



# A LEVEL

**Examiners' report** 

# MATHEMATICS B (MEI)

**H640** For first teaching in 2017

# H640/02 Summer 2018 series

Version 1

www.ocr.org.uk/mathematics

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# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

# Paper H640/02 series overview

This was the second paper for this new A Level and all the candidates had prepared for this examination in one year. The marks were generally very good as many candidates are also Further Mathematics candidates. This paper contributes 36.4% of the total A-level and assesses content from pure mathematics and statistics.

Candidates are expected to have studied statistics using the large data set and to have routinely used spreadsheets, graphing software and calculators when studying this course.

To do well in this component, candidates need to be able to apply their knowledge of the syllabus content in a variety of modelling and statistical contexts, and to make efficient use of calculator technology.

Notation for sample variance and sample standard deviation

Sample variance

$$s^{2} = \frac{1}{n-1}S_{xx}$$
 where  $S_{xx} = \sum (x_{i} - \overline{x})^{2} = \sum x_{i}^{2} - \frac{(\sum x_{i})^{2}}{n} = \sum x_{i}^{2} - n\overline{x}^{2}$ 

Standard deviation

$$s = \sqrt{\text{variance}}$$

The notations  $s^2$  and s for sample variance and sample standard deviation, respectively, are written into both British Standards (BS3534-1, 2006) and International Standards (ISO 3534). In this specification the usage will be consistent with these definitions. Thus the meanings of 'sample variance', denoted by  $s^2$ , and 'sample standard deviation', denoted by s, are defined to be calculated with divisor (*n*-1).

Students should be aware of the variations in notation used by manufacturers on calculators and know what the symbols on their particular models represent

# Section A overview

Section A questions are designed to give all candidates an opportunity to do some of the questions on the paper as they require little reading or interpretation. Most candidates did very well in section A. The content was clearly understood, and the work was clearly and efficiently presented.

#### **Question 1**

1 Show that  $\sqrt{27} + \sqrt{192} = a\sqrt{b}$ , where *a* and *b* are prime numbers to be determined. [2]

This is a show that question; simply obtaining the values for *a* and *b* from the calculator, without any written justification, would not gain full credit.

#### Question 2

2 Solve the inequality |2x+1| < 5.

[3]

This question proved routine for the majority of candidates.

### Question 3 (i)

- 3 The probability that Chipping FC win a league football match is P(W) = 0.4.
  - (i) Calculate the probability that Chipping FC fail to win each of their next two league football matches.

[1]

[1]

#### A small minority of candidates found the probability to win the next two league football matches.

#### Question 3 (ii)

The probability that Chipping FC lose a league football match is P(L) = 0.3.

(ii) Explain why  $P(W) + P(L) \neq 1$ .

Generally answered well, although some mini essays seen.

# Question 4 (i)

4 A survey of the number of cars per household in a certain village generated the data in Fig. 4.

Number of cars	0	1	2	3	4
Number of households	8	22	31	27	7

#### Fig. 4

(i) Calculate the mean number of cars per household.

Whilst it is acceptable to calculate the mean using a formal written method, there is an expectation in the new specification that candidates will use the statistical functions on their calculators for parts (i) and (ii), hence the single mark allocation.

#### Question 4 (ii)

(ii) Calculate the standard deviation of the number of cars per household.

Candidates who did well in this question made efficient use of the appropriate calculator function. Candidates who did less well made laborious calculations and slipped up in the arithmetic.

**Misconception** The choice of appropriate formula (using *n*, *n*-1 or other denominator corrections) is beyond the scope of this A Level Maths qualification, and the use of a single formula for all contexts is expected. The H640 OCR B (MEI) specification and formulae sheet makes clear that the divisor (*n*-1) should be used

# Question 5 (i) (A)

5 (i) (A) Sketch the graph of  $y = 3^x$ .

Sketches should make clear that the function tends towards y=0 but does not cut the x-axis

#### Question 5 (i) (B)

(B) Give the coordinates of any intercepts.

A clear indication of the coordinates (0,1) was required and not just y=1.

#### Question 5 (ii)

The curve y = f(x) is the reflection of the curve  $y = 3^x$  in the line y = x.

(ii) Find f(x).

There was an expectation that candidates would use base 3 logarithms, but a variety of correct functions were given and gained credit.

[1]

[1]

[1]

[1]

[1]

### Question 6 (i)

6 (i) Express  $7\cos x - 24\sin x$  in the form  $R\cos(x+\alpha)$ , where  $0 < \alpha < \frac{\pi}{2}$ . [3]

The majority of candidates gained full credit, with careless arithmetic resulting in dropped accuracy marks on this routine item.

#### Question 6 (ii)

(ii) Write down the range of the function

 $\mathbf{f}(x) = 12 + 7\cos x - 24\sin x , \qquad 0 \le x \le 2\pi .$ 

[2]

[4]

Some candidates answered their own question, taken directly from part (i).

#### **Question 7**

7 Find 
$$\int \left(4\sqrt{x} - \frac{6}{x^3}\right) dx$$
.

Generally answered well, but the + c was often missed.

# Section B overview

Candidates who did well in this section were able to present clear arguments with supporting calculations. They were familiar with the appropriate technology and were able to critique statistical diagrams.

Candidates who did less well did not show full details of their working. In some cases, it appeared that the demand of the question had not been properly understood, and some candidates did not appear to be familiar with all the functionality of their calculator.

# Question 8 (i)

- 8 Every morning before breakfast Laura and Mike play a game of chess. The probability that Laura wins is 0.7. The outcome of any particular game is independent of the outcome of other games. Calculate the probability that, in the next 20 games,
  - (i) Laura wins exactly 14 games,

This was answered well when clear working was shown. Mistakes may have been due to going straight to the calculator and mistakenly finding at least 14 rather than exactly 14.

# Question 8 (ii)

(ii) Laura wins at least 14 games.

Candidates who did well in this question made efficient use of the binomial distribution function in their calculator.

Candidates who did less well mistook "at least" for "greater than" in this part or made arithmetic slips when using traditional methods to calculate the probabilities.

# Question 9 (i)

- **9** At the end of each school term at North End College all the science classes in year 10 are given a test. The marks out of 100 achieved by members of set 1 are shown in Fig. 9.
  - 3
     5

     4
     0
     9

     5
     2
     3
     6

     6
     0
     1
     3
     5
     6

     7
     0
     1
     2
     5
     6
     8
     9
     9

     8
     3
     4
     6
     6
     8
     8
     9

     9
     5
     5
     5
     6
     7

Key 5 2 represents a mark of 52

Fig. 9

(i) Describe the shape of the distribution.

Some confusion between positive and negative skew was seen.

#### [1]

#### 9

[2]

[2]

#### Question 9 (ii)

(ii) The teacher for set 1 claimed that a typical student in his class achieved a mark of 95. How did he justify this statement?

[1]

#### A simple statement was expected, and not a mini essay. Almost every candidate gained this mark.

#### Question 9 (iii)

(iii) Another teacher said that the average mark in set 1 is 76. How did she justify this statement? [1]

A simple statement was expected, and not a mini essay. The majority of candidates gained the mark

#### Question 9 (iv)

Benson's mark in the test is 35. If the mark achieved by any student is an outlier in the lower tail of the distribution, the student is moved down to set 2.

(iv) Determine whether Benson is moved down to set 2.

[2]

Candidates who did well in this question used the lower quartile and the interquartile range to determine whether Benson's mark is an outlier.

Candidates who did less well used the median in conjunction with the lower quartile.

#### Question 10 (i)

10 The screenshot in Fig. 10 shows the probability distribution for the continuous random variable X, where  $X \sim N(\mu, \sigma^2)$ .

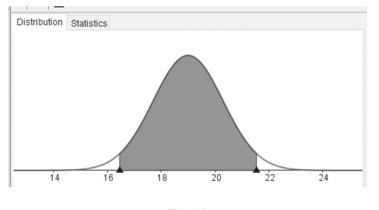


Fig. 10

The area of each of the unshaded regions under the curve is 0.025. The lower boundary of the shaded region is at 16.452 and the upper boundary of the shaded region is at 21.548.

(i) Calculate the value of  $\mu$ .

[1]

Candidates who did well in this question wrote down the mean by symmetry.

#### Question 10 (ii)

(ii) Calculate the value of  $\sigma^2$ .

Candidates who did well on this part made efficient use of the standard Normal variable to find the variance. Candidates who did less well confused the mean with the variance, or used a wrong value for z (usually 1.645) to find the variance.

#### Question 10 (iii) (A)

- (iii) *Y* is the random variable given by Y = 4X + 5.
  - (A) Write down the distribution of Y.

A common error in this part was to calculate 4× 1.69 or 16× 1.69 + 5 instead of 16×1.69 .

#### Question 10 (iii) (B)

(*B*) Find P(Y > 90).

This part required efficient use of the calculator.

#### Question 11 (i)

11 The discrete random variable X takes the values 0, 1, 2, 3, 4 and 5 with probabilities given by the formula

P(X=x) = k(x+1)(6-x).

(i) Find the value of k.

Candidates who did well in this question found *k* successfully and used the result in fractional form in part (ii) to calculate the requested probability.

#### Question 11 (ii)

In one half-term Ben attends school on 40 days. The probability distribution above is used to model *X*, the number of lessons per day in which Ben receives a gold star for excellent work.

(ii) Find the probability that Ben receives no gold stars on each of the first 3 days of the half-term and two gold stars on each of the next 2 days.
 [2]

11

The most common error was for candidates to add the two probabilities instead of multiplying.

[3]

[1]

[3]

[2]

### Question 11 (iii)

(iii) Find the expected number of days in the half-term on which Ben receives no gold stars.

[2]

There was a significant number of candidates that spoiled their answer to this part by rounding to the nearest whole number.

#### Question 12

12 You must show detailed reasoning in this question.

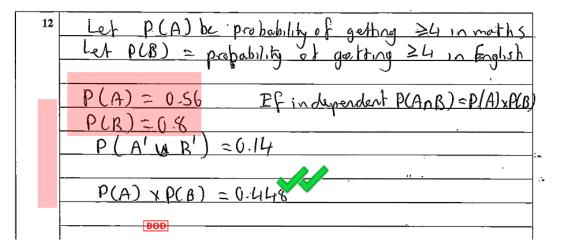
In the summer of 2017 in England a large number of candidates sat GCSE examinations in **both** mathematics **and** English. 56% of these candidates achieved at least level 4 in mathematics and 80% of these candidates achieved at least level 4 in English. 14% of these candidates did not achieve at least level 4 in either mathematics or English.

Determine whether achieving level 4 or above in English and achieving level 4 or above in mathematics were independent events. [5]

Candidates who did well in this question defined appropriate events and set out a clear, reasoned argument. They usually worked with a Venn diagram and clearly showed that  $p(A \text{ and } B) \neq p(A) \times p(B)$  in this case.

Candidates who did less well wrote down relevant ideas but were unable to draw them together successfully. They sometimes stated results instead of giving details of the calculation.

#### Exemplar 1



This candidate has been given BOD M1A1 for the calculation of  $p(A) \times p(B)$  being correctly done, as highlighted by the statements on the LHS. The events are clearly defined.

No attempt was made to find the actual value of p(A and B), so no further progress was made.

# Question 13 (i)

13 Each weekday Keira drives to work with her son Kaito. She always sets off at 8.00 a.m. She models her journey time, x minutes, by the distribution  $X \sim N(15, 4)$ .

Over a long period of time she notes that her journey takes less than 14 minutes on 7% of the journeys, and takes more than 18 minutes on 31% of the journeys.

(i) Investigate whether Keira's model is a good fit for the data.

[3]

Candidates who did well in this question made appropriate calculations, showed full working and explained the significance of their results.

Candidates who did less well gave incomplete reasoning and/or made slips in their calculations.

#### Question 13 (ii)

Kaito believes that Keira's value for the variance is correct, but realises that the mean is not correct.

(ii) Find, correct to two significant figures, the value of the mean that Keira should use in a refined model which does fit the data.
 [2]

Candidates who did less well in this question ignored the request for an answer correct to two significant figures.

#### Question 13 (iii)

Keira buys a new car. After driving to work in it each day for several weeks, she randomly selects the journey times for n of these days. Her mean journey time for these n days is 16 minutes. Using the refined model she conducts a hypothesis test to see if her mean journey time has changed, and finds that the result is significant at the 5% level.

(iii) Determine the smallest possible value of *n*.

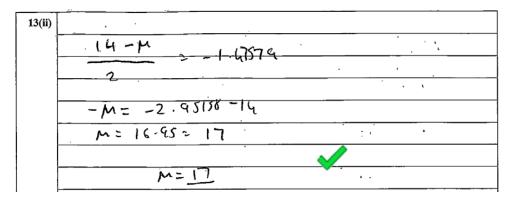
[5]

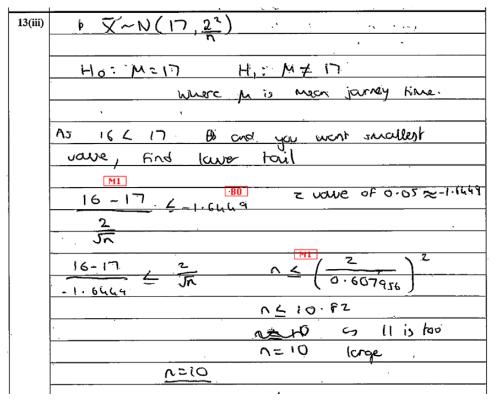
Candidates who did well in this question recognised that the hypothesis test was two-tailed and worked with  $\sqrt{n}$  in their calculations.

Candidates who did less well worked with an incorrect *z* value or an incorrect form of the standard deviation.

#### Exemplar 2

13(i) 
$$P(X \leq 14) = 0.07$$
  $P(X > 18) = 0.31$   
 $14 - M$   
 $5 = -1.41574$   $15 - M$   
 $14 - 15 = -0.5 \times 1.5$   
 $16 - M$   
 $18 - 15 = 0.49385$   
 $18 - 15 = 1.5$   
 $5 = 1.5$   
 $5 = 1.5$   
 $15 - 15 = 1.5$   
 $5 = 1.5$   
 $1.5 = 1.5$   
 $1.5 \times 1.5$   
 $2 \times 1.5$   
 $1.5 \times 1.5$   
 $2 \times 1.5$   





Note that in part (i) the candidate has chosen a less frequently seen approach, identified in the RH column of the mark scheme.

Part (ii) is well done.

In part (iii) the wrong choice of *z* proved costly.

# Question 14 (i) (A)

14 The pre-release material includes data on unemployment rates in different countries. A sample from this material has been taken. All the countries in the sample are in Europe. The data have been grouped and are shown in Fig 14.1.

Unemployment rate	0—	5–	10–	15–	20–	35–50
Frequency	15	21	5	5	2	2

Fig.	14.1
1.15.	1.4.1

A cumulative frequency curve has been generated for the sample data using a spreadsheet. This is shown in Fig. 14.2.

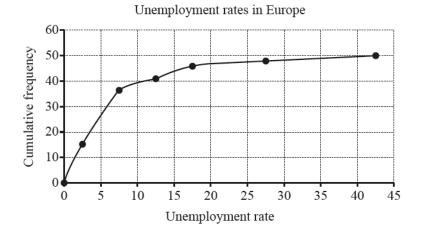


Fig. 14.2

Hodge used Fig. 14.2 to estimate the median unemployment rate in Europe. He obtained the answer 5.0. The correct value for this sample is 6.9.

- (i) (A) There is a systematic error in the diagram.
  - Identify this error.
  - State how this error affects Hodge's estimate.

[2]

Candidates who did well in this question recognised that the cumulative frequencies had been plotted at the mid-point of the intervals instead of at the upper limit.

#### Question 14 (i) (B)

- (B) There is another factor which has affected Hodge's estimate.
  - · Identify this factor.
  - State how this factor affects Hodge's estimate.

[2]

Candidates understood that grouping the data affects the accuracy of the result and commented accordingly.

Candidates who did less well made comments about whether the points had been joined by straight lines or a curve.

#### Question 14 (ii)

Use your knowledge of the pre-release material to give another reason why any estimation of the median unemployment rate in Europe may be unreliable. [1]

Candidates who did less well based comments on general geographical or economic ideas, rather than specifically related to issues related to the estimation of median values.

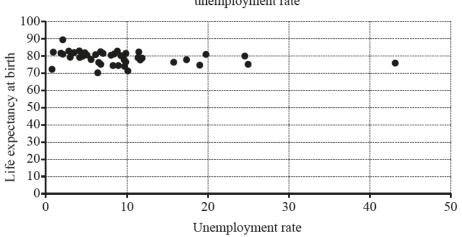
#### Question 14 (iii)

(iii) Use your knowledge of the pre-release material to explain why it is very unlikely that the sample has been randomly selected from the pre-release material. [1]

Candidates who did well on part (ii) and part (iii) were familiar with the pre-release material made appropriate comments.

#### Question 14 (iv)

The scatter diagram shown in Fig. 14.3 shows the unemployment rate and life expectancy at birth for the 47 countries in the sample for which this information is available.



Scatter diagram to show life expectancy at birth against unemployment rate



The product moment correlation coefficient for the 47 items in the sample is -0.2607. The *p*-value associated with r = -0.2607 and n = 47 is 0.0383.

(iv) Does this information suggest that there is an association between unemployment rate and life expectancy at birth in countries in Europe?
 [2]

Candidates who did well commented on the nature of the correlation. They compared the *p*-value with a significance level and then made an appropriate deduction. Candidates who did less well compared the correlation coefficient with the *p*-value or did not comment on the association at all.

Hodge uses the spreadsheet tools to obtain the equation of a line of best fit for this data.

(v) The unemployment rate in Kosovo is 35.3, but there is no data available on life expectancy. Is it reasonable to use Hodge's line of best fit to estimate life expectancy at birth in Kosovo? [1]

Candidates who did well commented on the nature of the scatter or the position of 35.3 relative to the given values to justify their comment.

#### **Question 15**

#### 15 You must show detailed reasoning in this question.

The equation of a curve is

$$y^3 - xy + 4\sqrt{x} = 4$$

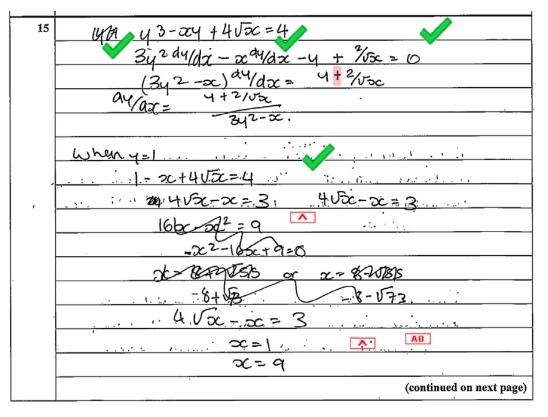
Find the gradient of the curve at each of the points where y = 1.

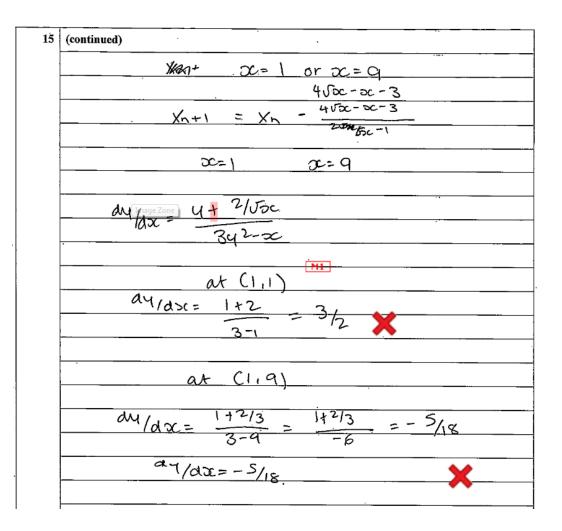
[9]

Candidates who did well in this question explained their reasoning clearly, in particular they showed how the values of *x* were obtained and how  $\frac{dy}{dx}$  was obtained.

Candidates who did less well made algebraic and arithmetic slips. They did not fully explain their reasoning, especially when finding the values of *x*.

#### **Exemplar 3**





This candidate differentiated correctly, but made a sign error when rearranging, which cost accuracy marks at the end. Note that detailed reasoning was required in this question. It was not clear how the (correct) values of *x* were obtained so A marks were withheld.

#### Question 16 (i)

16 In the first year of a course, an A-level student, Aaishah, has a mathematics test each week. The night before each test she revises for *t* hours. Over the course of the year she realises that her percentage mark for a test, *p*, may be modelled by the following formula, where *A*, *B* and *C* are constants.

$$p = A - B(t - C)^2$$

- Aaishah finds that, however much she revises, her maximum mark is achieved when she does 2 hours revision. This maximum mark is 62.
- Aaishah had a mark of 22 when she didn't spend any time revising.
- (i) Find the values of A, B and C.

[3]

Candidates who did well in this question recognised that the maximum value of p has to be 62, and that this occurs when t = 2, thus obtaining A and C. The value of B soon follows.

This who did less well wrote down three equations in three variables and often went astray.

#### Question 16 (ii)

(ii) According to the model, if Aaishah revises for 45 minutes on the night before the test, what mark will she achieve?
 [2]

Candidates who did well made the correct substitution in their formula.

Candidates who did less well substituted t = 45.

#### Question 16 (iii)

(iii) What is the maximum amount of time that Aaishah could have spent revising for the model to work?

[2]

Candidates who did well understood that the value of *p* couldn't be negative and obtained a value for *t* accordingly.

Candidates who did less well worked with < instead of = or >.

#### Question 16 (iv)

In an attempt to improve her marks Aaishah now works through problems for a total of t hours over the three nights before the test. After taking a number of tests, she proposes the following new formula for p.

$$p = 22 + 68(1 - e^{-0.8t})$$

For the next three tests she recorded the data in Fig. 16.

t	1	3	5
р	59	84	89



(iv) Verify that the data is consistent with the new formula.

[2]

Candidates who did well made correct substitutions and provided a supporting comment.

Candidates who did less well neglected to substitute all three values or made no comment on what their calculations showed.

#### Question 16 (v)

(v) Aaishah's tutor advises her to spend a minimum of twelve hours working through problems in future.
 Determine whether or not this is good advice.

Candidates who did well evaluated p at t = 12 and either at t = 5 or a value in between 5 and 12 and commented appropriately.

Candidates who did less well made no supporting calculations but supplied a comment, or vice versa.

#### Question 17 (i)

17 (i) Express 
$$\frac{(x^2 - 8x + 9)}{(x+1)(x-2)^2}$$
 in partial fractions.

Candidates who did well recognised the correct form of partial fractions and were able to work successfully to find the coefficients.

Candidates who did less well made algebraic slips in clearing the fractions or made slips in arithmetic when finding the coefficients.

#### Question 17 (ii)

(ii) Express y in terms of x given that

$$\frac{dy}{dx} = \frac{y(x^2 - 8x + 9)}{(x+1)(x-2)^2} \text{ and } y = 16 \text{ when } x = 3.$$

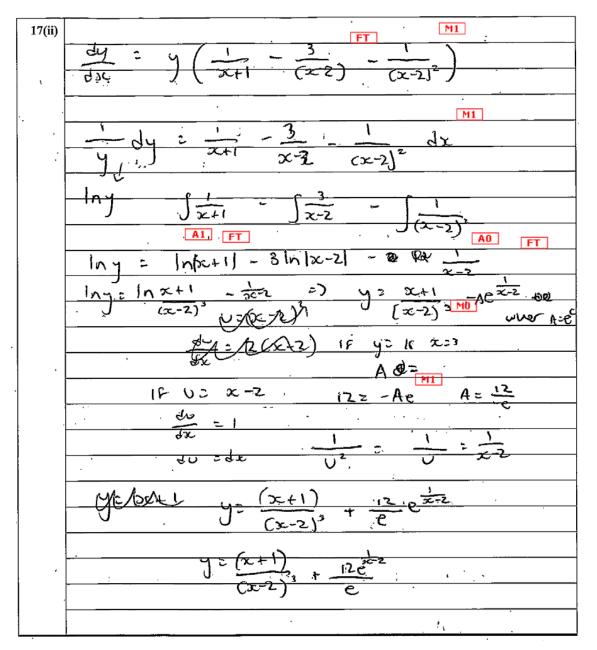
[7]

Candidates who did well recognised the need to use their result from part (i). They separated the variables successfully and were then able to integrate and substitute the values of *x* and *y* to find the constant of integration. Candidates who did very well were able to go on and find a correct expression for *y*.

Candidates who did less well rearranged incorrectly when they attempted to separate the variables, or were unable to integrate the quadratic term correctly. They made slips in exponentiating both sides of their equation, usually assuming that the operation is distributive.

[5]

#### Exemplar 4



In this response FT marks have been credited for the use of their partial fractions and separation of variables. One A mark has been credited FT, but the integration of the quadratic term went astray.

The exponentiation of both sides was incorrect, but in spite of this, the method mark for substitution was subsequently earned.

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