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Examiners' report

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y531/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y531/01 series overview

Y531 is the mandatory paper for AS Further Mathematics. It is taken alongside two other papers which can be freely chosen from a choice of 4 other papers. It tests knowledge of proof, complex numbers, matrices, vectors and further algebra as well as testing the understanding of the overarching themes of mathematical argument, problem solving and modelling. To do well on this paper candidates need a thorough understanding of the techniques covered and they need to support their answers with detailed working. They also need to have good algebraic and numerical manipulation skills.

Candidates should be aware that if they make multiple attempts at a question, then it is the last attempt that will be marked. If it is unclear which is the last attempt, for example if one attempt is in the response space in the main paper and one is on extra sheets, then the examiner will choose which they believe to be the last attempt, which may not be to the advantage of the candidate. Candidates should make it clear to the examiner which attempt they want to be marked.

Results given, or proved, in the "stem" of a question can be used throughout the question, but results given or proved in a "part" of a question cannot (usually) be assumed in other parts.

Candidates seemed to be well prepared for this paper, and there were a considerable number of excellent scripts. For the most part, candidates presented the work well but in a few cases poor handwriting led to work being hard to follow or candidate making mistakes through being unable to read what they had written. Apart from question 8(iii) there were very few questions being omitted in the scripts.

The specification includes a section on the meaning of some of the command words used in questions to indicate the required level of written justification needed for full credit. However, candidates should be reminded of the importance of showing their working (even when the question does not demand it) as it is not possible to give method marks for a wrong answer if no method is shown. Whilst candidates should be encouraged to make efficient use of their calculators, and develop confidence with all the available functions, the specification makes mention of the standard advice regarding writing down any explicitly any expressions to be evaluated; stating the values of any parameters and variables used; and using correct mathematical notation.

Candidate Performance Overview

Candidates who did well on this paper:

- Had fluent algebraic and numerical manipulation skills, especially when dealing with indices and negative numbers
- Showed full working to support their answers, and their presentation was clear
- Ensured that they showed detailed reasoning when the question asked for this, or when the question was a "show that"
- Used the more straightforward techniques out of those possible for a question (especially on questions 2, 5, 7, and 8)

Candidate who did less well on this paper:

- Made errors in manipulation
- Showed little working, or working that was hard to follow
- Assumed results they were trying to prove, or assumed that results given in one part of the question were still valid in another part

Question 1(i)

1 **(i)** Find a vector which is perpendicular to both $\begin{pmatrix}1\\3\\-2\end{pmatrix}$ and $\begin{pmatrix}-3\\-6\\4\end{pmatrix}$. **[2]**

The majority of candidates answered this question correctly. Most common errors were multiplying two negative numbers to get a negative, or getting the wrong sign for the **j** component of the answer.

Some candidates who got the wrong answer showed no working to support their answer, which meant they could not be given any credit.

Question 1(ii)

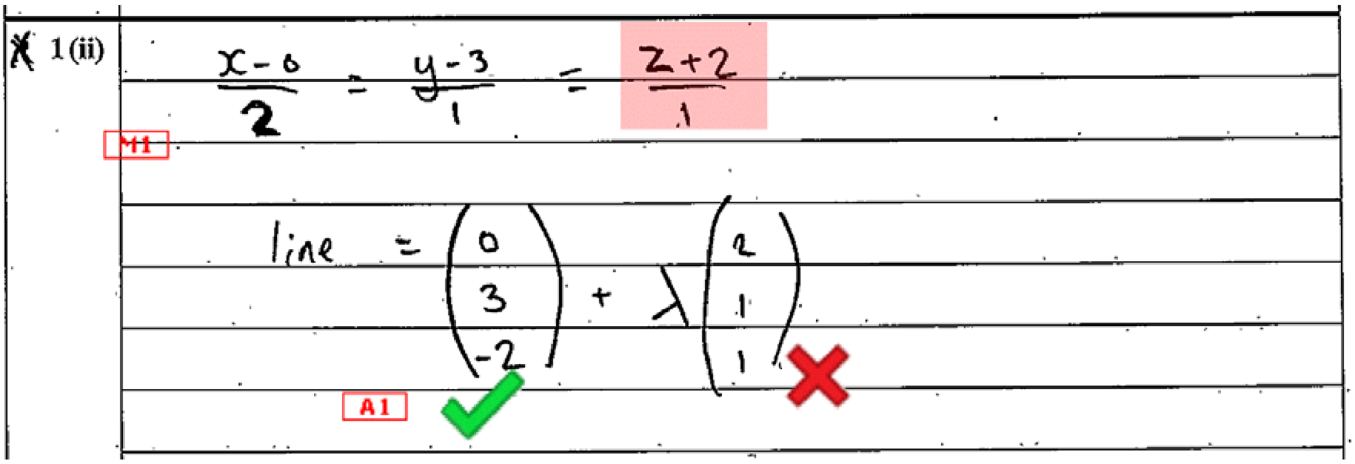
(ii) The cartesian equation of a line is $\frac{x}{2} = y - 3 = 2z + 4$.

Express the equation of this line in vector form. **[3]**

This was also a very well answered question, where the majority of candidates gained full credit (slightly fewer than in the previous part). The most common mistakes here were to fail to deal with the $2z+4$ term correctly, or to write $y-3 = \frac{y-3}{0}$ rather than $y-3 = \frac{y-3}{1}$

Probably the most successful technique (though rarely used) was to equate each of the expressions to λ and then rearrange to get $y = 3 + \lambda$ etc.

Exemplar 1



This response shows a typical incorrect answer to this question. The $2z+4$ term has been dealt with incorrectly, which is the aspect that candidates were most likely to make a mistake with.

Question 2

2 **In this question you must show detailed reasoning.**
The cubic equation $2x^3+3x^2-5x+4=0$ has roots α, β and γ. By making an appropriate substitution, or otherwise, find a cubic equation with integer coefficients whose roots are $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. **[3]**

Another well answered question, with the majority of candidates gaining full credit. There were two main techniques used to answer the question. The more straightforward technique of substituting $x=\frac{1}{u}$ was usually the most successfully applied technique, although some candidates got no further than the substitution, so only gaining the first mark, whilst others multiplied throughout by u^3 but omitted the "=0" and so did not gain the last mark as they did not have an equation as their answer.

The other technique used was to use the formulae for the coefficients of the equation in terms of the roots. Many candidates applied this well, but as it is a much more involved technique they often made a mistake along the way, and needed to keep careful track of the negative signs. Candidates using this technique were less likely to write their cubic equation with integer coefficients, and so not gaining full credit.

Question 3(i)

3 **In this question you must show detailed reasoning.**
The complex numbers z_1 and z_2 are given by $z_1=2-3\text{i}$ and $z_2=a+4\text{i}$ where a is a real number.

(i) Express z_1 in modulus-argument form, giving the modulus in exact form and the argument correct to 3 significant figures. **[3]**

The request to "show detailed reasoning" means that candidates must show working to support their answer. Most candidates gained a mark for finding the modulus, but to gain credit for finding the argument they needed to show some method to support their angle.

Many different forms of the modulus argument form were seen, but if candidates are using the $\left[\sqrt{13}, -0.983\right]$ form, they must be careful to use square brackets and not curved ones.

Question 3(ii)

(ii) Find z_1z_2 in terms of a, writing your answer in the form $c+\text{i}d$. **[2]**

Lots of very good answers here, but some candidates did not gain the second mark, as they did not write the answer in the required form, i.e. gathering the imaginary terms as $(8-3a)\text{i}$. Some candidates left two imaginary terms in the final answer, whilst others gathered the *a* terms instead.

Question 3(iii)

(iii) The real and imaginary parts of a complex number on an Argand diagram are x and y respectively. Given that the point representing z_1z_2 lies on the line $y=x$, find the value of a. **[2]**

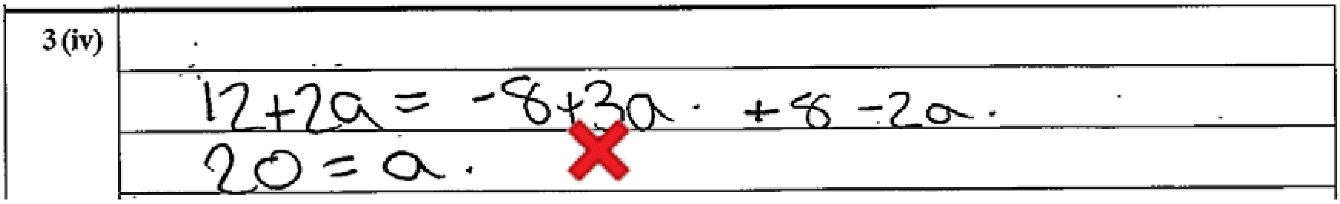
Lots of very good answers here, and follow through from the previous part was allowed for full credit. The most common mistake here was to keep the "i" present when equating real and imaginary parts.

Question 3(iv)

(iv) Given instead that $z_1z_2 = (z_1z_2)^*$ find the value of a. **[2]**

Lots of good answers here, but fewer than in part (iii). Most candidates who got this incorrect assumed that the condition in part (iii) still held and hence that $(z_1z_2)^*$ also lies on the line $y = x$. This is despite this question starting with "Given instead".

Exemplar 2



3(iv)

$12+2a = -8+3a \quad +8 \ -2a$

$20 = a$

This response shows the most common error for this question. Instead of equating (z_1z_2) and $(z_1z_2)^*$, the candidate has assumed that $(z_1z_2)^*$ lies on the line $y = x$.

Question 4(i)

4 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & a \end{pmatrix}$.

(i) Show that $\det\mathbf{A} = 6 - 3a$. **[2]**

Many good answers here. Most candidates used a row or column to find the determinant, but a few used the sum of the "forward diagonals" - the sum of the "backwards diagonals". As the answer was given candidates needed to show full working to support their answer.

Question 4(ii)

(ii) State the value of a for which **A** is singular. **[1]**

Almost all candidates gained the mark here, a few made mistakes when solving $6 - 3a = 0$ (usually to get $a = -2$), or said that for a matrix to be singular the determinant is equal to 1.

Question 4(iii)

(iii) Given that **A** is non-singular find $\mathbf{A}^{-1}$ in terms of a. **[4]**

There was quite a bit of room for arithmetical errors in this question, especially when dealing with negative signs. The most common mistakes were to fail to include the "sign pattern" on the cofactors, or to multiply the cofactors with the corresponding element in the original matrix, possibly mixing up the cofactor and finding the determinant techniques.

Question 5(i)

5 **In this question you must show detailed reasoning.**

(i) Express $(2+3\mathrm{i})^3$ in the form $a+ib$. **[3]**

Quite a few candidates lost marks here through not showing sufficiently "detailed working". For full credit the candidates needed to show all the terms of the binomial expansion and that $\mathrm{i}^2=-1$ and $\mathrm{i}^3=-\mathrm{i}$, or to show full working to justify $(2+3\mathrm{i})^2=-5+12\mathrm{i}$ and $(-5+12\mathrm{i})(2+3\mathrm{i})=-46+9\mathrm{i}$.

Question 5(ii)

(ii) Hence verify that $2+3\mathrm{i}$ is a root of the equation $3z^3-8z^2+23z+52=0$. **[3]**

Again, candidates often did not show sufficient "detailed working". The most common way to not gain full credit was to not show that both the real and imaginary terms separately sum to zero. A few decided to ignore the "hence" and try to divide out by a factor of $z-(2+3\mathrm{i})$, they were not usually successful and if using this method the candidate needs to draw attention to the zero remainder. Some other candidates assumed what they were trying to show, and hence that $2-3\mathrm{i}$ is also a root. These candidates needed to find the third root and then expand convincingly if they were to gain credit for this technique.

Question 5(iii)

(iii) Express $3z^3-8z^2+23z+52$ as the product of a linear factor and a quadratic factor with real coefficients. **[4]**

Most candidates were able to state that $2-3\mathrm{i}$ is also a root. Candidates who then expanded $(z-(2+3\mathrm{i}))(z-(2-3\mathrm{i}))$ to find the quadratic factor and then found the linear factor by inspection usually gained full credit, but a few made arithmetical mistakes, or did not write their final answer as a product of the linear and quadratic factor.

Some candidates found the real root first. This tended to be a less successful technique and the most common mistakes were to leave the linear factor as $\left(z+\frac{4}{3}\right)$, or to make mistakes when dividing the cubic by $(3z+4)$ or $\left(z+\frac{4}{3}\right)$.

A few candidates tried to use the formulae connecting the roots and the coefficients of the cubic. This tended not to be a successful strategy.

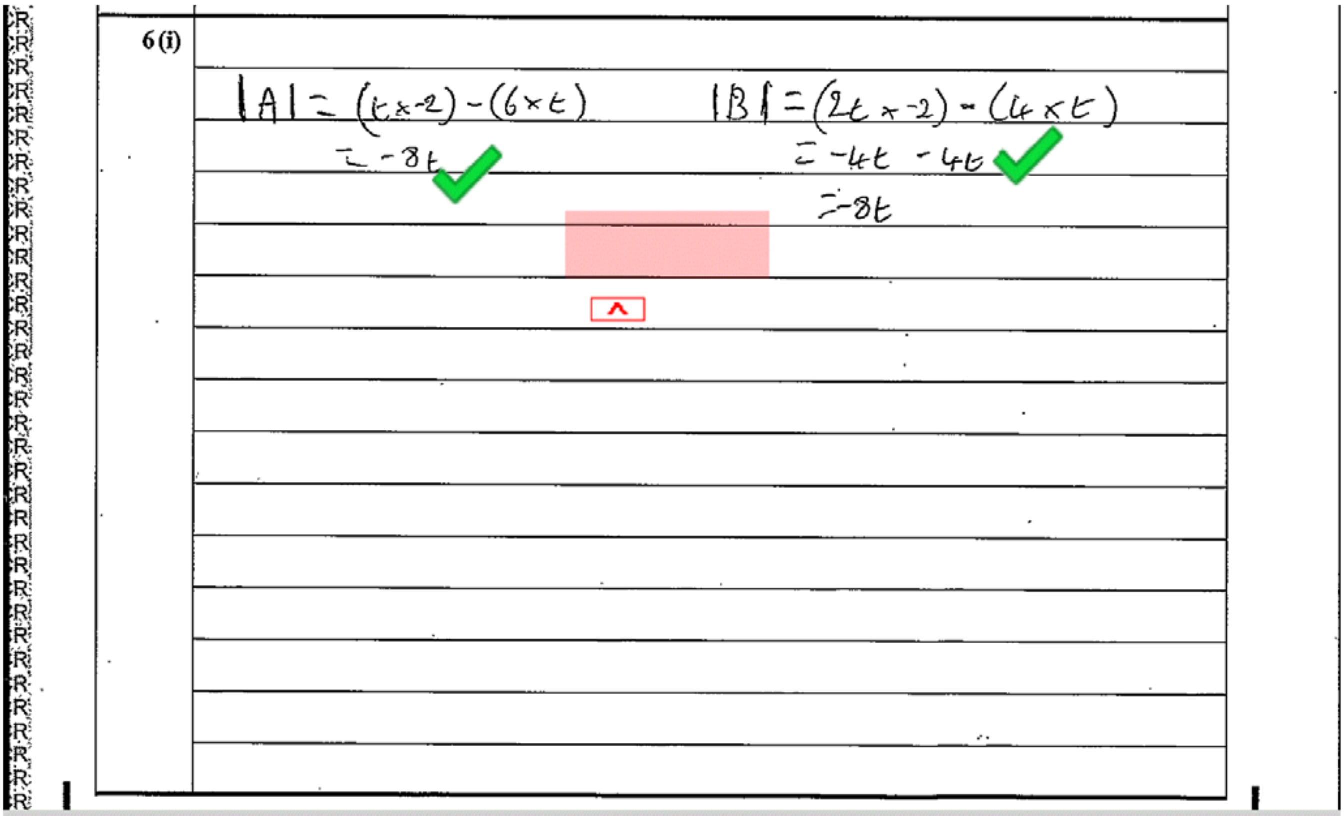
Question 6(i)

6 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$ where t is a constant.

(i) Show that $|\mathbf{A}| = |\mathbf{B}|$. **[2]**

Most candidates knew how to tackle this question and could calculate the two determinants correctly, however quite a few did not make a clear statement of equality and so did not gain the second mark. A few candidates made mistakes when dealing with the negative signs. Some candidates were a bit uncertain of the determinant notation, and stripped off the negative signs as they confused |**A**| with a modulus sign.

Exemplar 3



6(i)

$|A| = (t \times -2) - (6 \times t)$

$= -8t$

$|B| = (2t \times -2) - (4 \times t)$

$= -4t - 4t$

$= -8t$

Both determinants have been found correctly but no statement of equality or other indication that the candidate realises that they are equal. (Question was a "show that")

This is a typical response from a candidate. Both determinants have been found correctly, but there is no statement of equality, so only 1 mark out of 2 is gained.

Question 6(ii)

(ii) Verify that $|\mathbf{AB}| = |\mathbf{A}||\mathbf{B}|$. **[3]**

A few candidates got the rows and columns mixed up, and so did not gain full credit even though they had found the "correct" determinant. In many cases the conclusion |**AB**| = |**A**||**B**| was not stated.

Question 6(iii)

(iii) Given that $|\mathbf{AB}| = -1$ explain what this means about the constant t. **[2]**

Most candidates answered this well, and many benefited from the follow through mark.

Question 7

7 Prove by induction that $2^{n+1} + 5 \times 9^n$ is divisible by 7 for all integers $n \geqslant 1$. **[6]**

This question was found to be more difficult than the previous ones. Most candidates gained the first two marks for showing the *n*=1 case is divisible by 7 and assuming that the *n*=*k* case is divisible by 7. The most successful candidates usually wrote, "$2^{k+1} + 5 \times 9^k = 7A$" rather than "is divisible by 7" in words. Some candidates did not state what they were assuming for the *n*=*k* case, just writing down the given expression with *n*=*k* substituted.

Some candidates work was marred by a misunderstanding of how indices work, usually by writing $5 \times 9^k = 45^k$ but other similar mistakes were made. These candidates usually gained only the first two marks.

A few candidates took the route of considering f(*k*+1)-f(*k*) and trying to show that this was divisible by 7. Many of these got unstuck when dealing with the indices, others manages to show that the difference was a multiple of 7 but then did not explain why this meant that f(*k*+1) was a multiple of 7. There were a handful of candidates who used this approach to complete a convincing proof by induction.

Only the higher achieving candidates gained full marks for this question, and many did not gain the final mark, as their conclusion of the induction process was not clear. A typical incorrect conclusion was "since it is true for *n*=1, *n*=*k* and *n*=*k*+1 ..."

Question 8(i)

8 The 2×2 matrix $\mathbf{A}$ represents a transformation T which has the following properties.

- The image of the point $(0, 1)$ is the point $(3, 4)$.
- An object shape whose area is 7 is transformed to an image shape whose area is 35.
- T has a line of invariant points.

(i) Find a possible matrix for $\mathbf{A}$. **[8]**

This was found by the candidates to be the second hardest question part on the paper (after question 8(iii)). Most candidates gained the first two marks, for using the image of (0,1) and for setting the determinant equal to 5 (those who set it equal to -5 or ±5 also gained this mark).

Many candidates found the line of invariant points hard to interpret, many assuming that the matrix had an invariant line instead.

Another common mistake was to make arithmetical mistakes when solving the simultaneous equations, such as sign errors.

Some candidates stated the correct answer with very little working. As this question was not a "show that" and did not require "detailed reasoning" then this was allowed full credit. This is a risky strategy as if the answer was wrong then no marks for correct method can be given.

Question 8(ii)

The transformation S is represented by the matrix **B** where $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$.

(ii) Find the equation of the line of invariant points of S. **[2]**

This part was found to be slightly easier than the other parts in question 8. The most common mistake was to try and find the invariant lines rather than the line of invariant points. A few candidates only considered the *x* coordinate and not the *y* coordinate.

Exemplar 4

8(ii)

$B = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$

$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} u \\ mu \end{pmatrix}$ $\quad u' = 3u + mu$, $\quad v' = 2u + 2mu$

$\cancel{3}\ 2\cancel{u} + 2m\cancel{u} = m(3\cancel{u} + m\cancel{u})$

$2 + 2m = 3m + m^2 \quad m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$

$m = -2$ or $m = 1$ $\quad y = x$ or $y = -2x$.

Candidate has not used the definition of an invariant point (has used invariant lines of the form y=mx). If they went on to show that y=-2x is a line of invariant points and y=x is not this would gain credit. As it stands, no progress towards finding line of invariant points

This candidate has confused "line of invariant points" with "an invariant line", and they have also assumed that all invariant lines have the form $y = mx$. A candidate using this approach can gain full credit if they then show that the line $y = -2x$ is an invariant line (i.e. by evaluating $\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} a \\ -2a \end{pmatrix} = \begin{pmatrix} a \\ -2a \end{pmatrix}$)

Question 8(iii)

(iii) Show that any line of the form $y = x + c$ is an invariant line of S. **[3]**

This was found to be the hardest question part in the paper, and it was also the one with the highest omit rate (20% did not attempt this question). Many candidates did not use the form of the line given in the question, but instead tried to use a general line such as $y = mx + c$ (or $y = mx$ but this was not a valid method). Candidates using $y = mx + c$ could gain full credit for this question, but considerably more work was needed than if they evaluated $\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ x+c \end{pmatrix}$.

?

Candidates got confused between lines of invariant points and invariant lines. In addition to this, many candidates assumed that an invariant line had to be of the form $y=mx$, i.e. that it passes through the origin.

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