds. [3]
- 7 Two independent observations X_1 and X_2 are made of a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{\theta} & 0 \leq x \leq \theta, \\ 0 & \text{otherwise,} \end{cases}$$

where θ is a parameter whose value is to be estimated.

- (i) Find $E(X)$. [1]
- (ii) Show that $S_1 = X_1 + X_2$ is an unbiased estimator of θ . [1]
- L is the larger of X_1 and X_2 , or their common value if they are equal.
- (iii) Show that the probability density function of L is $\frac{2l}{\theta^2}$ for $0 \leq l \leq \theta$. [4]
- (iv) Find $E(L)$. [1]
- (v) Find an unbiased estimator S_2 of θ , based on L . [1]
- (vi) Determine which of the two estimators S_1 and S_2 is the more efficient. [7]

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