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Examiners' report

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y534/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y534/01 series overview

Y534/01 is one of the four optional examination components for H235, the new revised AS Examination for GCE Further Mathematics A. This component tests applications of Discrete Mathematics to solving real-world problems of existence, construction, enumeration and optimisation using contexts from graphs and networks, algorithms, critical path analysis, linear programming and game theory.

Candidate performance overview

Candidates mostly coped well on this first paper from the new specification, demonstrating a good understanding of the new material and being well prepared for problem solving questions and problems involving mathematical modelling.

Examiners will try to interpret candidates' intentions but sometimes it is impossible to read a response.

- A few candidates had messy or very tiny handwriting that was difficult to read, even when enlarged.
- Some candidates had overwritten answers leaving the result impossible to read.

Most candidates were able to attempt every question, although a few may have been short of time on the final few parts.

Question 1(i)

1 Some jars need to be packed into small crates.

There are 17 small jars, 7 medium jars and 3 large jars to be packed.

- A medium jar takes up the same space as four small jars.
- A large jar takes up the same space as nine small jars.

Each crate can hold:

- at most 12 small jars,
- or at most 3 medium jars,
- or at most 1 large jar (and 3 small jars),
- or a mixture of jars of different sizes.

(i) One strategy is to fill as many crates as possible with small jars first, then continue using the medium jars and finally the large jars.

Show that this method will use seven crates. **[2]**

This part was answered successfully by most of the candidates.

Some candidates used a coding, such as 1 = S, 4 = M, 9 = L, this was fine provided a key was given.

Question 1(ii)

The jars can be packed using fewer than seven crates.

(ii) The jars are to be packed in the minimum number of crates possible.

- Describe how the jars can be packed in the minimum number of crates.
- Explain how you know that this is the minimum number of crates. **[3]**

The most successful answers were those that listed the packing in six crates and then either stated that all the jars had been packed and all the crates were full or calculated the minimum number of crates as $72 \div 12 = 6$.

A few candidates left out some of the jars.

Question 1(iii)

Some other numbers of the small, medium and large jars need to be packed into boxes.

The number of jars that a box can hold is the same as for a crate, except that

- a box cannot hold 3 medium jars.

(iii) Describe a packing strategy that will minimise the number of boxes needed. **[1]**

Candidates needed to use a strategy that would both minimise the number of boxes and ensure that no box had more than 2 medium jars.

Exemplar 1

1(iii) pack medium jars first in groups of 2, then 1 singular.
- then pack large jars in boxes.
- then fill gaps w/ small jars.

This candidate packs pairs of medium jars, putting the remaining medium jar and the large jars into separate boxes, and then fills up the spaces with small jars.

Question 2 (i)

2 Mo eats exactly 6 doughnuts in 4 days.

(i) What does the pigeonhole principle tell you about the number of doughnuts Mo eats in a day? **[1]**

Many excellent answers, saying that Mo must eat at least two doughnuts on at least one day.

Some candidates made statements about the number of doughnuts eaten that were not consequences of the pigeonhole principle (such as stating the average number of doughnuts eaten per day).

Question 2 (ii)

Mo eats exactly 6 doughnuts in 4 days, eating at least 1 doughnut each day.

(ii) Show that there must be either two consecutive days or three consecutive days on which Mo eats a total of exactly 4 doughnuts. **[3]**

Candidates needed to discuss all the various options. Some candidates described the different possibilities and explained how each fitted the requirements. Several candidates listed the various possibilities and then, for example, underlined the days when there were exactly 4 doughnuts.

Exemplar 2

2(ii)

pigeonhole = days (4)

Eats at least one a day.

The other 2 doughnuts can go in any.

If at ~~ea~~ either end, it will take 3 consecutive days.

If remaining 2 in any day, it will take 2 consecutive days.

Those are the only ways, so it must take either 2 or 3 days for him to eat 4.

This candidate has only given one example of each type. This is not sufficient to get any marks.

Question 2 (iii)

Mo eats exactly 3 identical jam doughnuts and exactly 3 identical iced doughnuts over the 4 days.

The number of jam doughnuts eaten on the four days is recorded as a list, for example 1, 0, 2, 0. The number of iced doughnuts eaten is not recorded.

(iii) Show that 20 different such lists are possible. **[3]**

Often candidates tried to record all the possibilities, many of these were successful but inevitably sometimes some were omitted or repeated.

Some candidates gave calculations that led to the given value 20 but with no justification for why these were a valid way to count the possibilities.

Exemplar 3

2(iii)

4 days

$\frac{4!}{2} = 12.$

2 → Because there are 2 zeros for list 1, 0, 2, 0

A different list could be 0, 0, 3, 0.

$\frac{4!}{3!} = 4.$

Another list can be 1, 1, 1, 0

$= 4.$

$12 + 4 + 4 = 20.$

This candidate has considered the three types of list and then counted how many permutations there are for each.

Question 3 (i)

3 In the pay-off matrix below, the entry in each cell is of the form (r, c), where r is the pay-off for the player on rows and c is the pay-off for the player on columns when they play that cell.

	P	Q	R
X	(1, 4)	(5, 3)	(2, 6)
Y	(5, 2)	(1, 3)	(0, 1)
Z	(4, 3)	(3, 1)	(2, 1)

(i) Show that the play-safe strategy for the player on columns is P. **[2]**

There is no need to convert the table in any way. The expectation was that candidates would consider the worst outcome for each column and hence show that column P has the largest value, ie the best worst outcome for P. Some candidates tried to make a zero-sum game, for example by subtracting the scores in each cell, or assumed that the pay-offs for the player on columns were the negatives of either r or c (for example finding the column minimax).

Those who used the pay-offs for the player on columns were usually successful.

Exemplar 4

3(i)

	P	Q	R
X	(1, 4)	(5, 3)	(2, 6)
Y	(5, 2)	(1, 3)	(0, 1)
Z	(4, 3)	(3, 1)	(2, 1)
worst outcome (lowest pay-off)	2	1	1

2 is highest - lowest pay-off (Maximin)
therefore P is the columns play safe strategy

This candidate has shown the 'worst outcome' in each column and explained that column P is the maximin.

Question 3 (ii)

(ii) Demonstrate that the game is not stable. **[2]**

Several candidates assumed that the game was zero-sum and discussed the values of what they thought were the play-safe strategies.

The play-safe for rows was strategy Z, where, for the player on rows, the row minima (1, 0, 2) are maximised. However, cell (Y, P) gives the player on rows a better pay-off (of 5) than cell (Z, P).

Question 3 (iii)

The pay-off for the cell in row Y, column P is changed from (5, 2) to (y, p), where y and p are real numbers.

(iii) What is the largest set of values A, so that if $y \in A$ then row Y is dominated by another row? **[1]**

The set *A* was presented in various ways. Some candidates assumed that the value of *y* had to be non-negative and some assumed that the value of *y* needed to be an integer. Any reasonable form was allowed.

Because of strong/weak dominance examiners accepted both $A = \{y: y < 4\}$ and $A = \{y: y \leq 4\}$.

Question 3 (iv)

(iv) Explain why column P can never be redundant because of dominance. **[1]**

Some candidates showed that P dominates one of Q or R but did not show that P was not itself dominated by the other. Many candidates were confused about whether they should be using the pay-offs for the player on rows or the player on columns. Some gave apparently correct comparisons but did not explain which row and columns they were comparing.

Some candidates said that P cannot be redundant because it is the play-safe strategy, but the value of *p* is no longer necessarily 2.

If the player on rows plays strategy Z, the player on columns does better by playing P than by playing either Q or R, since 3 > 1, so P cannot be dominated by either Q or R, and hence is not redundant.

Question 4 (i)

4 The complete bipartite graph $K_{3,\,4}$ connects the vertices $\{2, 4, 6\}$ to the vertices $\{1, 3, 5, 7\}$.

(i) How many arcs does the graph $K_{3,\,4}$ have? **[1]**

This was just a test of specification item 7.02e, knowing that $K_{m,n}$ has *mn* arcs. The answer was 12.

Question 4 (ii)

(ii) Deduce how many different paths are there that pass through each of the vertices once and once only. The direction of travel of the path does not matter. **[3]**

Many candidates realised that the path needed to 'zig-zag' between the two sets, starting at one of the four vertices {1, 3, 5, 7}. Quite a few candidates achieved the value 144 (or 36 if the initial choice of a value from four was overlooked). The direction of travel does not matter so each path has been counted twice and the total number of paths is 72.

Question 4 (iii) (a)

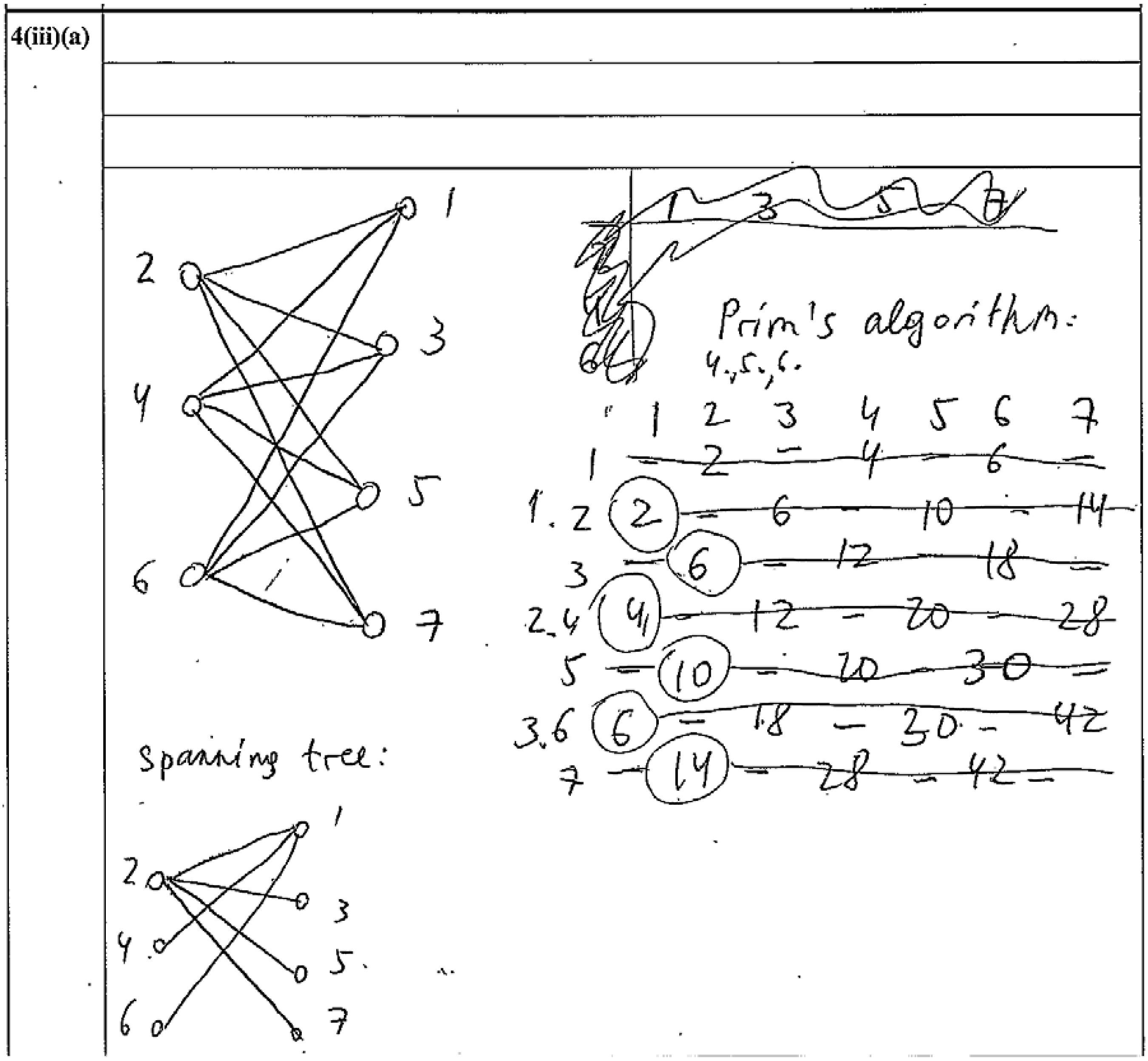
The arcs are weighted with the product of the numbers at the vertices that they join.

(iii) (a) Use an appropriate algorithm to find a minimum spanning tree for this network. **[3]**

Some candidates used Kruskal's algorithm and some used Prim's algorithm (often in tabular/matrix form). Candidates needed to at least state the algorithm being used or show sufficient working to make the method obvious.

A few candidates relabelled the vertices as A, B, C, … which was accepted provided a key was provided. Some candidates forgot that the arcs always connect an odd number to an even number.

Exemplar 5



This candidate has shown the arc weights in the table, has used Prim's algorithm and has also drawn the minimum spanning tree.

Question 4 (iii) (b)

(b) Give the weight of the minimum spanning tree. **[1]**

The MST had weight 42.

Question 5 (i)

5 Greetings cards are sold in luxury, standard and economy packs.

The table shows the cost of each pack and number of cards of each kind in the pack.

Pack	Cost (£)	Handmade cards	Cards with flowers	Cards with animals	Other cards	Total number of cards
Luxury	6.50	10	5	5	0	20
Standard	5.00	5	10	5	10	30
Economy	4.00	0	10	10	20	40

Alice needs 25 cards, of which at least 8 must be handmade cards, at least 8 must be cards with flowers and at least 4 must be cards with animals.

(i) Explain why Alice will need to buy at least two packs of cards. **[2]**

Candidates needed to show how just one pack of each of three different types did not fulfil the requirements. One pack of luxury cards only has 5 cards with flowers when 8 are needed, or only has 20 cards in total when 25 are needed. One pack of standard cards or one pack of economy cards do not have 8 handmade cards.

Some candidates just repeated the wording from the question and quite a few misread the number of cards with animals as 8 instead of 4.

Question 5 (ii) (a)

Alice does not want to spend more than £12 on the cards.

(ii) (a) List the combinations of packs that satisfy all Alice's requirements. **[2]**

Many correct solutions.

Some with one combination missing or an extra, incorrect, combination (such as EEE).

Question 5 (ii) (b)

(b) Which of these is the cheapest? **[1]**

Candidates usually identified SS as the cheapest, although some claimed LE.

Question 5 (iii)

Ben offers to buy any cards that Alice buys but does not need. He will pay 12 pence for each handmade card and 5 pence for any other card.

Alice does not want her net expenditure (the amount she spends minus the amount that Ben pays her) on the cards to be more than £12.

(iii) Show that Alice could now buy two luxury packs. **[2]**

Most candidates realised that Alice could sell (up to) 12 handmade cards to Ben. Some candidates showed that if she did this she would end up with a net expenditure of £11.56. A few candidates realised that Alice only needed to sell Ben 9 handmade cards to end up with a net expenditure that is less than £12.00.

Many candidates tried to find the minimum net expenditure for Alice. This was £11.41 by selling Ben 12 handmade cards and 3 cards with flowers or animals.

However, many candidates forgot that Alice needed 25 cards so that the maximum that Ben could buy was 15 cards; this usually led to an incorrect answer where Ben bought 12 handmade cards and 8 cards with flowers or animals.

Exemplar 6

5(iii)	Handmade = 20 (~~[illegible]~~ 12+12) (12 left)
	flowers = 10 (2 left)
	animals = 10 (6 left)
	12p × 12 = £1.44
M1	5p × (6+2) = 40p
	£1.44 + 0.40 = 1.88 (6.50 × 2) − 1.88 = £11.12 > £12

In this solution Ben buys back 12 handmade cards, 2 cards with flowers and 6 cards with animals.

Alice ends up with only 20 cards.

Exemplar 7

5(iii)	2 2 packs = £13.~~50~~00 money Ben pays 12 × 12p = £1.44
	cards she doesn't need: ~~money~~ 2 × 5p = £0.10
	H : 12 40 − 12 1 × 5p = £0.05
	F : ~~3~~2 = 28 = 26 £13.~~50~~00 − £1.59 £1.59
	A : ~~2~~1 < 25 = £11.41 so she can afford

In this solution Ben buys back 12 handmade cards, 2 cards with flowers and 1 card with animals.

£11.41 is the minimum net expenditure for Alice.

Exemplar 8

5(iii)	
	2 luxury = 20 handmade, 10 flowers & 10 animals.
	20 − 8 = 12 10 − 10 = 0 10 − 7 = 3
	12 × 0.12 = 1.44 0.15 + 1.44 = 1.59
	3 × 0.05 = 0.15 (2 × 6.50) − 1.59 = 11.41 < 12

This candidate has used all three types of packs correctly and has simplified the expressions obtained.

Question 5 (iv)

Alice decides to buy exactly 2 packs, of which x are luxury packs, y are standard packs and the rest are economy packs.

(iv) Give an expression, in terms of x and y only, for the number of cards of each type that Alice buys. **[4]**

Candidates usually had the correct expression for the number of handmade cards, but often forgot to include the economy packs when calculating the other numbers of cards. The total number of packs is 2 so the number of economy packs is (2 – *x* – *y*).

Question 5 (v)

Alice wants to minimise her net expenditure.

(v) Find, and simplify, an expression for Alice's minimum net expenditure in pence, in terms of x and y. You may assume that Alice buys enough cards to satisfy her own requirements. **[3]**

Most candidates were able to find an expression for the cost of buying the packs and subtract an expression for the amount that Ben pays. To achieve the method marks.

However, the economy packs were often left out and, even when all three types of pack were used, candidates often had Ben buying 20 cards instead of 15.

Question 5 (vi)

(vi) Find Alice's minimum net expenditure. **[2]**

Some candidates made a fresh start and calculated Alice's net expenditure to buy SS, others used their expression from part (v) to calculate the cost of a valid combination. Quite a few set $x = 0$ and $y = 0$ to minimise the cost, but this corresponded to EE which has no handmade cards.

Question 6 (i)

6 Sheona and Tim are making a short film. The activities involved, their durations and immediate predecessors are given in the table below.

	Activity	Duration (days)	Immediate predecessors	S	T
A	Planning	2	–	✓	✓
B	Write script	1	A	✓	
C	Choose locations	1	A		✓
D	Casting	0.5	A	✓	
E	Rehearsals	2	B, D	✓	
F	Get permissions	1	C		✓
G	First day filming	1	E, F	✓	
H	First day edits	1	G		✓
I	Second day filming	0.5	G	✓	
J	Second day edits	2	H, I		✓
K	Finishing	1	J	✓	✓

(i) By using an activity network, find:

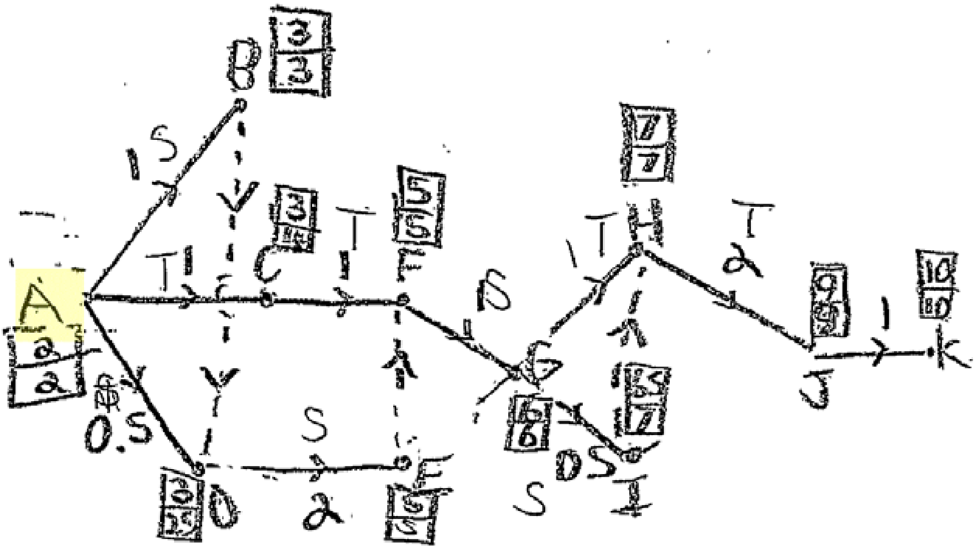
- the minimum project completion time
- the critical activities
- the float on each non-critical activity. **[7]**

The specification requires activity-on-arc. Candidates coped well with this, even though for some centres this will have been a new topic. In this particular case, examiners were tolerant of missing directions and missing or extra dummy activities, but this will not necessarily be the case on future papers.

Exemplar 9

6(i)

B0



M1 A1

Minimum project completion time = 10 mins

M1 A1

Critical activities: A, B, E, G, H, J, K

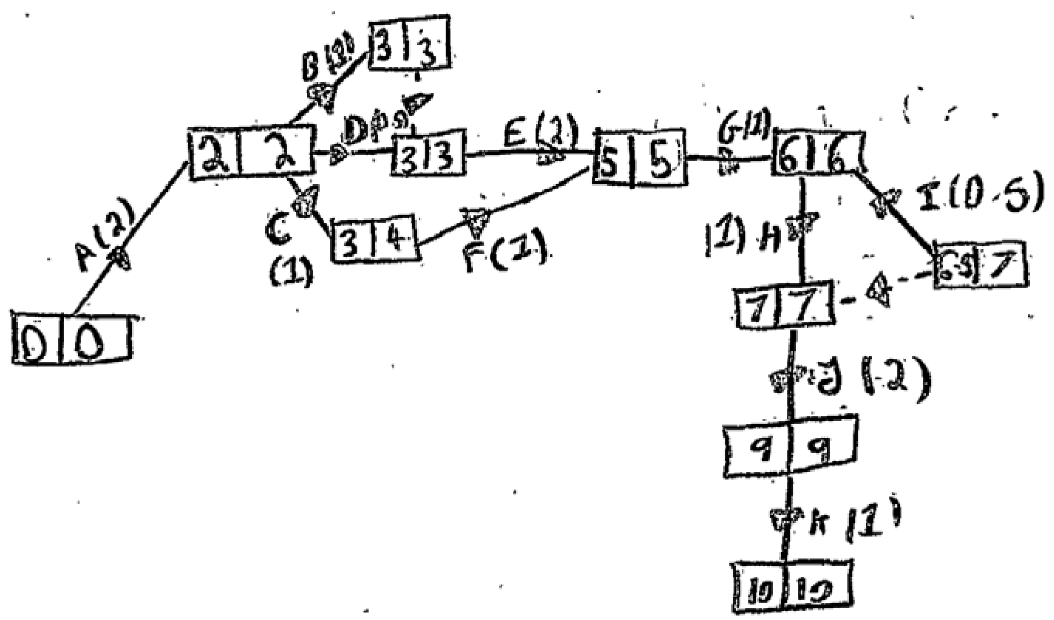
M1 A0

Activity	D	F	E	[illegible]	[illegible]		
Float (days)	0.5	1	0.5				

This candidate has used activity on node so gets 0 for the activity network, however the forward pass leads to the correct minimum project completion time (although the units are claimed as mins instead of days) and the backward pass leads to the correct critical activities. Activity C is missing from the list of floats.

Exemplar 10

6(i)



Minimum project completion time = 10

Critical activities: A B E G H J K

Activity	C	D	F	I			
Float (days)	1	0.5	1	0.5			

This is a fully correct solution.

Question 6 (ii)

(ii) Give two reasons why the filming may take longer than the minimum project completion time. **[2]**

Most candidates were able to give either a reason why the durations may be extended or a resourcing issue that would affect the order in which the activities can be carried out. Some candidates were able to give one of each.

Question 6 (iii)

Each activity will involve either Sheona or Tim or both.

- The activities that Sheona will do are ticked in the S column.
- The activities that Tim will do are ticked in the T column.
- They will do the planning and finishing together.
- Some of the activities involve other people as well.

An additional restriction is that Sheona and Tim can each only do one activity at a time.

(iii) Explain why the minimum project completion is longer than in part **(i)** when this additional restriction is taken into account. **[2]**

Usually candidates realised that two people cannot do activities B, C and D at the same time, or more specifically that Sheona cannot do B and D at the same time. This will delay the start of activity E, which is a critical activity, and hence delay the whole project.

Question 6 (iv) (a)

(iv) The project must be completed in 14 days. Find:

(a) the longest break that either Sheona or Tim can take, **[2]**

Part (iv) tested candidates' understanding of (total) float. Each person is busy for 8 of the 14 days. Sheona can rest for 6 days between I and K, Tim can rest for 6 days between F and H.

Question 6 (iv) (b)

(b) the longest break that Sheona and Tim can take together, **[2]**

Some candidates identified the longest break that Sheona and Tim can take together. The problem was sorting out when activities G, H and I should happen. Some candidates realised that the only 'clash' was between B and D so Sheona and Tim could complete the project in 10.5 days and then take a 3.5 day break.

Question 6 (iv) (c)

(c) the float on each activity. **[2]**

Most of the activities, taken in isolation, had 3.5 days of float. The activities that were previously non-critical had longer floats. The float on B and D depended on whether B came before or after D.

Whilst the application of scheduling and cascade charts would be a valid approach for the latter parts of question 6, this question does not require candidates to construct either a schedule or a cascade chart, which is beyond the AS content of the full A Level specification.

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